Lifshitz Space-Times for Schrødinger Holography

Quantum Field Theory, String Theory and Condensed Matter Physics

Crete, Kolymbari, September 5, 2014

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based on work with:

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1409.1519 & 1409.1522 & to appear

and

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1311.4794 (PRD) & 1311.6471 (JHEP)

Introduction

- holography as tool to study strong coupling physics

relevance for phenomenology:
holographic QCD, QG plasma, out-of-equilibrium dynamics, thermalization,
high T superconductors + other CM systems

done by considering deformations: e.g. temperature, chemical potential,

- surge of interest in applied holography
  - many new interesting AdS black hole solutions
  - construction of new types of holographic dualities involving non-asympt AdS e.g. Schroedinger, Lifshitz, hyperscaling violating

focus of this talk: Holography for Lifshitz spacetimes

Lifshitz symmetries

- Many systems in nature exhibit critical points with non-relativistic scale invariance

Includes in particular scale invariance with dynamical exponent z>1

$$t \to \lambda^z t \,, \qquad \vec{x} \to \lambda \vec{x}$$

Such systems typically have Lifshitz symmetries:

$$\begin{aligned} D_z &: \quad \vec{x} \to \lambda \vec{x} \qquad t \to \lambda^z t \,, \\ H &: \quad t \to t + a \,, \\ P_i &: \quad x^i \to x^i + a^i \,, \\ J_{ij} &: \quad x^i \to R^i{}_j x^j \,. \end{aligned}$$

Lifshitz algebra (non-zero commutators, not involving rotations)

$$[D_z, H] = -zH \,, \qquad [D_z, P_i] = -P_i \,.$$

Schroedinger symmetries

example of symmetry group that also cisplays non-relativistic scaling and contains Lifshitz is Schroedinger group

additional symmetries:
Galilean boosts $G_i \; (x^i \to x^i + v^i t)$
particle number symmetry M

Schroedinger algebra

$$
\begin{aligned}
[D_z, H] &= -zH\,, & [D_z, P_i] &= -P_i\,, & [D_z, M] &= (z-2)M \\
[D_z, G_i] &= (z-1)G_i\,, & [H, G_i] &= P_i\,, & [P_i, G_j] &= M\delta_{ij}
\end{aligned}
$$

for z=2: additional special conformal generator K

Lifshitz spacetimes

Aim: construct holographic techniques for (strongly coupled) systems with NR symmetries

Lifshitz holography $$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

[Kachru,Liu,Mulligan]
[Taylor]

> Lifshitz (or hyperscaling violating/Bianchi spaces) geometries have appeared as (IR) groundstate geometry of CM type systems

Note: here one can either have AdS or Lifshitz UV completion (possibly with hyperscaling violation) depending on one's interest

- IR geometries (near-horizon) of asympt. AdS backgrounds often involve Lifshitz scaling
  -> the IR geometry has its own holographic duality

Motivation and Goal

Why Lifshitz holography

•

Taylor/Danielson,Thorlacius/Ross,Saremi/Ross/
Baggio,de Boer,Holsheimer/Mann,McNees/
Griffin,Horava,Melby-Thompson/
Korovin,Skenderis,Taylor/
Cheng,Hartnoll,Keeler/Baggio/Holsheimer/
Christensen,Hartong,NO,Rollier
Chemissany, Papadimitriou

- how general is the holographic paradigm ?
  (nature of quantum gravity, black hole physics)
- extending to spacetimes that go beyond AdS
- applications of holography to strongly coupled condensed matter systems
  (non relativistic scaling necessitates spacetimes with different asymptotics)

Goal

- Want to understand more precisely the holographic dictionary in such cases
  (boundary geometry, holographic renormalization, 1-point functions,
  stress-energy tensor, …)

Main results/punchline

- For Lifshitz spacetimes we expect a non-relativistic structure on the boundary: Newton-Cartan geometry (or some generalization thereof):

- fnd that bdr. geometry is torsional Newton-Cartan geometry (novel extension of NC)
- find how TNC couples to vevs of the field theory (stress tensor, mass current)

(see also [Jensen] this week)

can provide new insights/tools into cond-mat (strongly-correlated electron system, FQH)

recent activity of using NC/TNC

[Son][Gromov,Abanov][Geracie,Son,Wu,Wu]
[Brauner,Endlich,Monin,Penco][Geracie,Son]
yesterday: [Wu,Wu] using HL-gravity

- find that the dual field theory has Schroedinger symmetries !

Plan

- Newton-Cartan geometry
- asymptotically locally Lifshitz space-times
- two arguments why dual field theory is
  Schroedinger invariant

1. sources (torsional NC geometry), vevs and Ward identities

[Hartong,Kiritsis,NO]1
[Bergshoeff,Hartong,Rosseel]

2. bulk vs. boundary Killing symmetries

[Hartong,Kiritsis,NO]2

Mini-intro to Newton-Cartan geometry

GR is a diff invariant theory whose tangent space is Poincare invariant

Newtonian gravity is diff invariant theory whose tangent space is the Bargmann algebra (non-rel limit of Poincare)

Andringa,Bergshoeff,Panda,de Roo

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}\,, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}\,,$$
$$[G_a, H] = -P_a\,, \qquad [G_a, P_b] = -\delta_{ab}Z\,,$$

centrally extended Galilean algebra

here: only interested in geometrical framework; not in EOMs
boundary geometry in holographic setup is non-dynamical

From Poincare to GR

GR is a diff invariant theory whose tangent space is Poincare invariant

- make Poincare local (i.e. gauge the translations and rotations)

$$
\begin{aligned}
A_\mu &= P_a e_\mu^a + \frac{1}{2} M_{ab} \omega_\mu{}^{ab} \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu \,, A_\nu] = P_a R_{\mu\nu}{}^a(P) + \frac{1}{2} M_{ab} R_{\mu\nu}{}^{ab}(M) \\
\delta A_\mu &= \partial_\mu \Lambda + [A_\mu \,, \Lambda] \,, \qquad \Lambda = \xi^\mu A_\mu + \frac{1}{2} M_{ab} \lambda^{ab}
\end{aligned}
$$

GR (Lorentzian geometry) follows from curvature constraint

$$
R_{\mu\nu}{}^a(P) = 0 \begin{cases} \omega_\mu{}^{ab} = \text{spin connection: expr. in terms of } e_\mu^a \\ \delta A_\mu = \mathcal{L}_\xi A_\mu + \frac{1}{2} M_{ab} \partial_\mu \lambda^{ab} + \frac{1}{2} [A_\mu \,, M_{ab}] \lambda^{ab} \\ R_{\mu\nu}{}^{ab}(M) = \text{Riemann curvature 2-form} \\ \nabla_\mu \ \text{ defined via vielbein postulate} \end{cases}
$$

From Bargmann to NC

Andringa,Bergshoeff,Panda,de Roo

Newtonian gravity is a diff invariant theory whose tangent space is Bargmann (make Bargmann local)

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$R_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$R_{\mu\nu}{}^a(P)$
boosts	G^a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$R_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$R_{\mu\nu}{}^{ab}(J)$
central charge transf.	Z	m_μ	$\sigma(x^\nu)$	$R_{\mu\nu}(Z)$

curvature constraints $$R_{\mu\nu}(H) = R_{\mu\nu}{}^a(P) = R_{\mu\nu}(M) = 0.$$

leaves as independent fields: $\tau_\mu,\ e^a_\mu,\ m_\mu$

transforming as

$$\begin{aligned}
\delta\tau_\mu &= \mathcal{L}_\xi \tau_\mu \\
\delta e^a_\mu &= \mathcal{L}_\xi e^a_\mu + \lambda^a \tau_\mu + \lambda^a{}_b e^b_\mu \\
\delta m_\mu &= \mathcal{L}_\xi m_\mu + \partial_\mu \sigma + \lambda_a e^a_\mu
\end{aligned}$$

Lessons from z=2 holographic Lifshitz model

[Christensen,Hartong,NO,Rollier]

considered first a specific z=2 example (in 4D) that can
be obtained by Scherk-Schwarz dim. reduction (null on bdry)
froma 5D AlAdS solution

[Donos,Gauntlet][Cassani,Faedo][Chemissany,Hartong]

counterterms and reduction: [Papadimitriou][Chemissany,Geisbuehler,Hartong,Rollier]

important lessons:

- use of vielbeins highly advised (see also [Ross])
- identification of sources requires appropriate lin. combo of timelike vielbein and bulk gauge field (-> crucial for boundary gauge field)
- bdr. geometry is torsional Newton-Cartan
- can compute unique gauge and tangent space inv. bdry stres tensor
- WIs take TNC covariant form
- conserved quantities from WIs and TNC (conformal) Killing vectors

EPD model and AlLif spacetimes

bulk theory

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{4}Z(\Phi)F^2 - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}W(\Phi)B^2 - V(\Phi)\right)$$

- admits Lifshitz solutions with z>1

For AlLif BCs useful to write:

$$ds^2 = \frac{dr^2}{R(\Phi)r^2} - E^0E^0 + \delta_{ab}E^aE^b\,, \qquad B_M = A_M - \partial_M\Xi$$

then AlLif BCs

[Ross],[Christensen,Hartong,NO,Rollier]
[Hartong,Kiritsis,NO]1

$$\begin{array}{rclrcl} E^0_\mu & \propto & r^{-z}\tau_\mu + \dots & E^a_\mu & \propto & r^{-1}e^a_\mu + \dots \\ A_\mu - \alpha(\Phi)E^0_\mu & \propto & r^{z-2}\tilde{m}_\mu + \dots & A_r & = & (z-2)r^{z-3}\chi + \dots \\ \Xi & = & r^{z-2}\chi + \dots & \Phi & = & r^{\Delta}\phi + \dots \end{array}$$

Transformation of sources

[Bergshoeff,Hartong,Rosseel]
[Hartong,Kiritsis,NO]1

use local bulk symmetries:
local Lorentz, gauge transformations and diffs preserving metric gauge

these symmetries induce an action on sources: $\tau_\mu,\ e^a_\mu,\ \tilde{m}_\mu,\ \chi$

= action of Bargmann algebra plus local dilatations = Schroedinger

there is thus a Schroedinger Lie algebra valued connection given by

$$A_\mu = H\tau_\mu + P_a e^a_\mu + M m_\mu + \frac{1}{2} J_{ab} {\omega_\mu}^{ab} + G_a {\omega_\mu}^a + D b_\mu$$

with $\tilde{m}_\mu = m_\mu - (z-2)\chi b_\mu$

with appropriate curvature constrains that reproduces trafos of the sources

Torsional Newton-Cartan (TNC) geometry

the bdry geometry is novel extension of NC geometry

source	ϕ	τ_μ	e_μ^a	v^μ	e_a^μ	$\tilde{m}_0$	$\tilde{m}_a$	χ
scaling dimension	Δ	$-z$	-1	z	1	$2z-2$	$z-1$	$z-2$

includes inverse veilbeins (v^μ, e_a^μ)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e_\mu^a = 0\,, \qquad e_a^\mu \tau_\mu = 0\,, \qquad e_a^\mu e_\mu^b = \delta_a^b$$

from (inverse) vielbeins and vector: $M_\mu = \tilde{m}_\mu - \partial_\mu \chi$

can build Galilean boost-invariants

$$\begin{aligned} h^{\mu\nu} &= \delta^{ab} e_a^\mu e_b^\nu\,, & \hat{v}^\mu &= v^\mu - h^{\mu\nu} M_\nu \\ \bar{h}_{\mu\nu} &= \delta_{ab} e_\mu^a e_\nu^b - \tau_\mu M_\nu - \tau_\nu M_\mu\,, & \Phi_N &= -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu \end{aligned}$$

affine connection of TNC

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma} \left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu} \right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\frac{1}{2} \hat{v}^\rho (\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

Vevs, EM tensor and mass current

assuming holographic renormalizability

-> general form of variation of on-shell action

$$\delta S^{os}_{ren} = \int d^3x e \left[-S^0_\mu \delta v^\mu + S^a_\mu \delta e^\mu_a + T^0 \delta \tilde{m}_0 + T^a \delta \tilde{m}_a + \langle O_\chi \rangle \delta \chi + \langle \tilde{O}_\phi \rangle \delta \phi - \mathcal{A}_{(\prime)} \frac{\delta r}{r} \right]$$

local bulk symmetries induce transformation on vevs (cf. sources)

-> exhibit again Schroedinger symmetry

from vevs & sources:

- bdyr EM tensor
- - mass current

$$\begin{aligned} \mathcal{T}^\mu{}_\nu &= -\left(S^0_\nu + T^0 \partial_\nu \chi\right) v^\mu + \left(S^a_\nu + T^a \partial_\nu \chi\right) e^\mu_a \\ T^\mu &= -T^0 v^\mu + T^a e^\mu_a \end{aligned}$$

tangent space projections provide

energy density, energy flux, momentum density, stress, mass density, mass current

$\mathcal{T}_\mu{}^\nu \tau_\nu \hat{v}^\mu$	$\mathcal{T}_\mu{}^\nu \tau_\nu e^\mu_a$	$\mathcal{T}_\mu{}^\nu \hat{e}^a_\nu \hat{v}^\mu$	$\mathcal{T}_\mu{}^\nu \hat{e}^a_\nu e^\mu_b$	$T^\mu \tau_\mu$	$T^\mu \hat{e}^a_\mu$
$z+2$	3	$2z+1$	$z+2$	$4-z$	3

Covariant Ward identities

Ward identities: (ignore for simplicity dilaton scalar)

$$
\begin{aligned}
0 &= -\hat{e}^a_\mu T^\mu + \tau_\mu e^{\nu a} \mathcal{T}^\mu{}_\nu && \text{boosts} \\
0 &= \hat{e}^a_\mu e^{\nu b} \mathcal{T}^\mu{}_\nu - (a \leftrightarrow b) && \text{rotations} \\
\mathcal{A} &= -z\hat{v}^\nu \tau_\mu \mathcal{T}^\mu{}_\nu + \hat{e}^a_\mu e^\nu_a \mathcal{T}^\mu{}_\nu + 2(z-1)\Phi_N \tau_\mu T^\mu && \text{dilatations} \\
\langle O_\chi \rangle &= e^{-1}\partial_\mu \left(e T^\mu \right) && \text{gauge trafos} \\
0 &= \nabla_\mu \mathcal{T}^\mu{}_\nu + 2\Gamma^\rho_{[\mu\rho]} \mathcal{T}^\mu{}_\nu - 2\Gamma^\mu_{[\nu\rho]} \mathcal{T}^\rho{}_\mu && \\
&\quad -T^\mu \hat{e}^a_\mu \mathcal{D}_\nu M_a + \tau_\mu T^\mu \partial_\nu \Phi_N && \text{diffs}
\end{aligned}
$$

- uses Galilean boost invariant vielbeins and density e

- ∇_μ contains affine TNC connection

- $\mathcal{D}_\mu$ contains Bargmann boost and rotation connections

TNC Killing vectors and flat NC spacetime

Conserved currents $\partial_\mu \left(e K^\nu \mathcal{T}^\mu{}_\nu \right) = 0$ [Kiritsis,Hartong,NO]2

whenever K is a TNC Killing vector:

$$
\begin{array}{rclrclrcl}
\mathcal{L}_K \tau_\mu & = & -z\Omega\tau_\mu \, , & \mathcal{L}_K \hat{v}^\mu & = & z\Omega\hat{v}^\mu \, , & \mathcal{L}_K \bar{h}_{\mu\nu} & = & -2\Omega\bar{h}_{\mu\nu} \\
\mathcal{L}_K h^{\mu\nu} & = & 2\Omega h^{\mu\nu} \, , & \mathcal{L}_K \Phi_N & = & 2(z-1)\Omega\Phi_N \, , & \mathcal{A}\Omega & = & 0
\end{array}
$$

notion of flat NC space-time

$$
\begin{array}{rcll}
\tau_\mu & = & \delta^t_\mu & \text{fixing diffs} \\
h^{\mu\nu} & = & \delta^{ij}\delta^\mu_i\delta^\nu_j & \text{fixing diffs and flat space} \\
v^\mu & = & -\delta^\mu_t & \text{fixing boosts} \\
h_{\mu\nu} & = & \delta_{ij}\delta^i_\mu\delta^j_\nu & \text{forced by other choices} \\
M_\mu & = & \partial_\mu M & \text{global inertial coordinates: } \Gamma^\rho_{\mu\nu} = 0 \\
\Phi_N & = & 0 & \text{no Newton potential}
\end{array}
$$

Conformal Killing vectors of flat NC spactime

the conformal Killing vectors are:

$$
\begin{aligned}
K^t &= a - z\lambda t - \alpha t^z \\
K^i &= a^i + v^i t + \lambda^i{}_j x^j - \lambda x^i - \alpha t^{z-1} x^i \\
\Omega &= \lambda + \alpha t^{z-1}
\end{aligned}
$$

provided we can solve

$$
\begin{aligned}
\mathcal{L}_K M &= v^i x^i - \frac{1}{2}(z-1)\alpha t^{z-2} x^i x^i + (z-2)\Omega M \\
0 &= \partial_t M + \frac{1}{2}\partial_i M \partial^i M \quad \text{due to } \Phi_N = 0 \text{ and } M_\mu = \partial_\mu M
\end{aligned}
$$

Two solutions:

$$
\begin{cases}
M = \text{cst} \rightarrow \text{conf. KVs: } H,\ D,\ P_i,\ J_{ij} \\
M = \dfrac{x^i x^i}{2t} \rightarrow \text{conf. KVs: } K,\ D,\ G_i,\ J_{ij}
\end{cases}
$$

Schr has outer automorphsim $(H, P_i, D, J_{ij}) \leftrightarrow (-K, G_i, -D, J_{ij})$

Field theory on TNC backgrounds

- Action for Schroedinger equation on TNC background

$$S = \int d^{d+1}x e \left[-i\phi^* \hat{v}^\mu \partial_\mu \phi + i\phi \hat{v}^\mu \partial_\mu \phi^* - h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + \phi\phi^* \Phi_N - V(\phi\phi^*)\right]$$

on flat NC background this becomes:

$$\begin{aligned} S &= \int d^{d+1}x \left[i\phi^* \left(\partial_t \phi + i\phi \partial_t M\right) - i\phi \left(\partial_t \phi^* - i\phi^* \partial_t M\right) \right. \\ &\quad \left. -\delta^{ij} \left(\partial_i \phi + i\phi \partial_i M\right)\left(\partial_j \phi^* - i\phi^* \partial_j M\right) - V(\phi\phi^*)\right] \end{aligned}$$

wave function ψ defined as $\phi = e^{-iM}\psi$.

$M = \text{cst}$ spacetime symmetries = Lifshitz subalgebra of Sch given by H, D, P_i and J_{ij}

$M = \frac{x^i x^i}{2t}$ spacetime symmetries = Lifshitz subalgebra of Sch given by K, D, $\dot{G}$ and J_{ij}

Bulk perspective of two solutions

$M = \text{cst}$ Lifshitz algebra and dual bulk

$$H = \partial_t \,, \quad P_i = \partial_i \,, \quad D = zt\partial_t + x^i\partial_i + r\partial_r \,, \quad J_{ij} = x^i\partial_j - x^j\partial_i$$

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^i dx^i}{r^2}$$

$M = \frac{x^i x^i}{2t}$ Lifshitz algebra and dual bulk

$$K = t^z\partial_t + t^{z-1}(x^i\partial_i + r\partial_r) \,, \quad G_i = t\partial_i \,, \quad D = zt\partial_t + x^i\partial_i + r\partial_r \,, \quad J_{ij}$$

$$ds^2 = \left(-\frac{1}{r^{2z}} + \frac{1}{t^2}\right) dt^2 - \frac{2drdt}{rt} + \frac{dr^2}{r^2} + \frac{1}{r^2}\left(dx^i - \frac{x^i}{t}dt\right)^2$$

Large diff relating the two bulk solutions !

$$\bar{t} = \frac{1}{1-z} t^{1-z} \,, \qquad \bar{x}^i = \frac{x^i}{t} \,, \qquad \bar{r} = \frac{r}{t}$$

Next steps

- subleading terms in asymtotic expansion and counterterms

- comparison to linearized perturbations

- adding other exponents: (logarithmic running of scalar) alpha/zeta-deformation

$$A_a = r^{-z-\zeta}\alpha_{(0)}\tau_{(0)a}$$

[Kiritsis,Goutereaux][Gath,Hartong,Monteiro,NO]

- adding charge

- 3D bulk (Virasoro-Schroedinger)

- applications to hydrodynamics: Lifsthiz hdyro: [Hoyos,Kim,Oz]
  black branes with zero/non-zero particle number density ?

- Schroedinger holography

- HL gravity and Einstein-aether theories

Discussion and Outlook

defined sources for AlLif spacetimes and shown that they

- transform under local Schroedinger group
- describe torsional NC boundary geometry
- lead to Sch Ward identities for bdry stress tensor and mass current

TNC of growing interest in cond-mat (str-el, mes-hall) literature

developments in Lifshitz holography can drive development of tools to study dynamics and hydrodynamics of non-rel. systems

(in parallel to progress in the last many years in relativistic fluids and superfluids inspired from the fluid/gravity correspondence in AdS)

TNC right ingredients to start constructing effective TNC theories and their coupling to matter (e.g. QH-effect)

The end