Fractional quantum Hall effect In Graphene : from SU(4) to SO(5)

Th. Jolicoeur
CNRS and Orsay University

Coworkers : F.C. Wu, I. Sodemann, Y. Araki, A. H. McDonald (Austin TX)

See arXiv : 1406.2330

Chania OAC - 2014

Outline :

- elementary facts about 2DEGs
- striking facts from FQHE (GaAs)
- Graphene and Landau levels
- SU(4) approximate symmetry
- explicit breaking of symmetry down to SO(5)
- aspects of ferromagnetism at filling factor 2
- conclusions

WHY study 2DEGs ???????

The answer is :
HEMTs

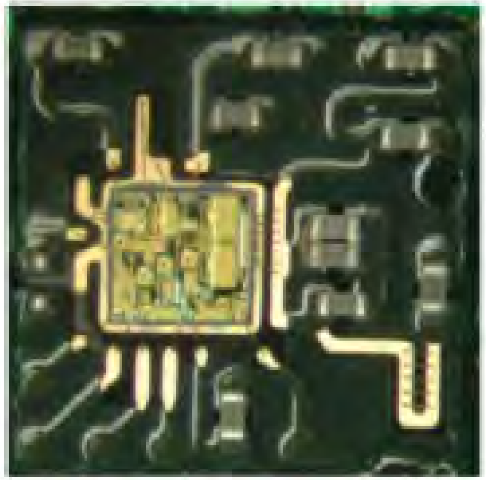
HEMT ICs

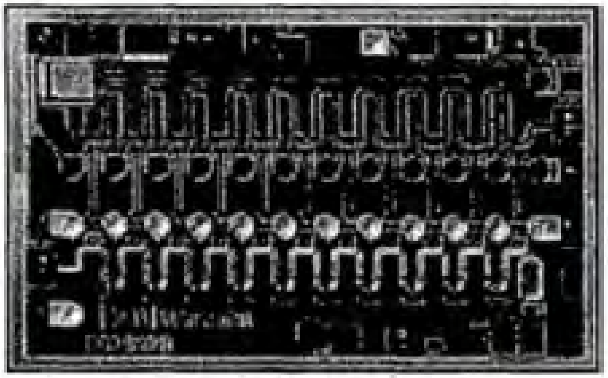
Saturday, February 26, 2011

TriQuint and Skyworks Power iPhone 5

UMTS-LTE PA module

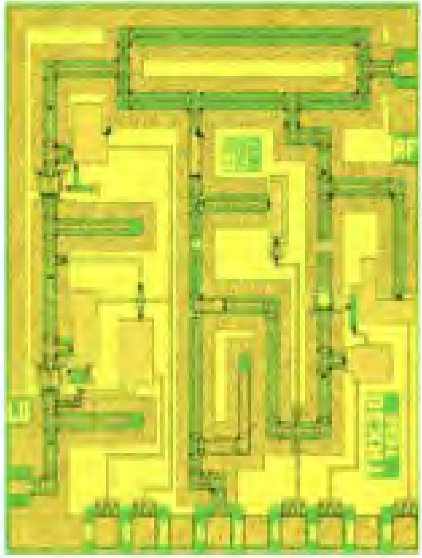
Chow, MTT-S 2008





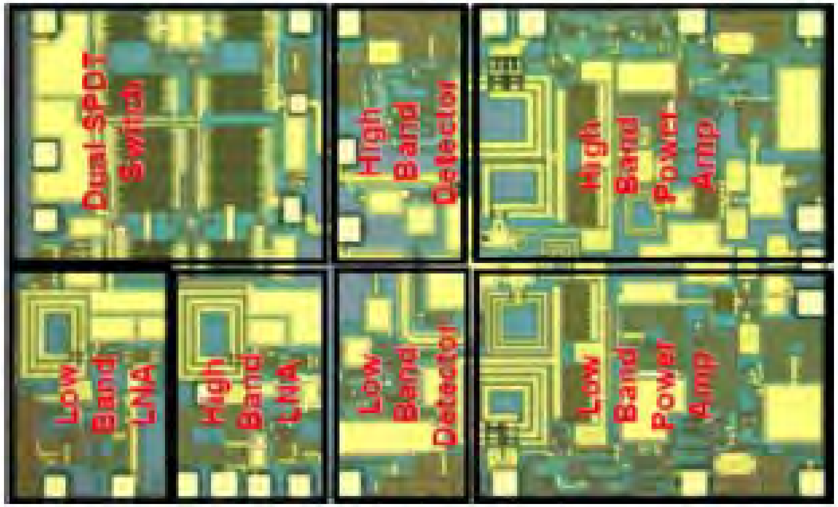
40 Gb/s modulator driver

Carroll, MTT-S 2002



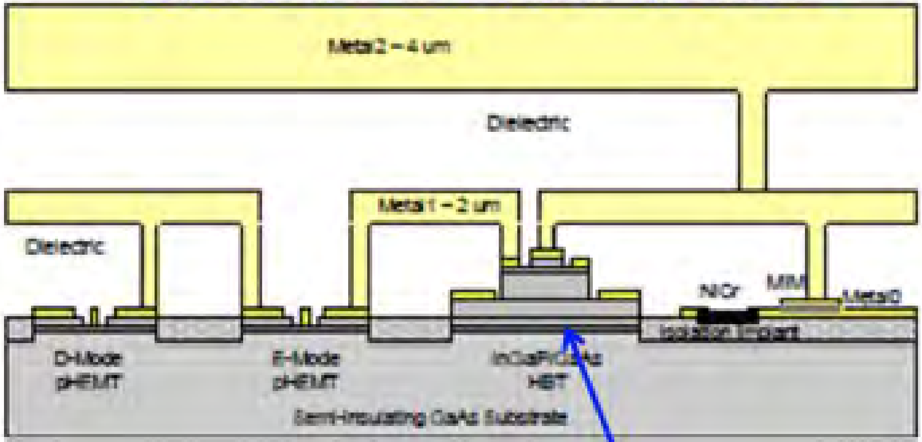
77 GHz transceiver

Tessmann, GaAs IC 1999



Single-chip WLAN MMIC, Morkner, RFIC 2007

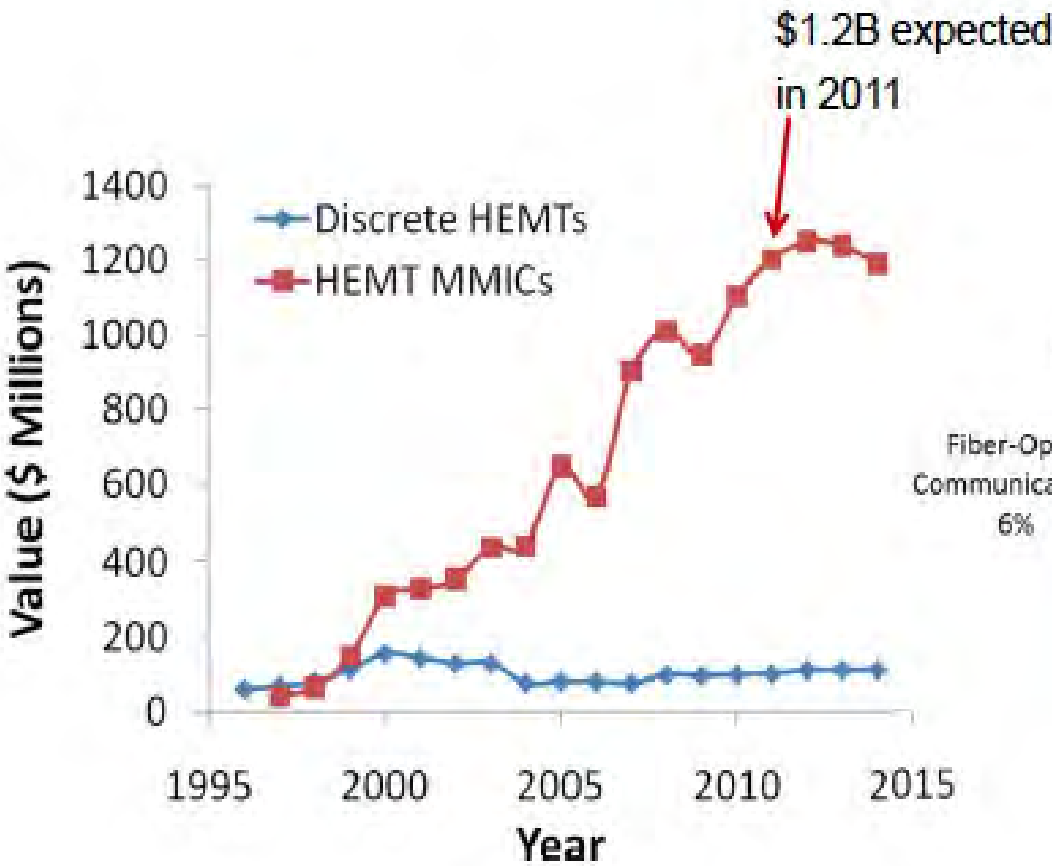
Bipolar/E-D PHEMT process

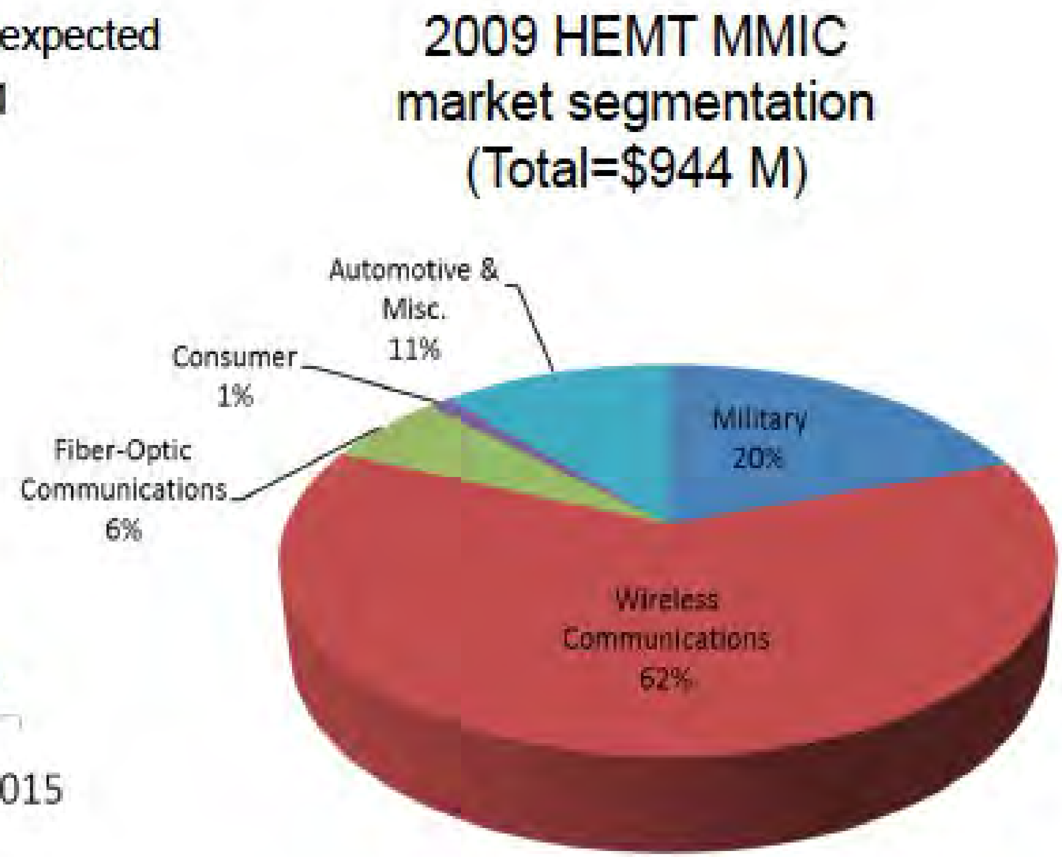


Henderson, Mantech 2007

Single MOCVD growth

HEMT markets



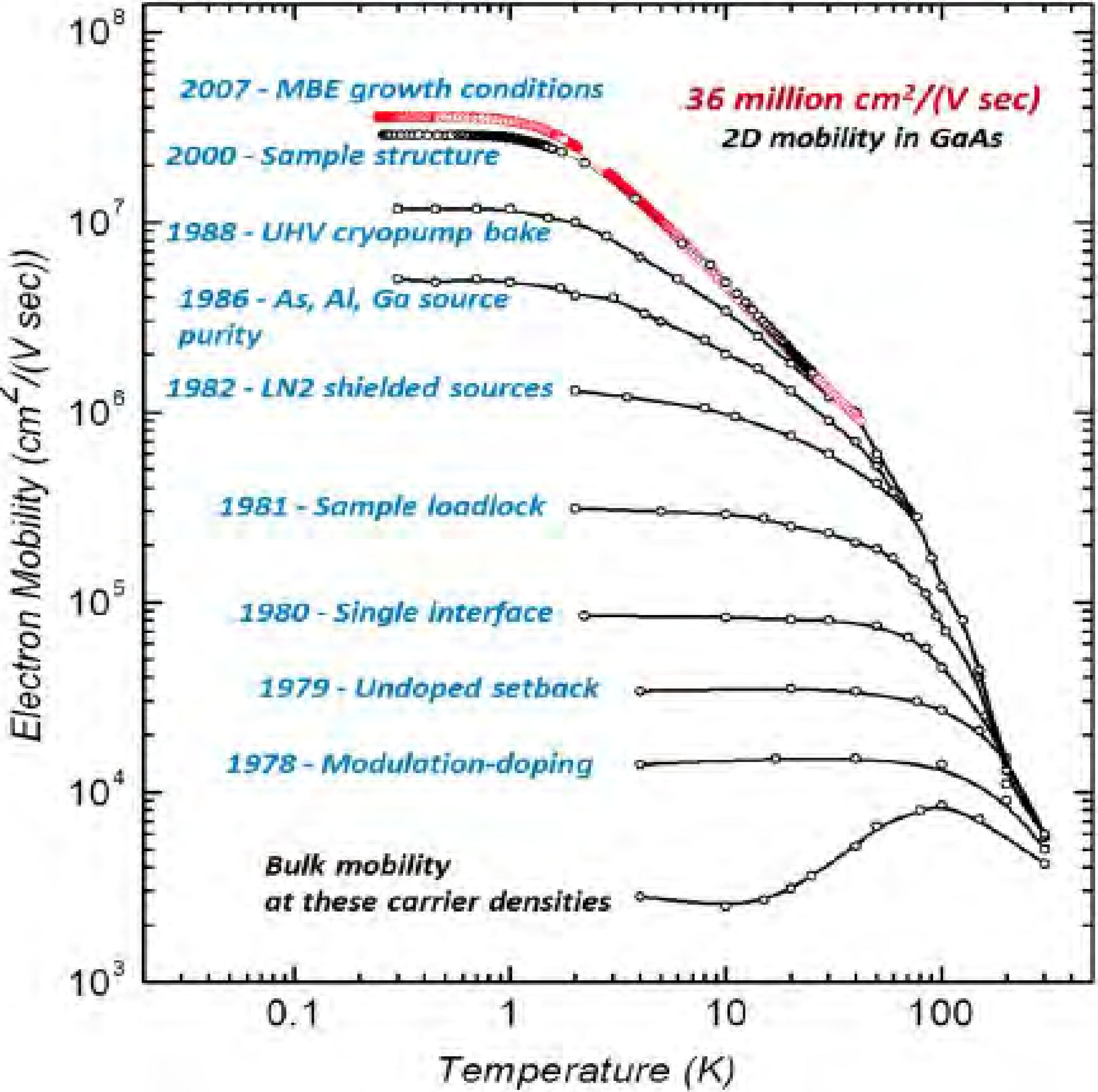


- Biggest market: wireless communications
- Biggest applications: cell phone handsets, WLAN, base stations and CATV

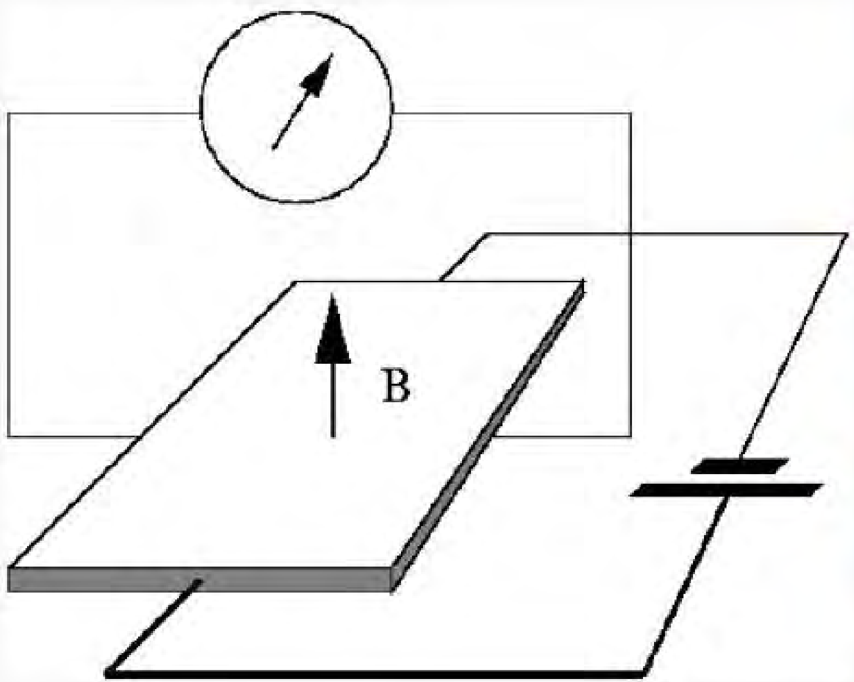
Better transistors require better *mobility* :

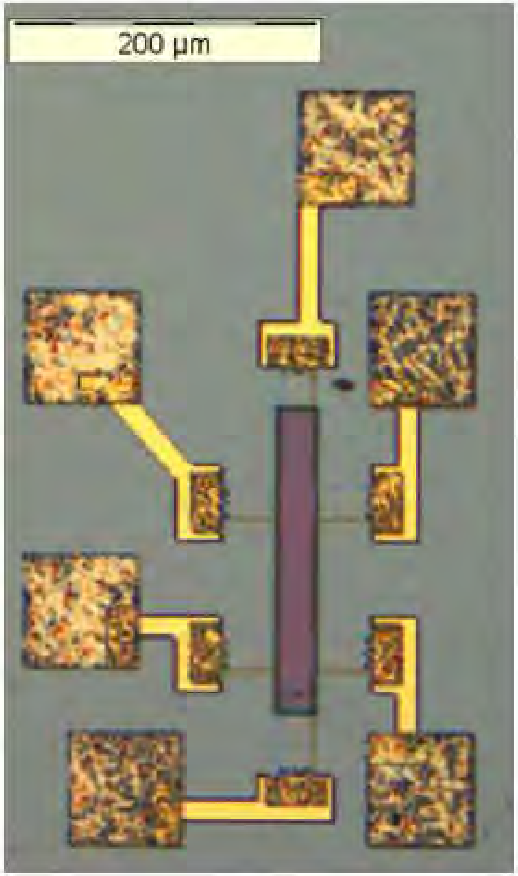
This is good for fundamental physics !!!!

$$v_d = \mu E$$

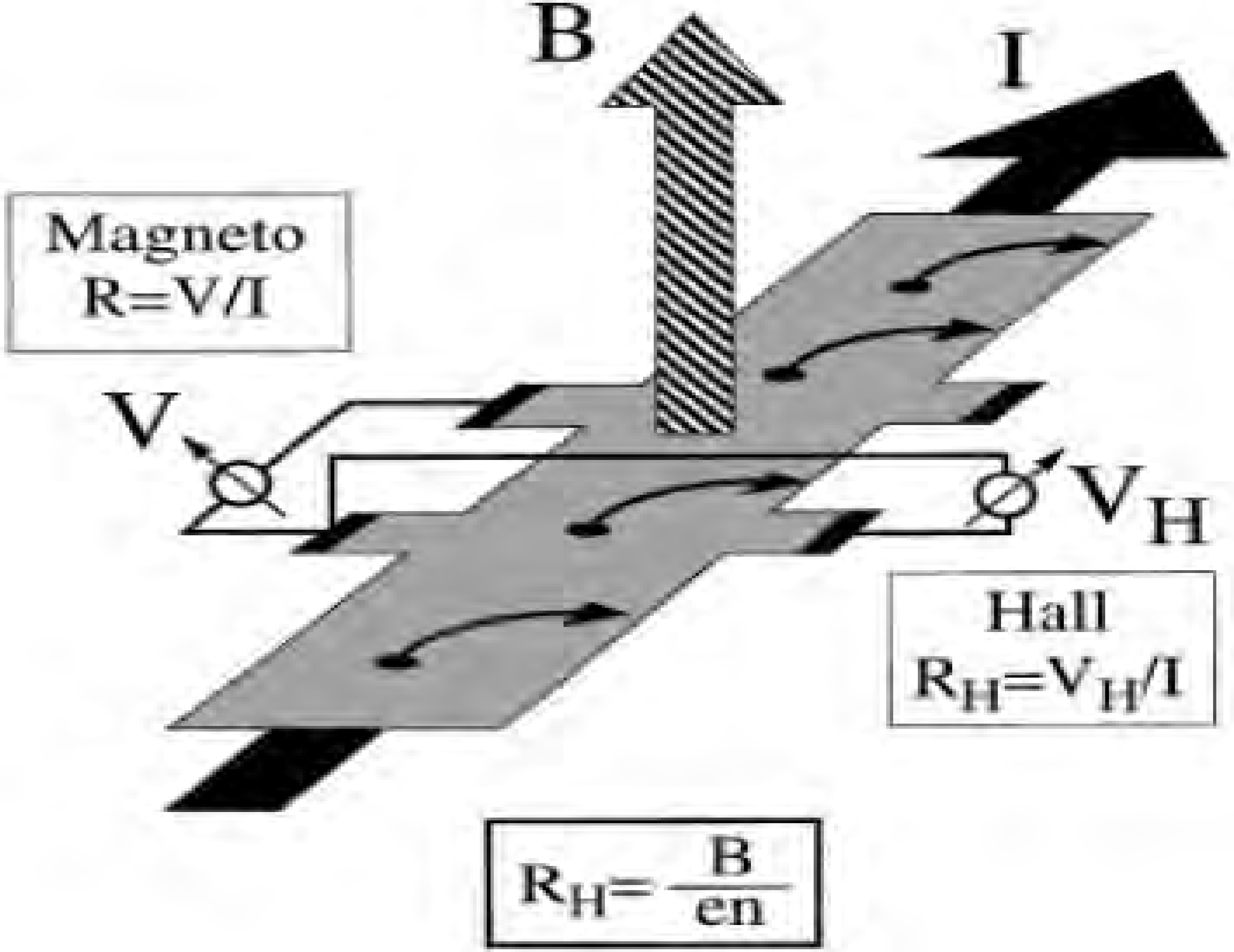


36 million cm²/(V sec)
2D mobility in GaAs
2007 - MBE growth conditions
2000 - Sample structure
1988 - UHV cryopump bake
1986 - As, Al, Ga source purity
1982 - LN2 shielded sources
1981 - Sample loadlock
1980 - Single interface
1979 - Undoped setback
1978 - Modulation-doping
Bulk mobility at these carrier densities
Electron Mobility (cm²/(V sec))
Temperature (K)
10⁸
10⁷
10⁶
10⁵
10⁴
10³
0.1
1
10
100

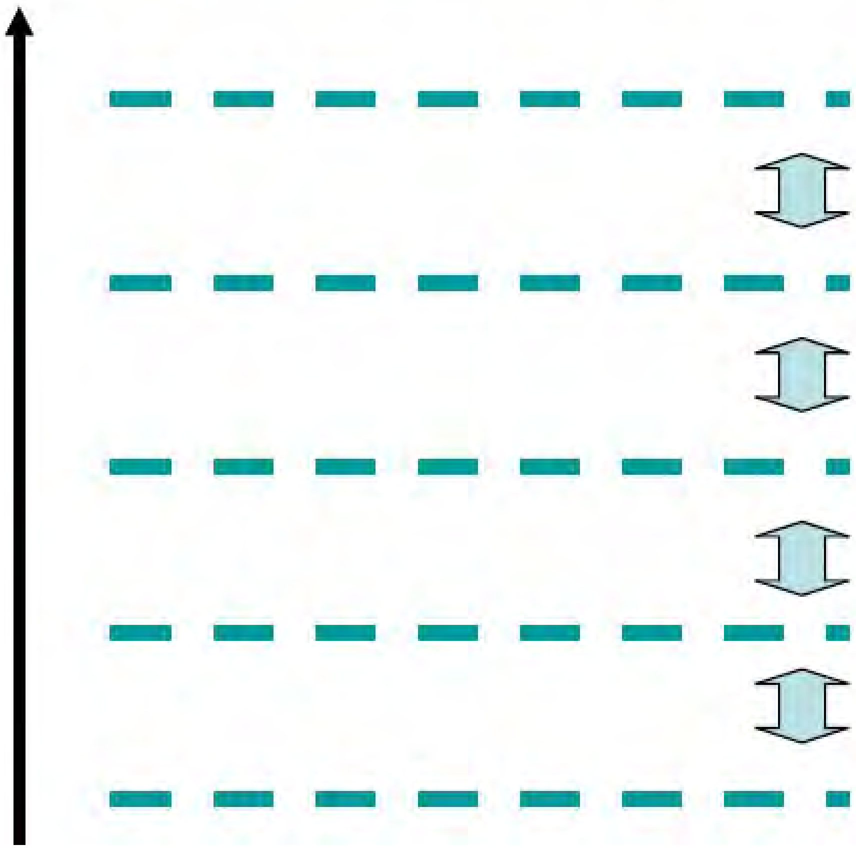




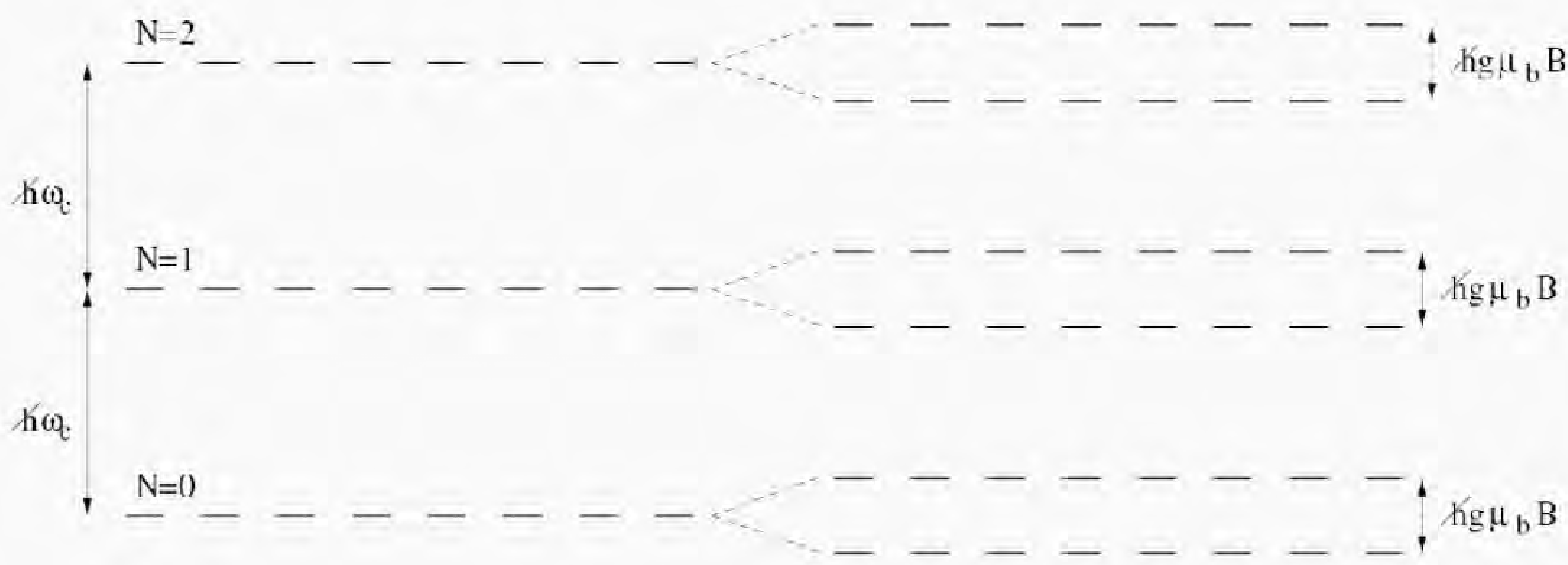
$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \qquad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$


B
I
Magneto
R=V/I
V
V_H
Hall
R_H=V_H/I
R_H= B/en

$$\nu = \frac{nh}{qB}$$

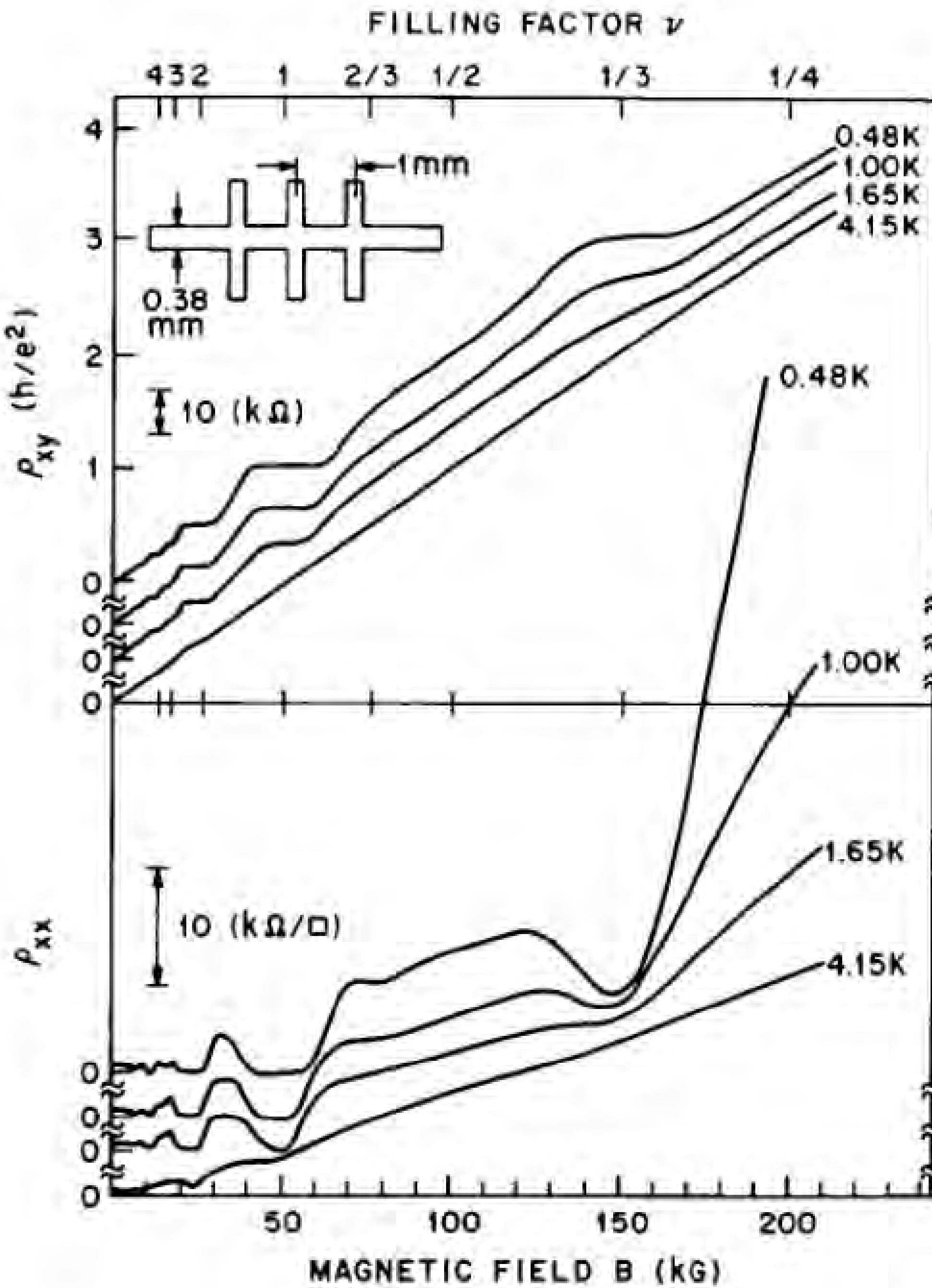


$$\hbar\omega_c = \frac{eB\hbar}{m}$$

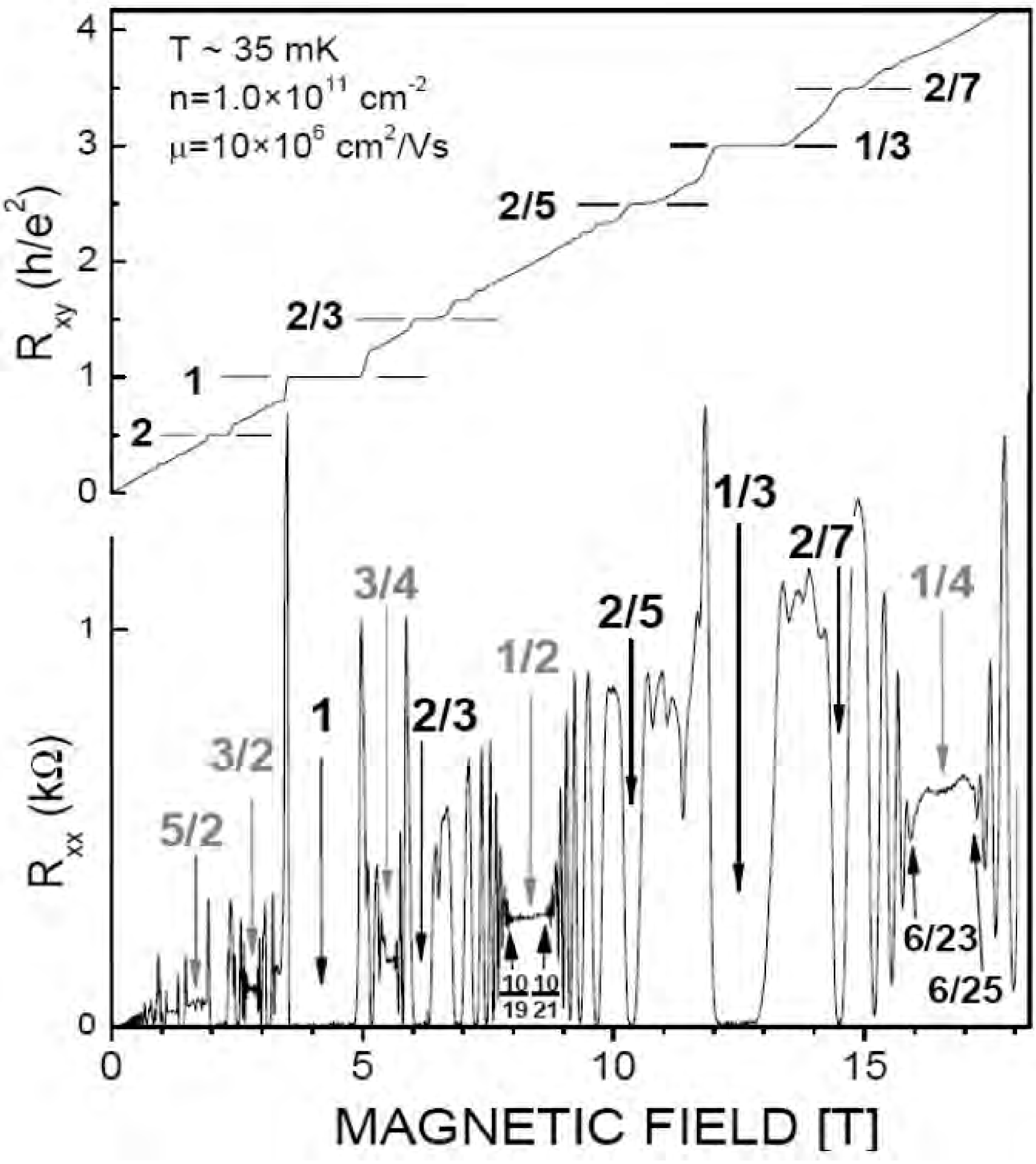


N=2
N=1
N=0
ħω_c
ħω_c
ħgμ_b B
ħgμ_b B
ħgμ_b B

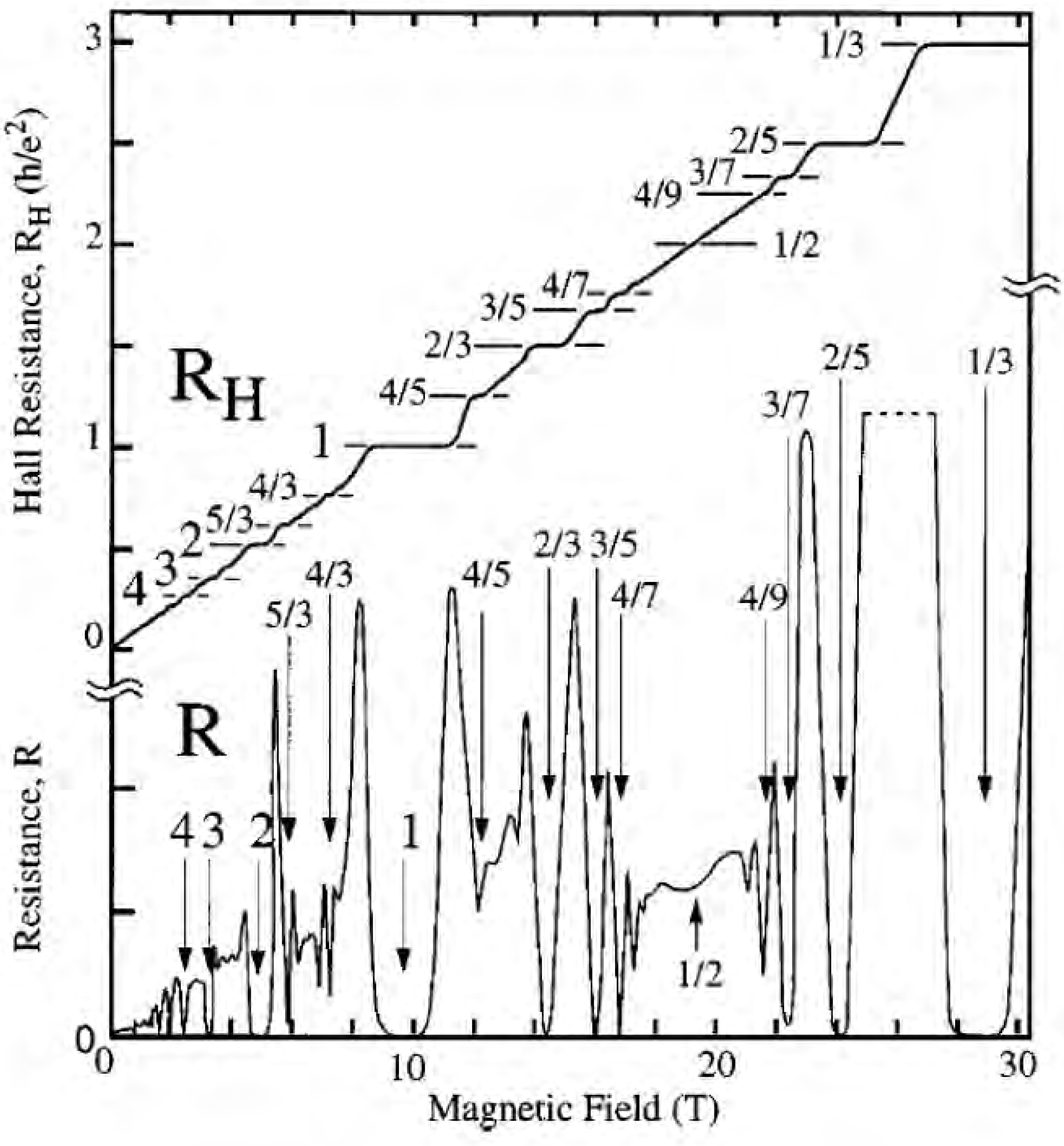
- The 2D situation under a field has no kinetic energy : instead highly degenerate Landau levels.
- Only interactions fix the nature of the ground state.
- For many rational fillings $\nu = p/q$ of the lowest Landau level, the ground state is a liquid with *gapped* excitations.
- Quasiholes and quasielectrons with fractional charge and statistics.



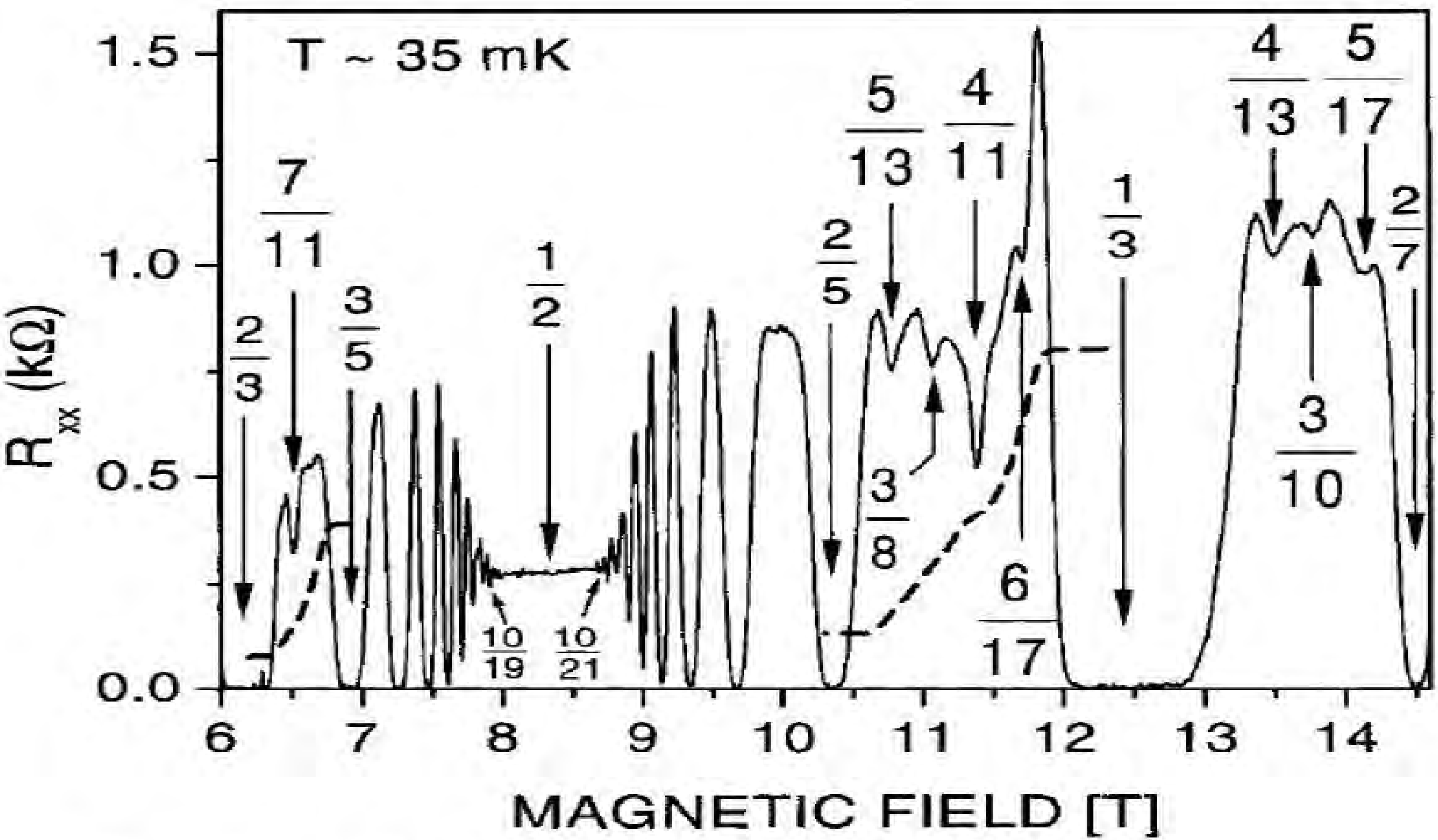
FILLING FACTOR ν
4 3 2 1 2/3 1/2 1/3 1/4
1mm
0.38 mm
10 (kΩ)
ρ_{xy} (h/e²)
0.48K
1.00K
1.65K
4.15K
10 (kΩ/□)
ρ_{xx}
0 50 100 150 200
MAGNETIC FIELD B (kG)



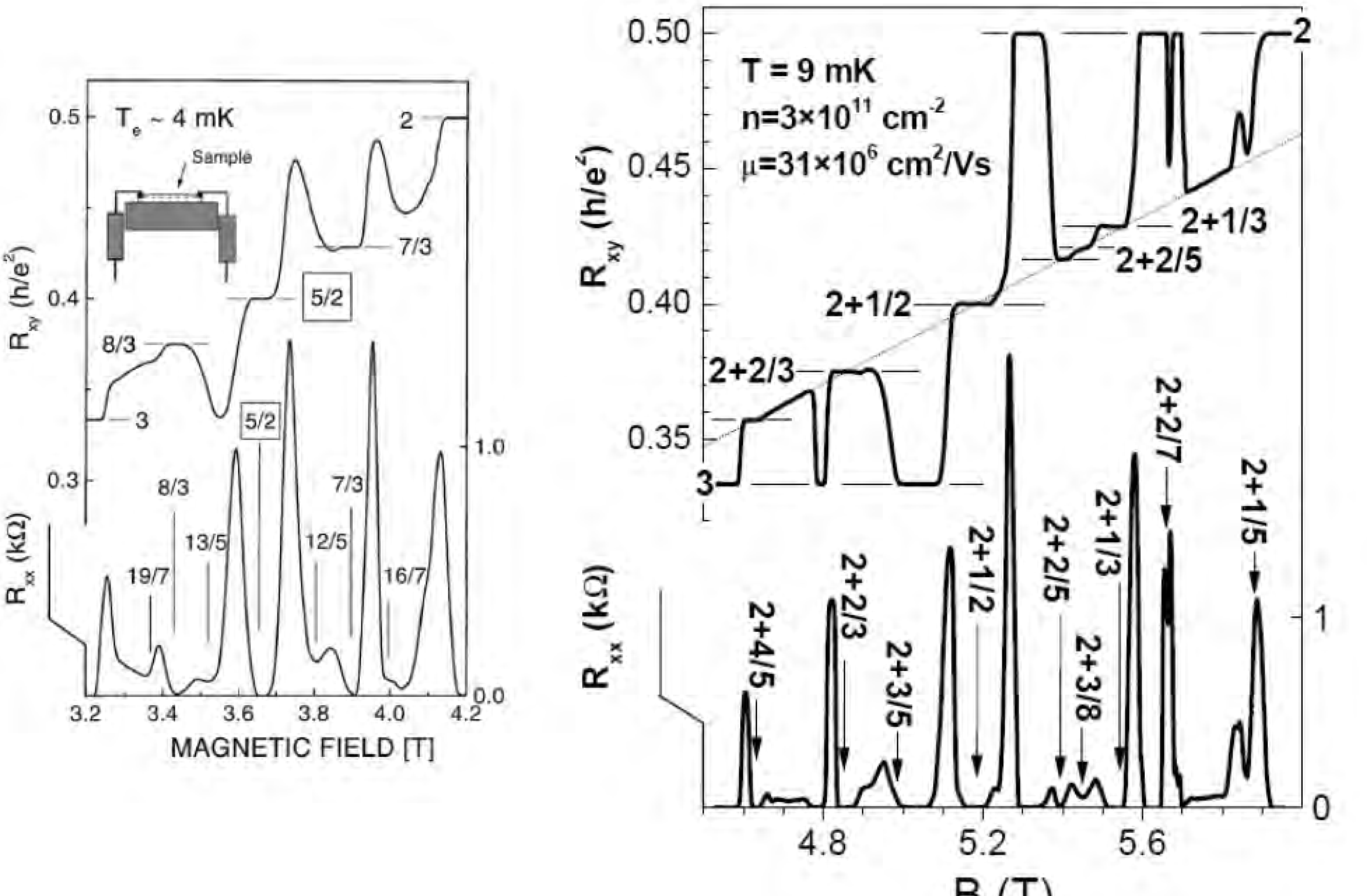
T ~ 35 mK
n=1.0×10^11 cm^-2
μ=10×10^6 cm^2/Vs
R_xy (h/e^2)
R_xx (kΩ)
MAGNETIC FIELD [T]
2
1
2/3
2/5
1/3
2/7
5/2
3/2
3/4
1/2
1/4
10/19
10/21
6/23
6/25



Hall Resistance, R_H (h/e^2)
Resistance, R
Magnetic Field (T)
R_H
R
1/3
2/5
3/7
4/9
1/2
4/7
3/5
2/3
4/5
1
4/3
5/3
2
3
4

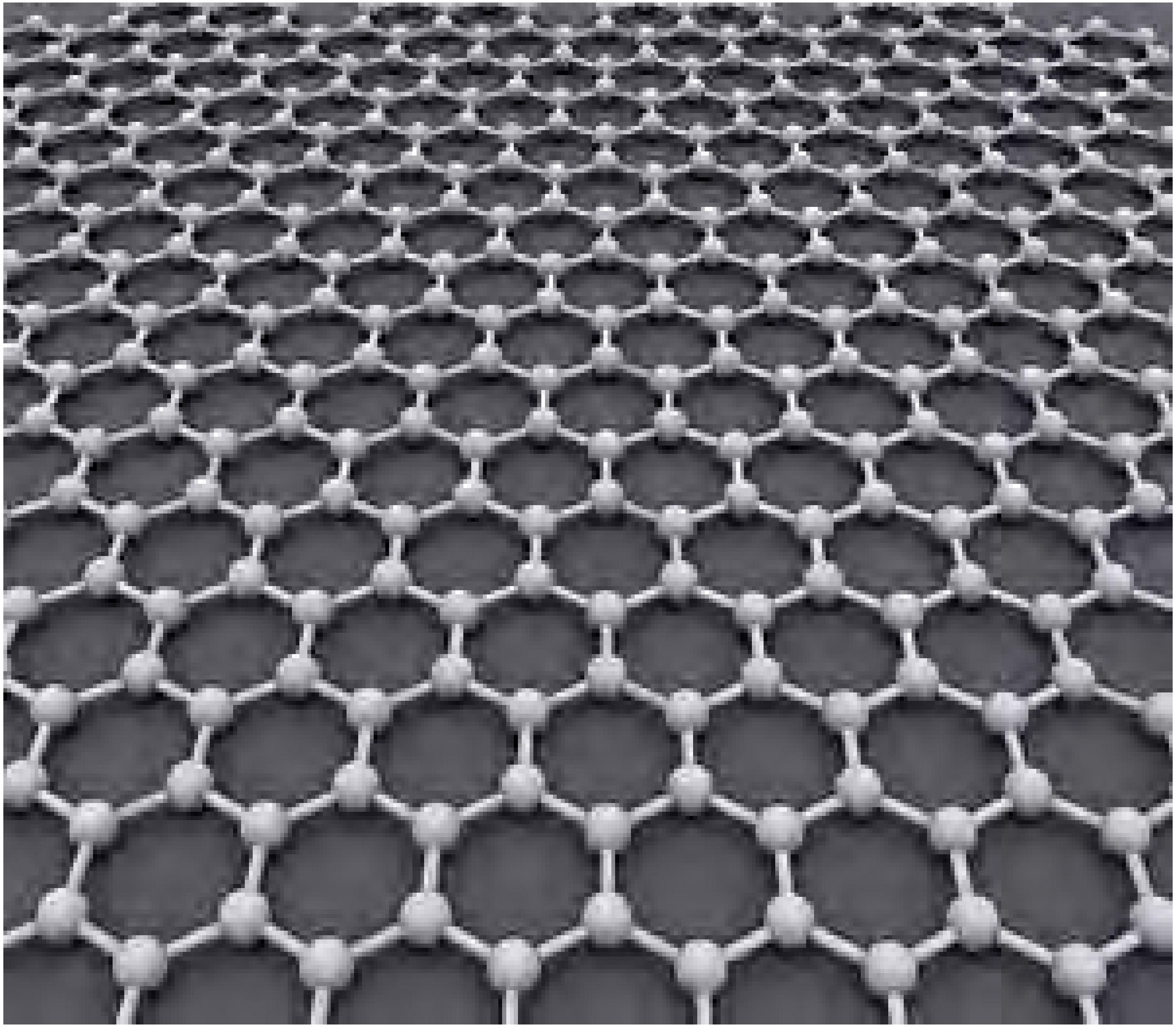


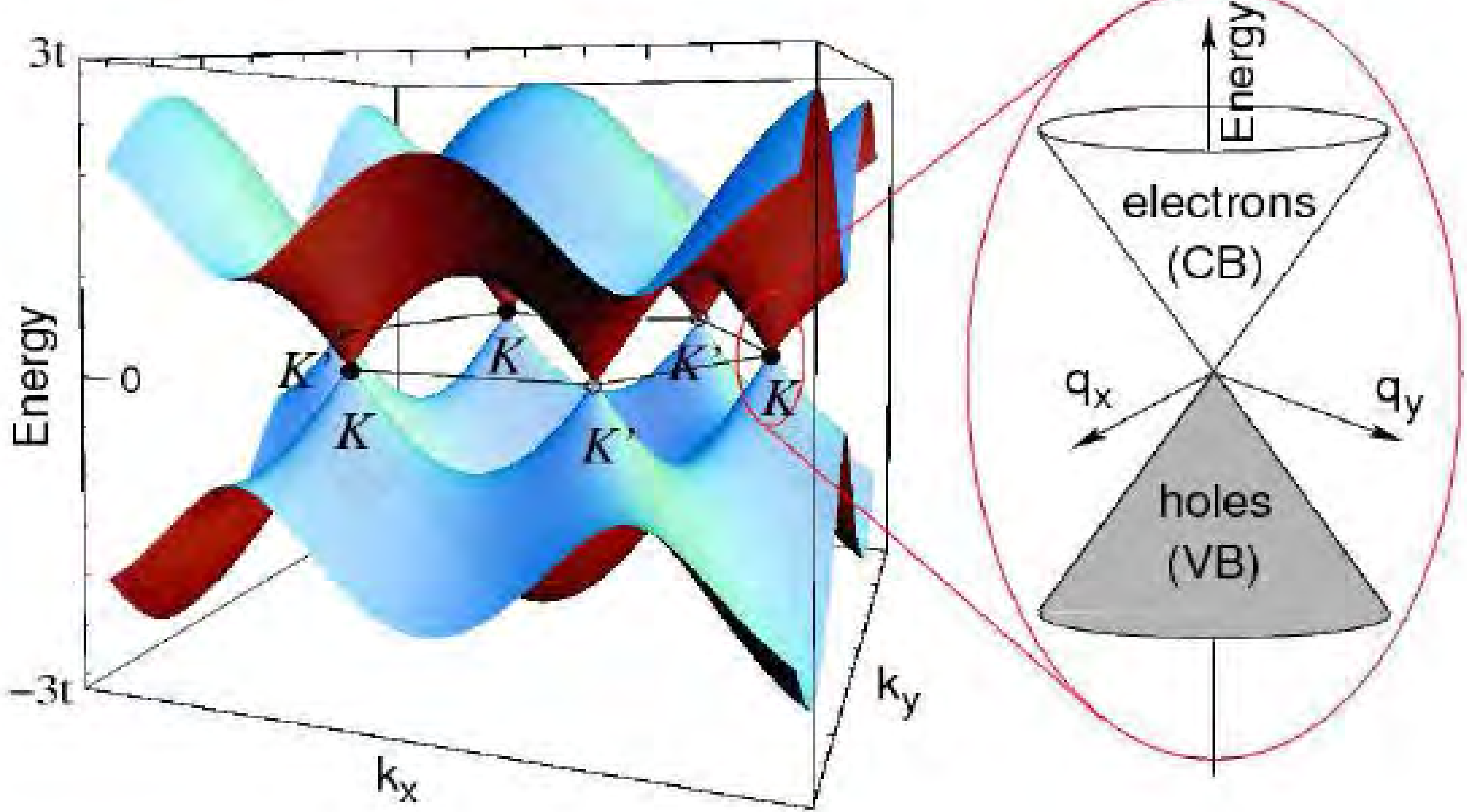
Pan et al, PRL90, 016801 (2003)



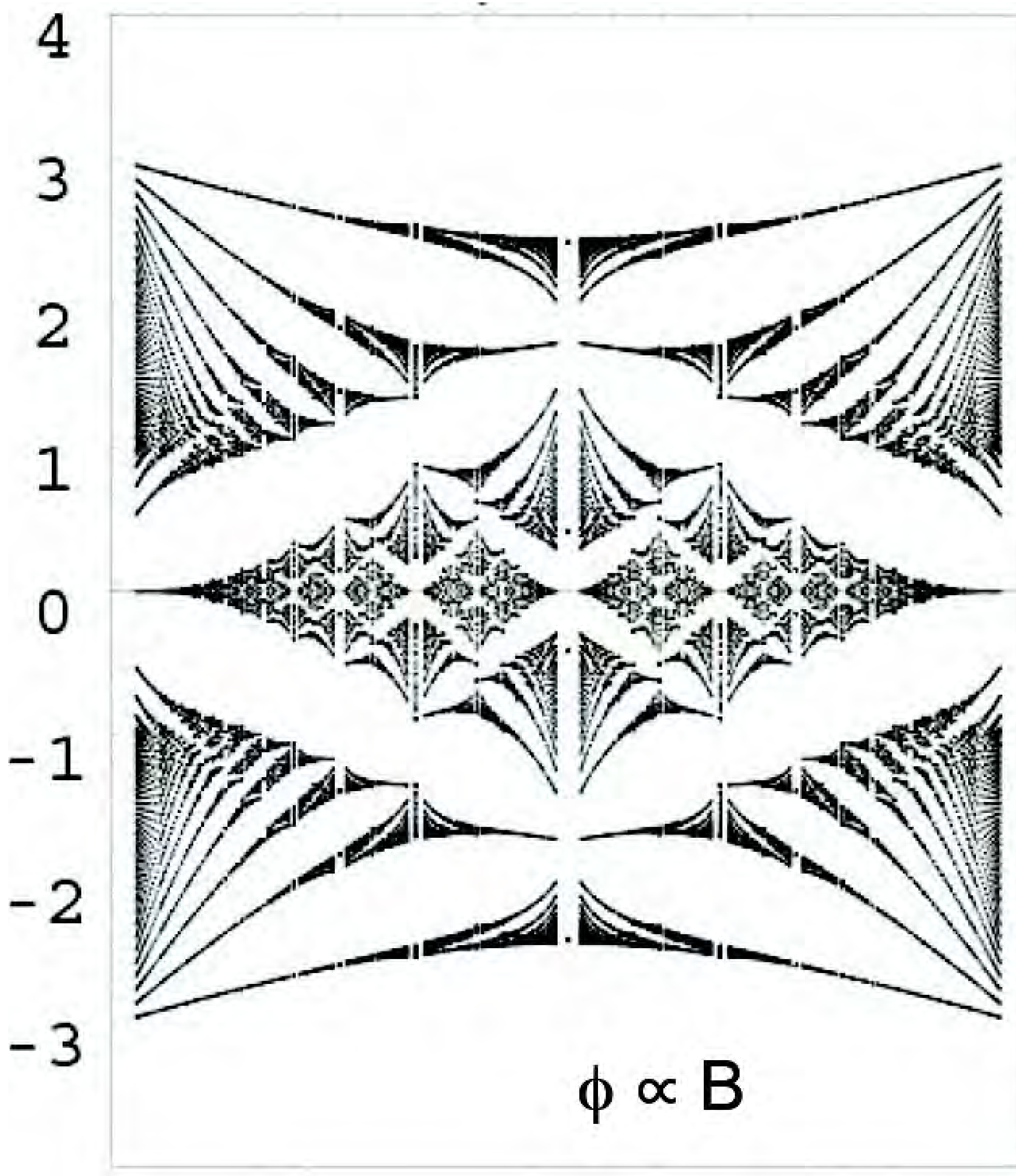
Tₑ ~ 4 mK
Sample
R_{xy} (h/e²)
R_{xx} (kΩ)
2
7/3
5/2
8/3
3
5/2
8/3
7/3
13/5
12/5
19/7
16/7
MAGNETIC FIELD [T]
T = 9 mK
n=3×10¹¹ cm⁻²
μ=31×10⁶ cm²/Vs
R_{xy} (h/e²)
R_{xx} (kΩ)
2
2+1/3
2+2/5
2+1/2
2+2/3
3
2+4/5
2+2/3
2+3/5
2+1/2
2+2/5
2+3/8
2+1/3
2+2/7
2+1/5
B (T)

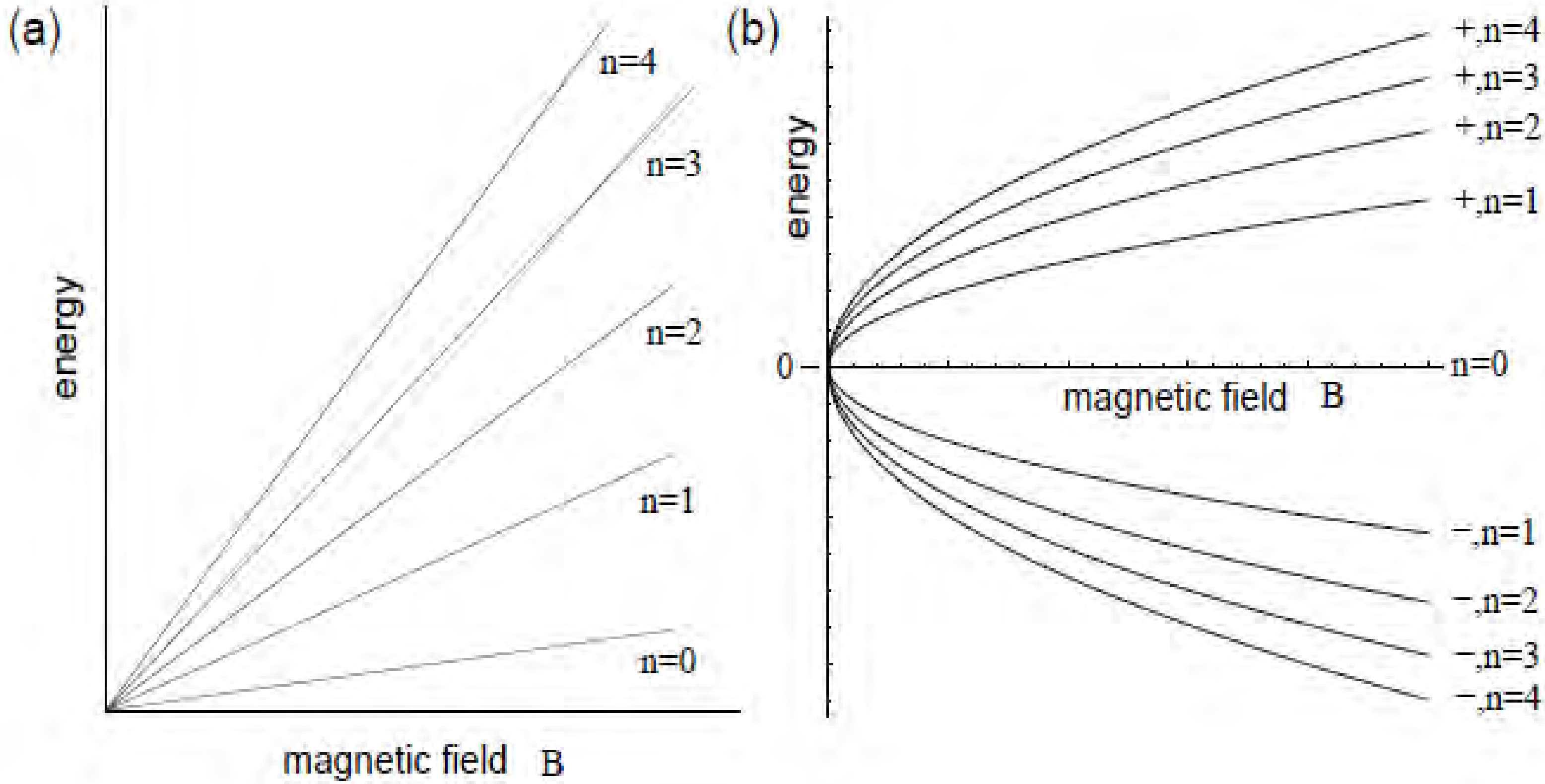
1/3	1/5	1/7	1/9	2/11	2/13	2/15	2/17	3/19	5/21	6/23	6/25
2/3	2/5	2/7	2/9	3/11	3/13	4/15	3/17	4/19	10/21		
4/3	3/5	3/7	4/9	4/11	4/13	7/15	4/17	5/19			
5/3	4/5	4/7	5/9	5/11	5/13	8/15	5/17	9/19			
7/3	6/5	5/7	7/9	6/11	6/13	11/15	6/17	10/19			
8/3	7/5	9/7	11/9	7/11	7/13	22/15	8/17				
	8/5	10/7	13/9	8/11	10/13	23/15	9/17				
	9/5	11/7	14/9	14/11	19/13						
	11/5	12/7	*25/9*	16/11	20/13						
	12/5	*16/7*		17/11							
	13/5(?)	*19/7*									*5/2*
	14/5										*7/2*
	16/5										*3/8(?)*
	19/5										*5/8(?)*
	21/5										*19/8*
	24/5										*3/10(?)*

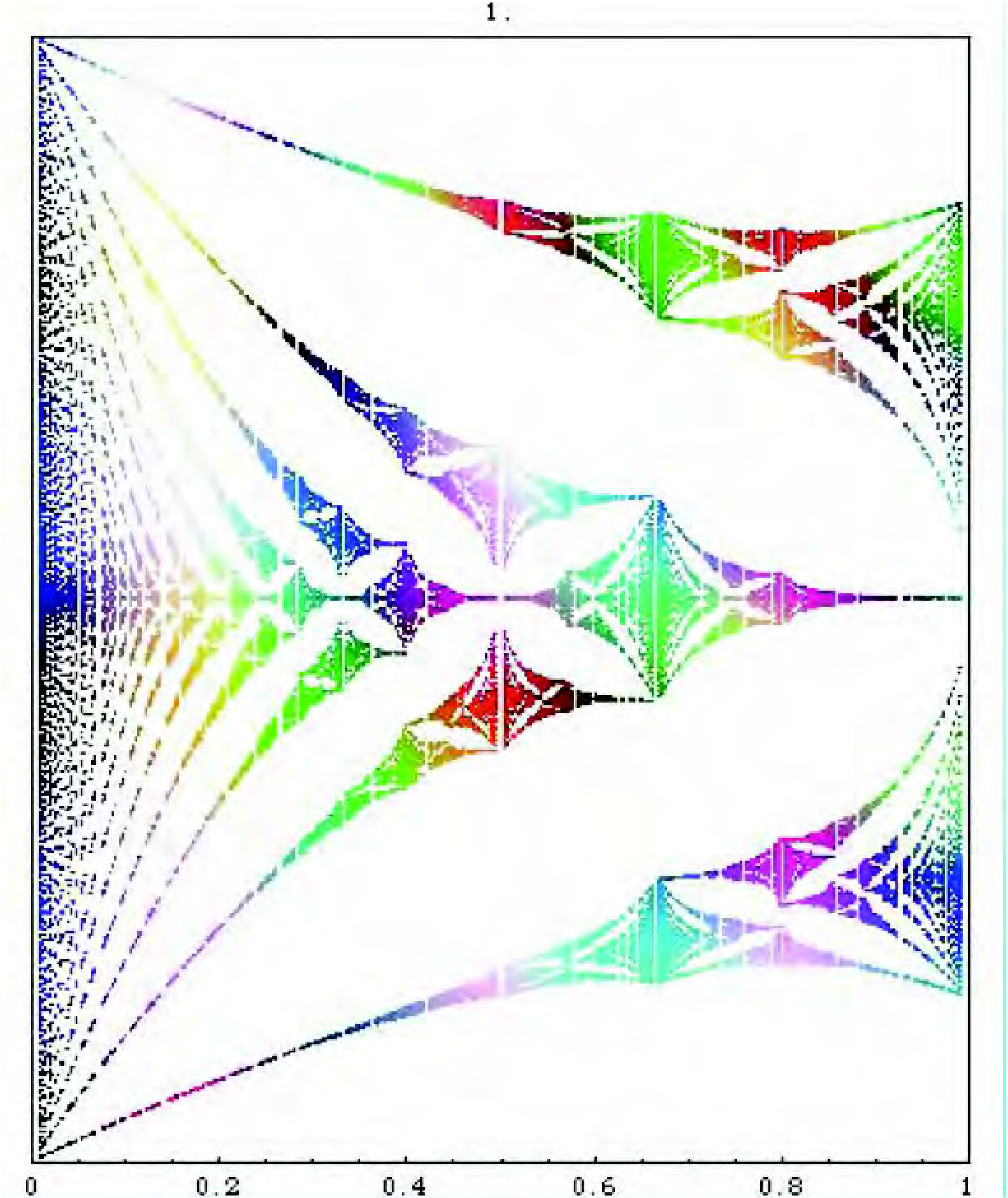




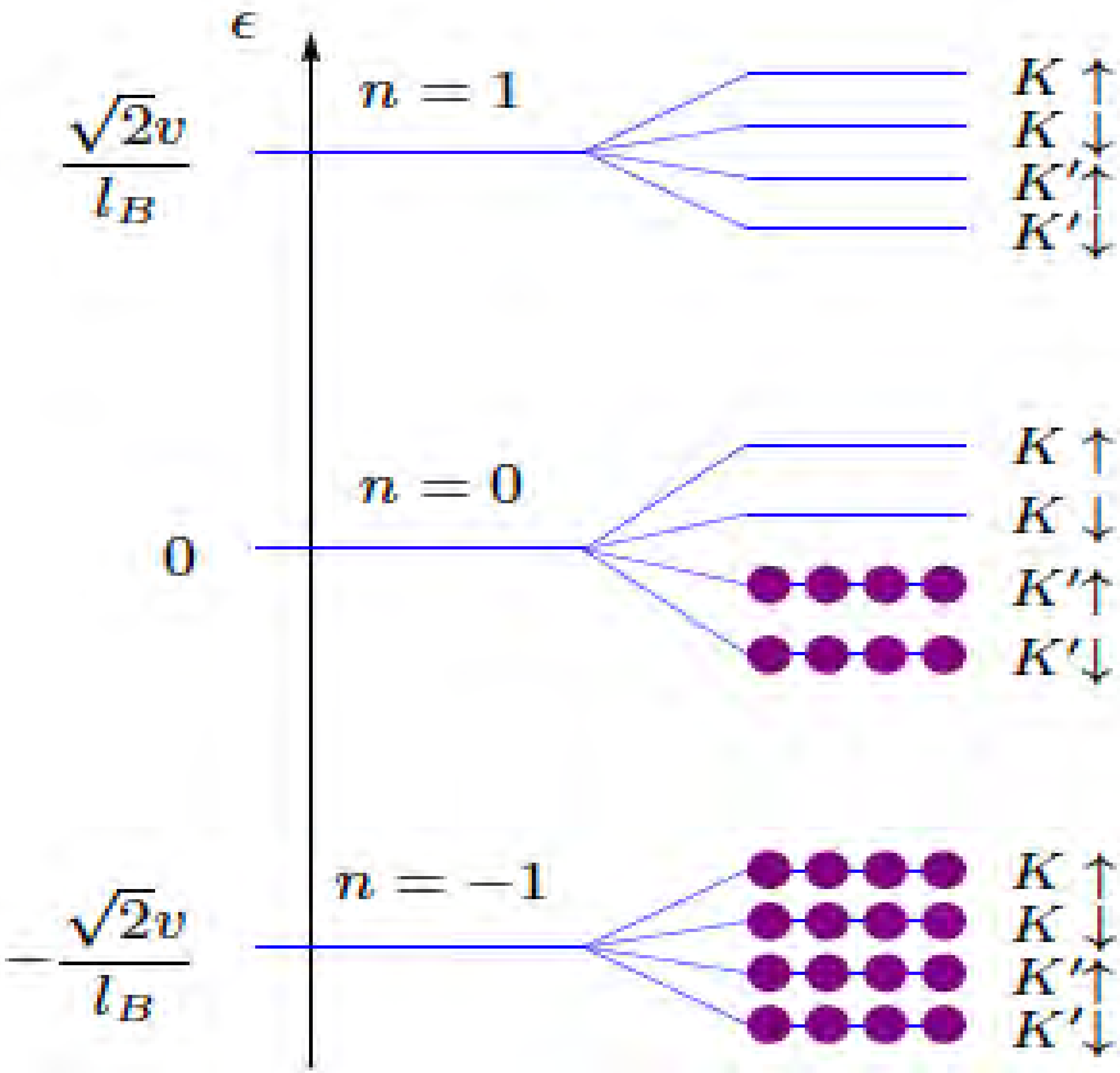
2+1 dimensional Dirac fermions with two flavors and real spin


4
3
2
1
0
-1
-2
-3
φ ∝ B

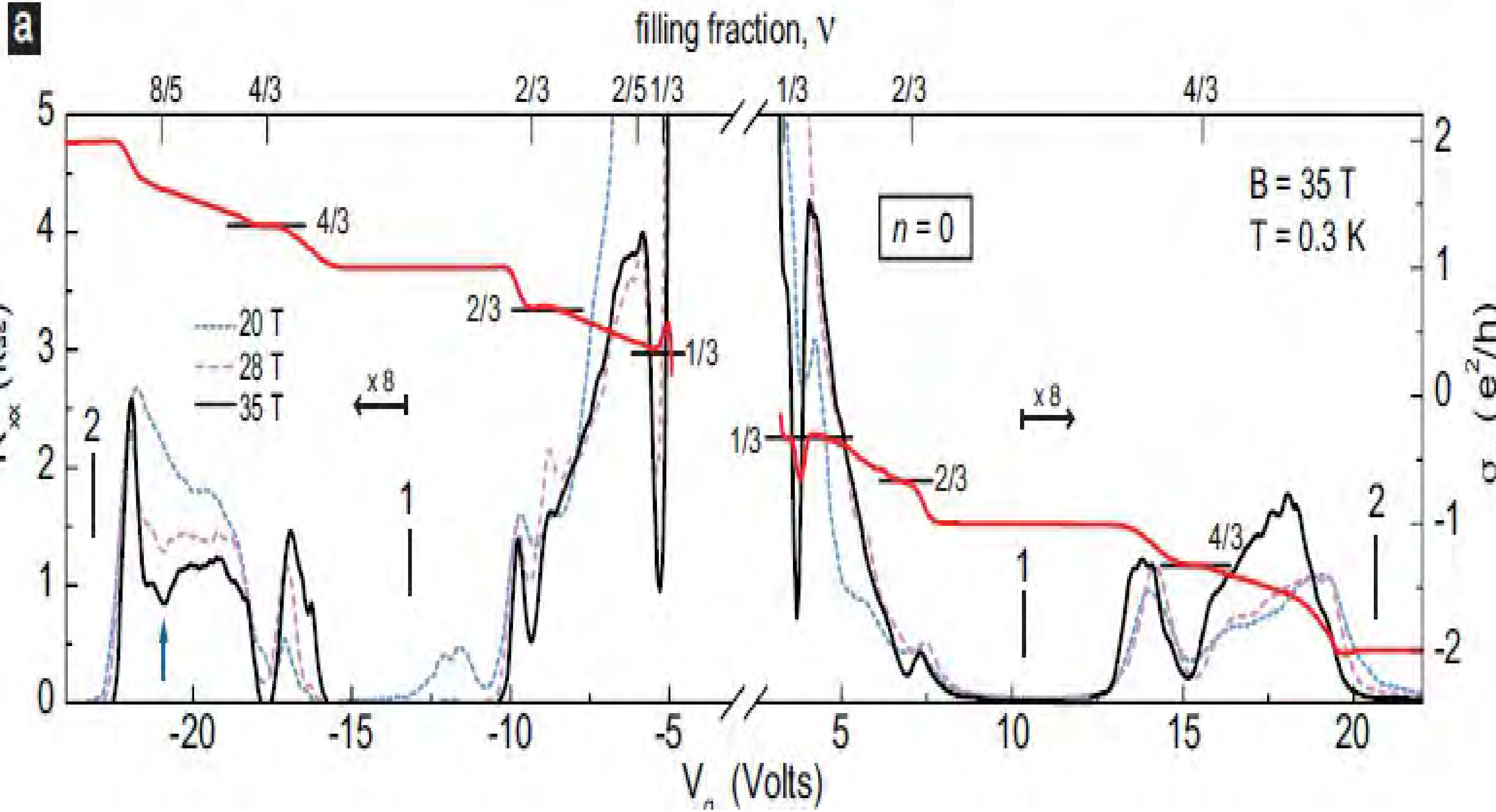

(a)
energy
n=4
n=3
n=2
n=1
n=0
magnetic field B
(b)
energy
+,n=4
+,n=3
+,n=2
+,n=1
0
n=0
magnetic field B
−,n=1
−,n=2
−,n=3
−,n=4



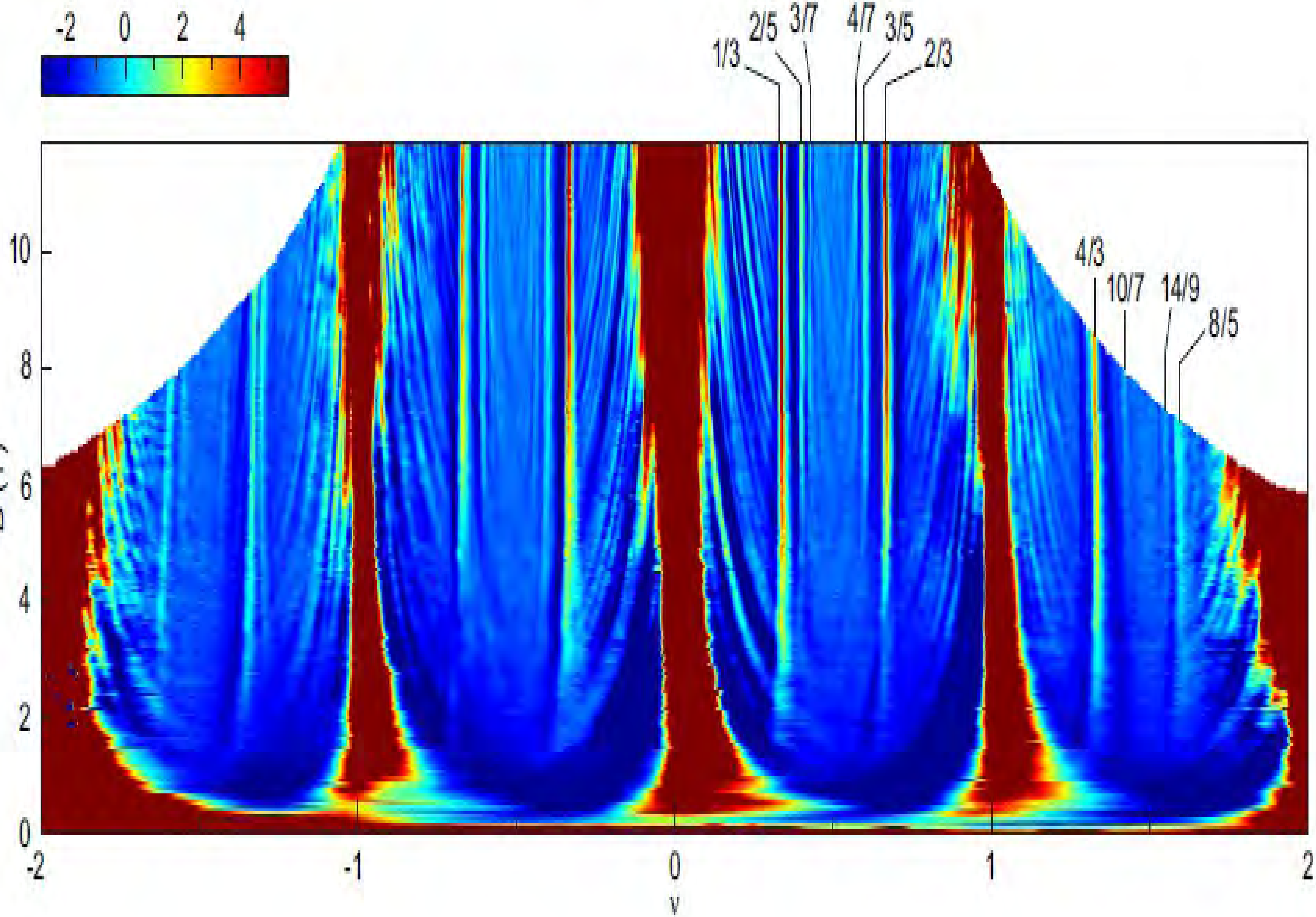
1.
0
0.2
0.4
0.6
0.8
1



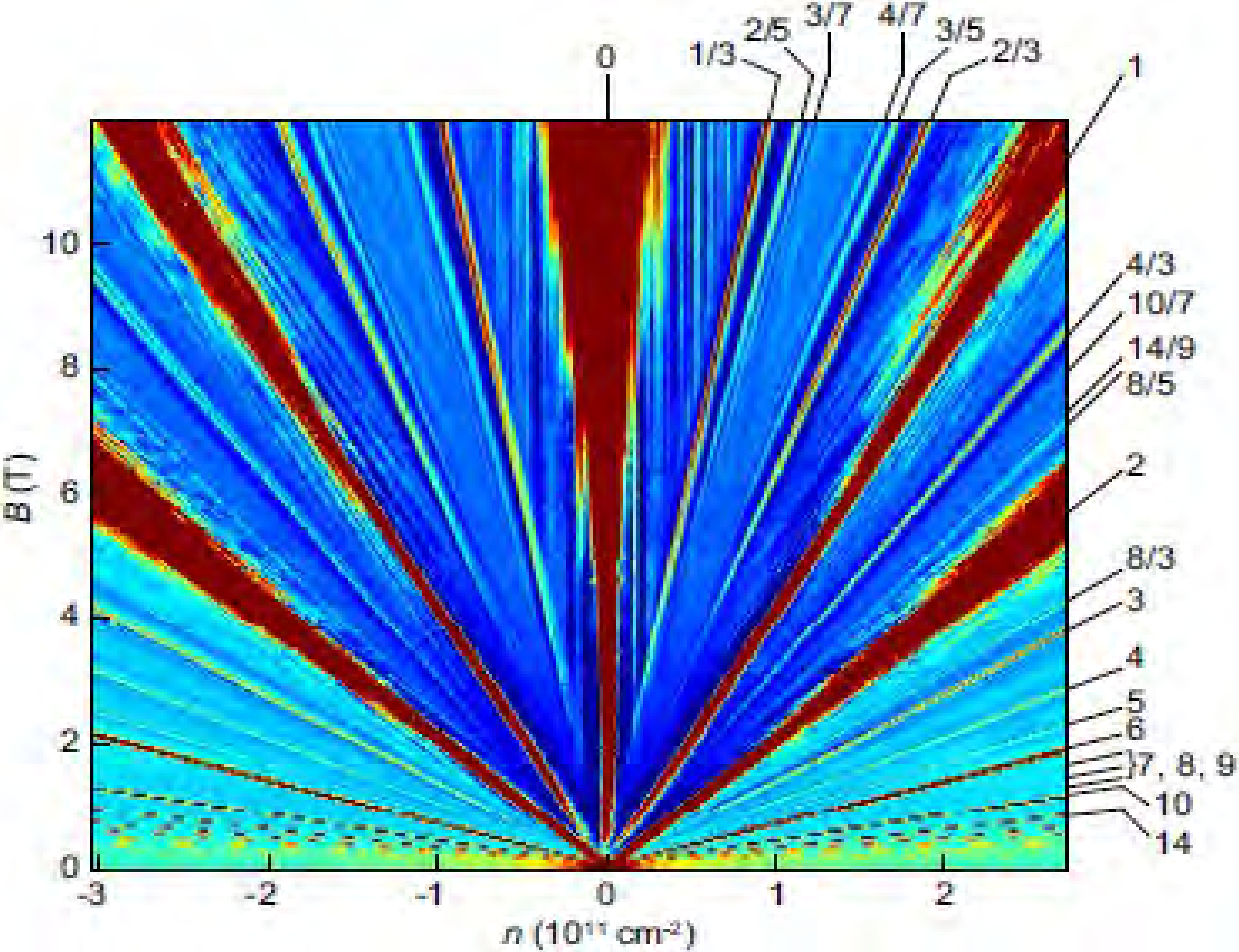
ϵ
$n = 1$
$\frac{\sqrt{2}v}{l_B}$
$K\uparrow$
$K\downarrow$
$K'\uparrow$
$K'\downarrow$
$n = 0$
0
$K\uparrow$
$K\downarrow$
$K'\uparrow$
$K'\downarrow$
$n = -1$
$-\frac{\sqrt{2}v}{l_B}$
$K\uparrow$
$K\downarrow$
$K'\uparrow$
$K'\downarrow$



a
filling fraction, ν
8/5
4/3
2/3
2/5
1/3
1/3
2/3
4/3
B = 35 T
T = 0.3 K
n = 0
4/3
2/3
1/3
1/3
2/3
4/3
20 T
28 T
35 T
x 8
x 8
2
1
1
2
R_{xx} (kΩ)
σ_{xy} (e^2/h)
V_g (Volts)



-2 0 2 4
1/3
2/5
3/7
4/7
3/5
2/3
4/3
10/7
14/9
8/5
B (T)
0
2
4
6
8
10
-2
-1
0
1
2
ν



0
1/3
2/5
3/7
4/7
3/5
2/3
1
4/3
10/7
14/9
8/5
2
8/3
3
4
5
6
7, 8, 9
10
14
B (T)
10
8
6
4
2
0
-3
-2
-1
0
1
2
n (10¹¹ cm⁻²)

Quantum Hall Ferromagnetism at neutrality

$$\Psi_{\alpha,\beta} = \prod_{k=1}^{N_\phi} c^\dagger_{k\alpha} c^\dagger_{k\beta} \, |0\rangle$$

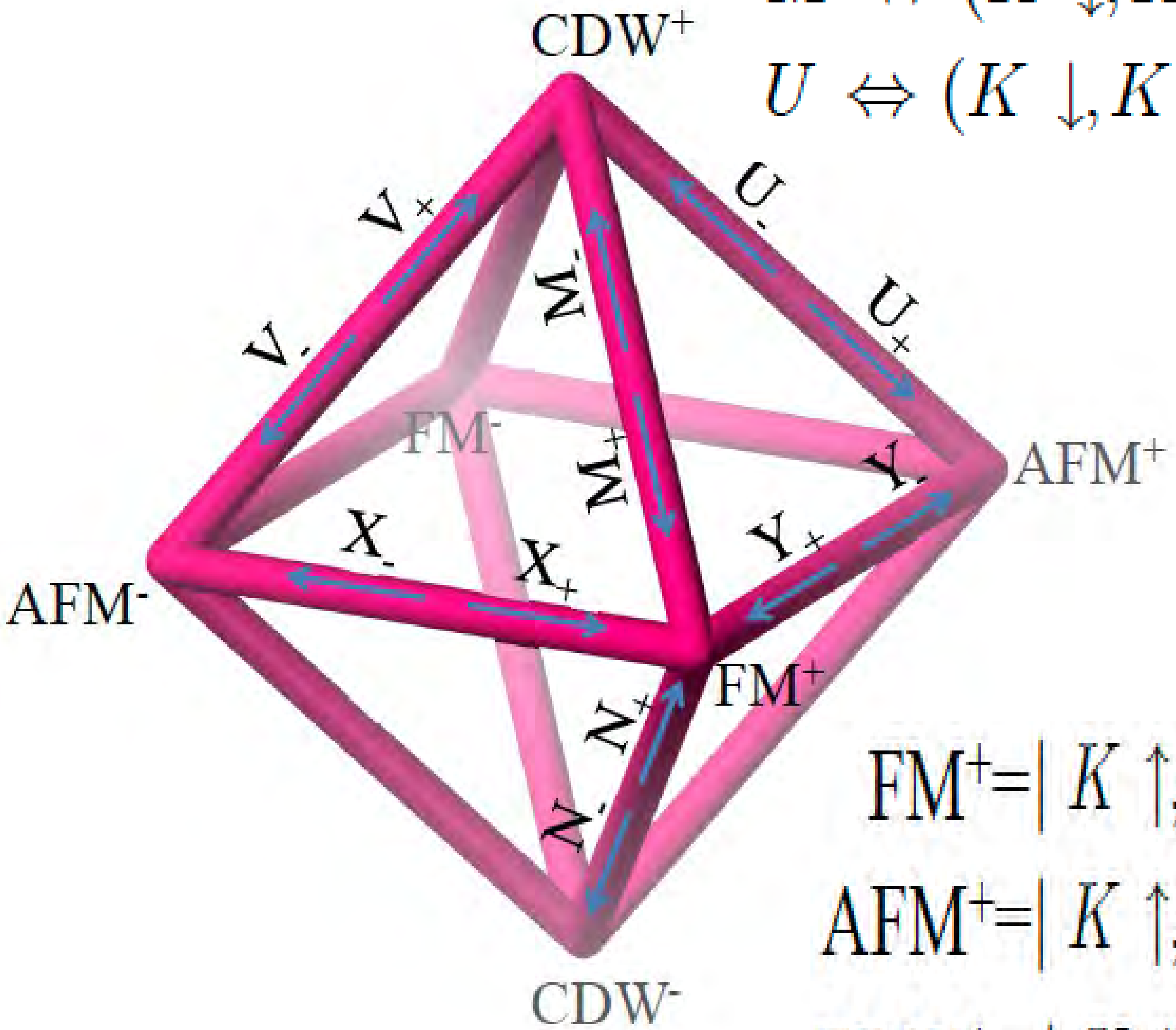
spinors $\phi_{\alpha,\beta}$ are in Span$\{K\uparrow, K\downarrow K'\uparrow, K'\downarrow\}$

EXACT eigenstates of the fully SU(4) symmetric Coulomb interaction

$$X \Leftrightarrow (K\uparrow, K\downarrow) \qquad Y \Leftrightarrow (K'\uparrow, K'\downarrow)$$

$$M \Leftrightarrow (K\downarrow, K'\uparrow) \qquad N \Leftrightarrow (K\uparrow, K'\downarrow)$$

$$U \Leftrightarrow (K\downarrow, K'\downarrow) \qquad V \Leftrightarrow (K\uparrow, K'\uparrow)$$



SU(4) irrep

$$\mathrm{FM}^+ = |K\uparrow, K'\uparrow\rangle \qquad \mathrm{FM}^- = |K\downarrow, K'\downarrow\rangle$$

$$\mathrm{AFM}^+ = |K\uparrow, K'\downarrow\rangle \qquad \mathrm{AFM}^- = |K\downarrow, K'\uparrow\rangle$$

$$\mathrm{CDW}^+ = |K\uparrow, K\downarrow\rangle \qquad \mathrm{CDW}^- = |K\downarrow, K\uparrow\rangle$$

Effective Hamiltonian in the nu = 0 Landau level :

$$
\begin{aligned}
H &= H_{\rm C} + H_{\rm v} + H_{\rm Z}, \\
H_{\rm C} &= \frac{1}{2}\sum_{i\neq j} \frac{e^2}{\epsilon|\vec{r}_i - \vec{r}_j|}, \\
H_{\rm v} &= \frac{1}{2}\sum_{i\neq j} \left(g_z \tau_z^i \tau_z^j + g_\perp(\tau_x^i\tau_x^j + \tau_y^i\tau_y^j)\right)\delta(\vec{r}_i - \vec{r}_j), \\
H_{\rm Z} &= -\epsilon_{\rm Z}\sum_i \sigma_z^i .
\end{aligned}
$$

$$g_\perp = g\cos\theta, \quad g_z = g\sin\theta$$

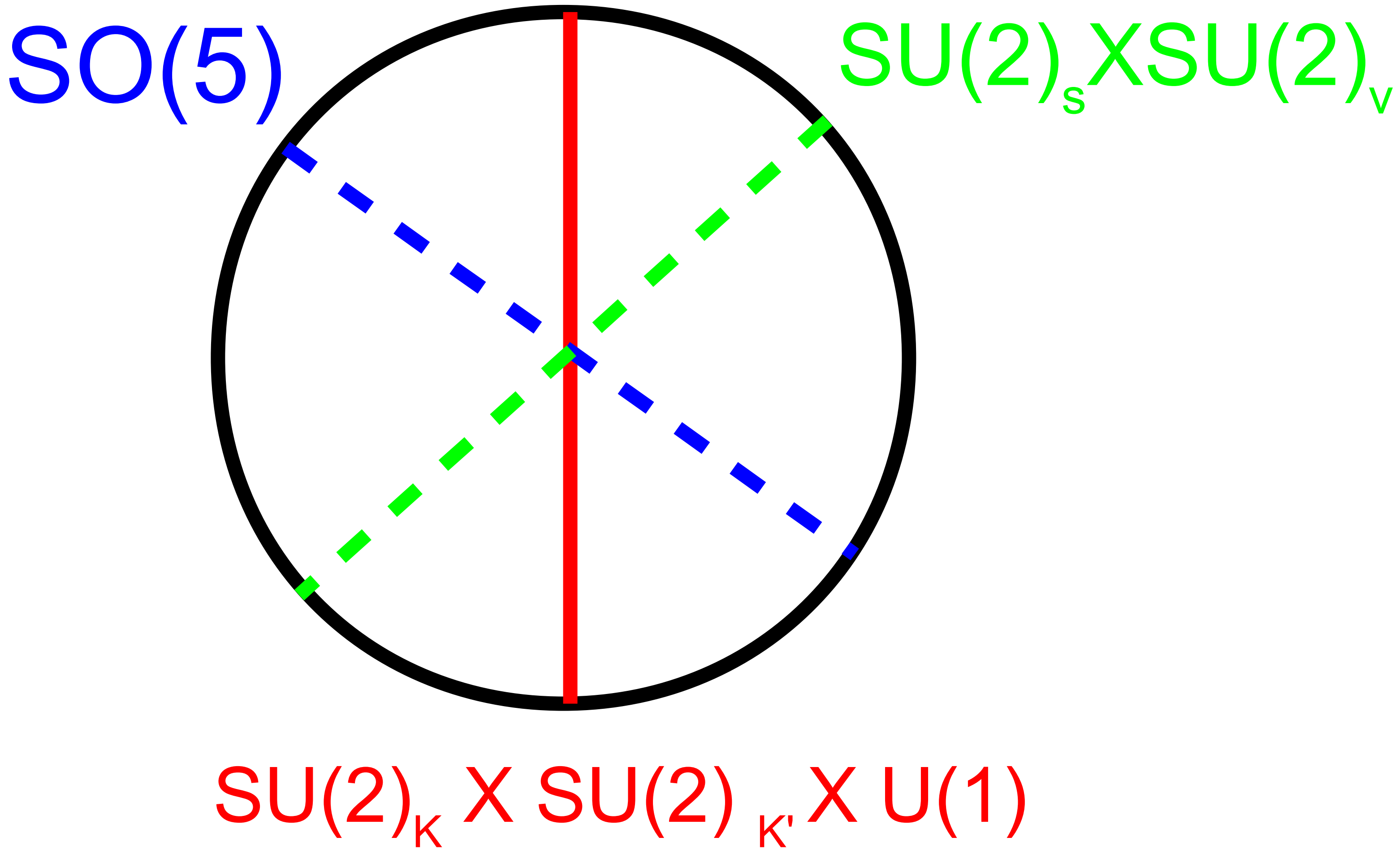
$$S_\alpha = \frac{1}{2}\sum_i \sigma_\alpha^i, \qquad T_\alpha = \frac{1}{2}\sum_i \tau_\alpha^i,$$

$$N_\alpha = \frac{1}{2}\sum_i \tau_z^i \sigma_\alpha^i, \quad \Pi_\alpha^\beta = \frac{1}{2}\sum_i \tau_\beta^i \sigma_\alpha^i,$$

Coulomb interaction is SU(4) symmetric : 15 generators

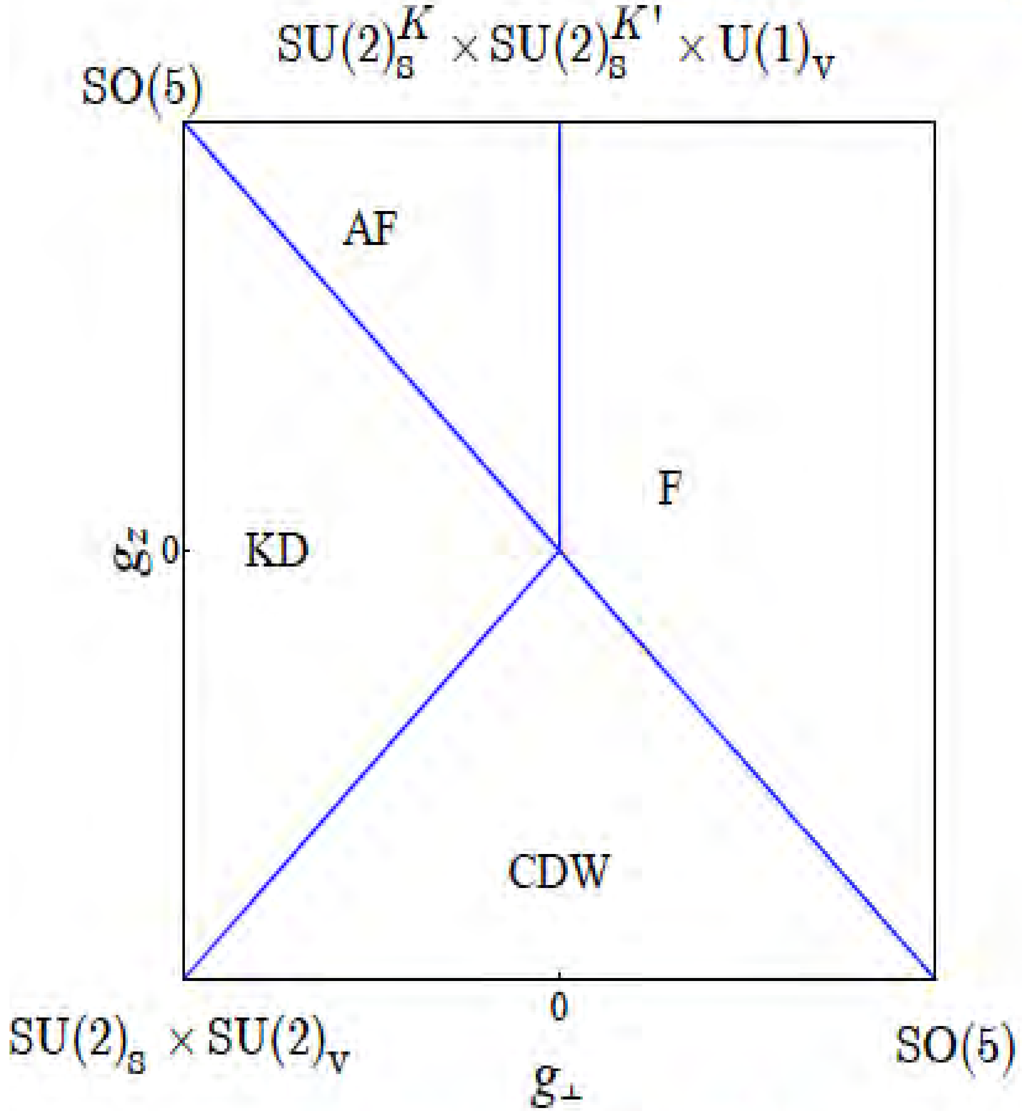
	Symmetry of $H_C + H_v$	generators
$g_\perp = 0$	$SU(2)_s^K \times SU(2)_s^{K'} \times U(1)_v$	S_α, N_α, T_z
$g_\perp = g_z$	$SU(2)_s \times SU(2)_v$	S_α, T_α
$g_\perp + g_z = 0$	$SO(5)$	S_α, T_z, Π_α^x, Π_α^y

$$g_\perp = g\cos\theta, \quad g_z = g\sin\theta$$

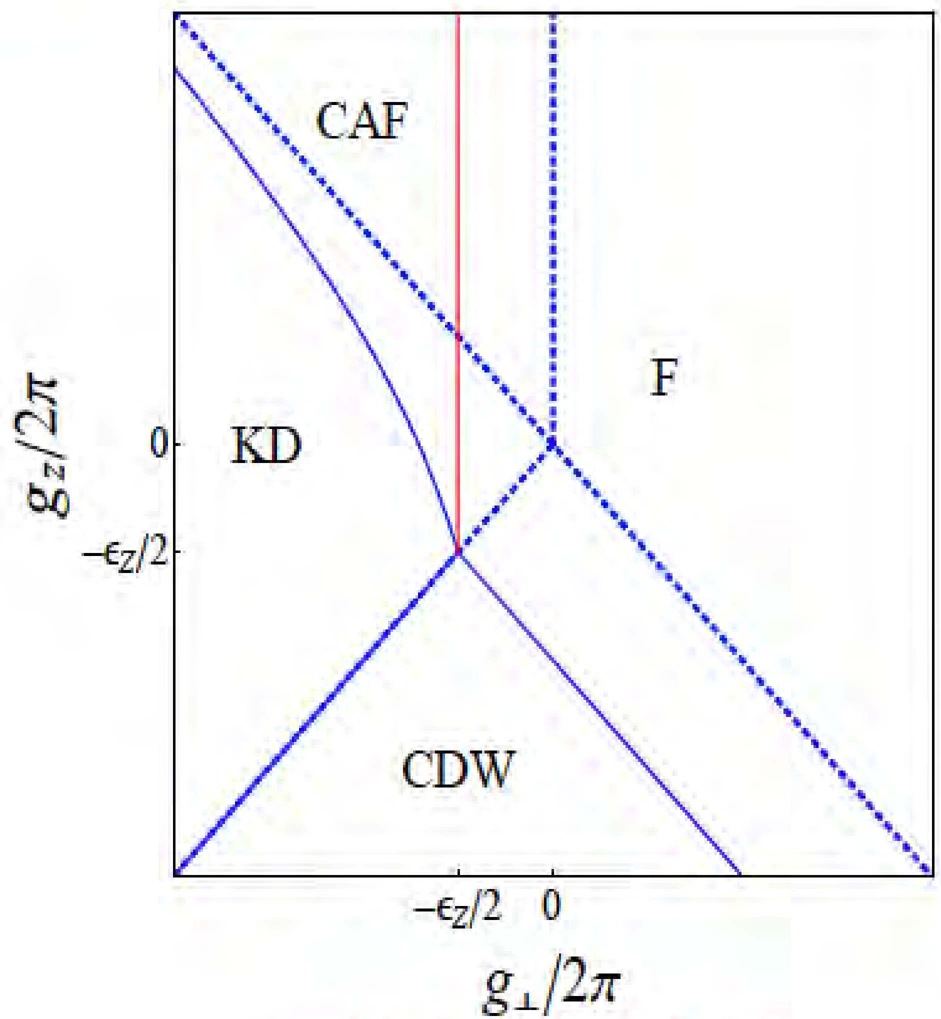


SU(2) X U(1) elsewhere....

Mean-field phase diagram

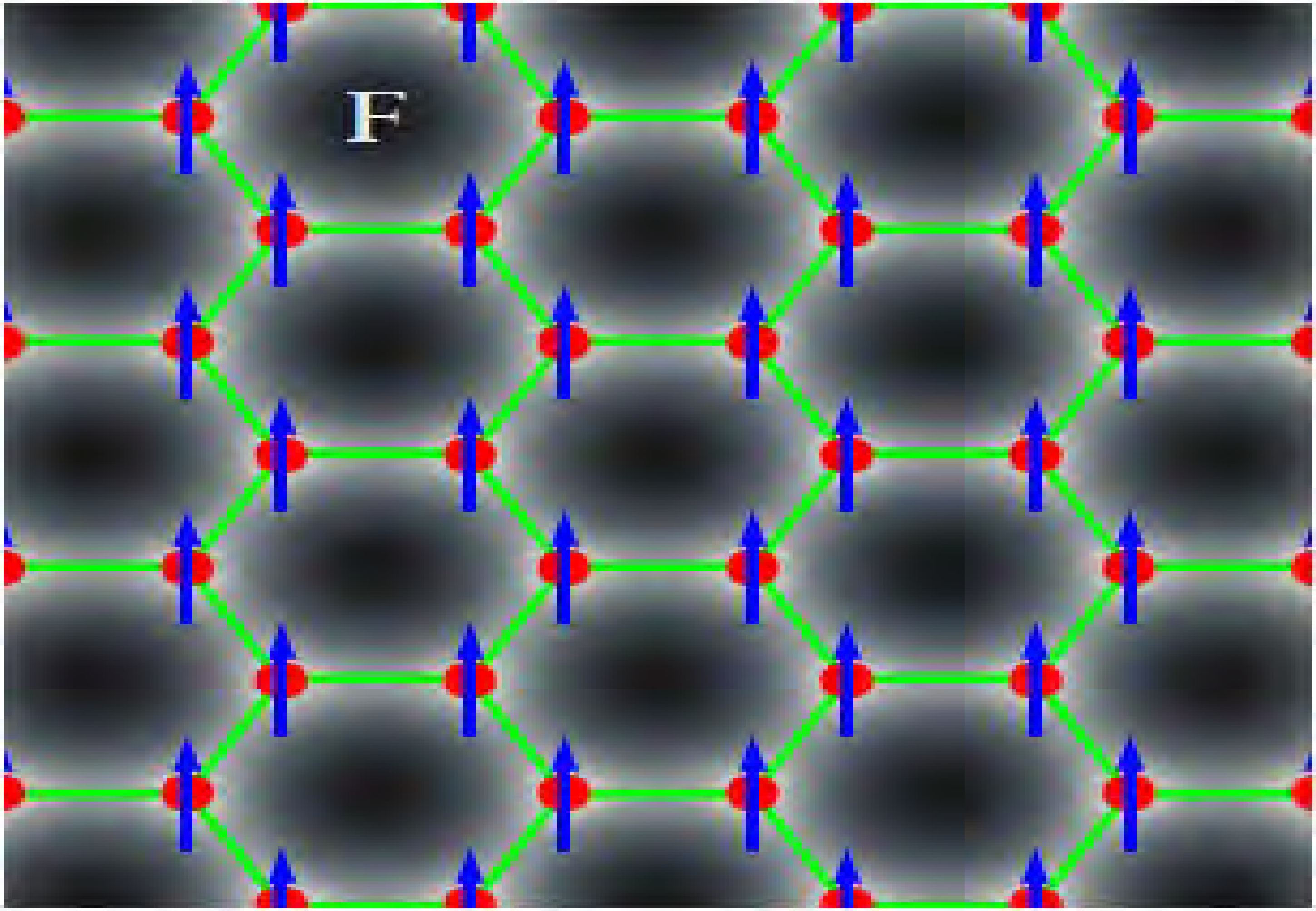


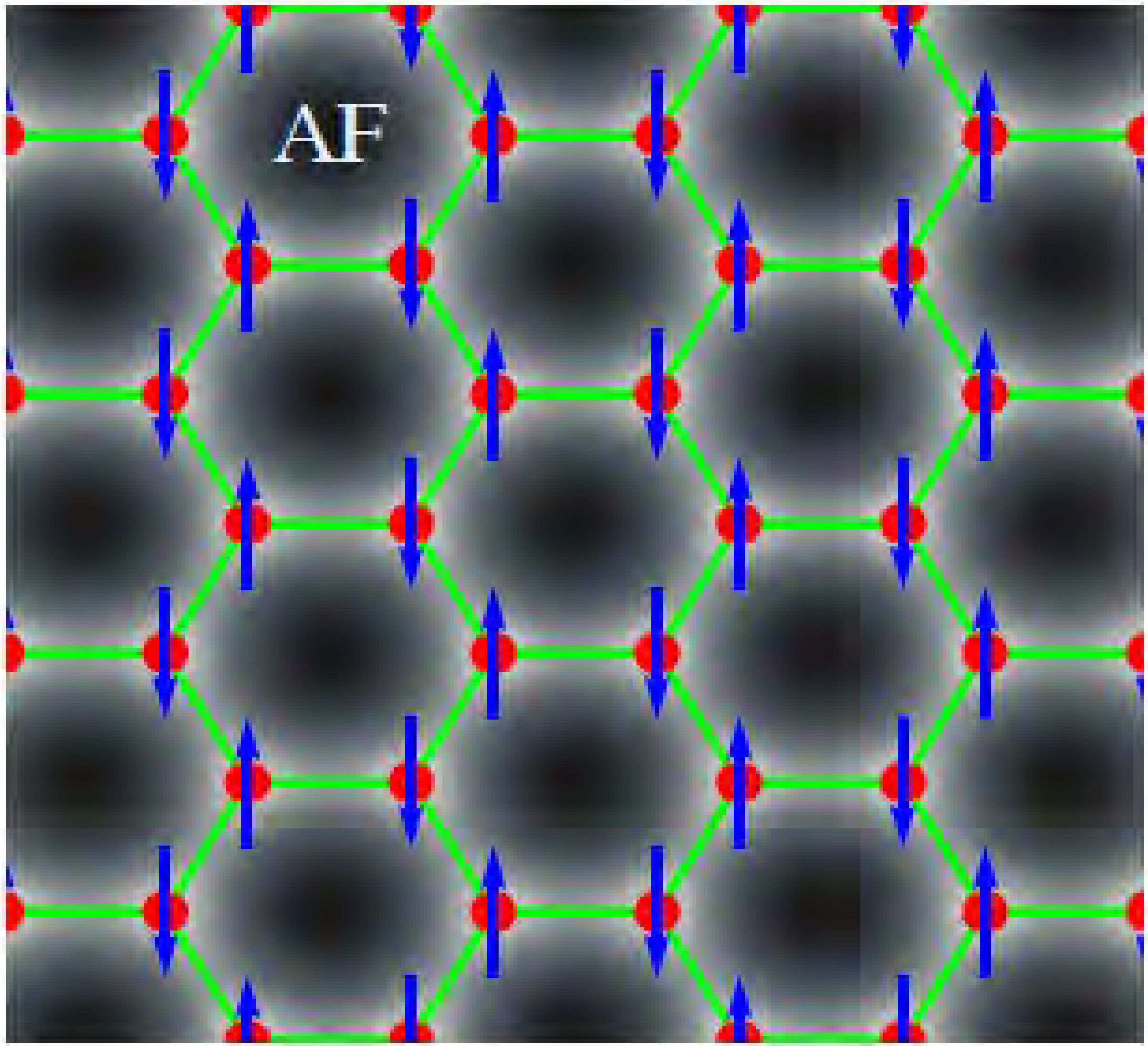
without Zeeman field

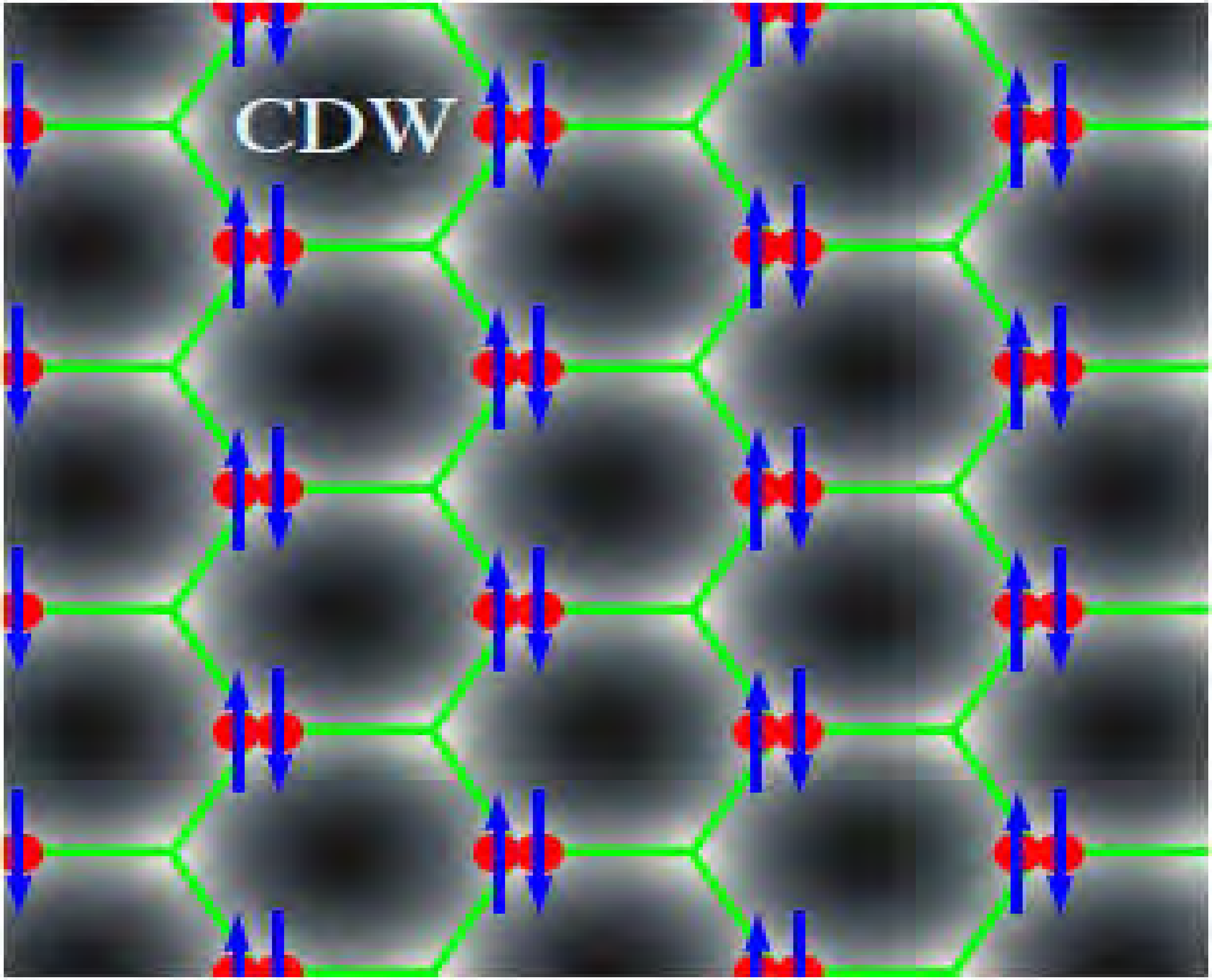


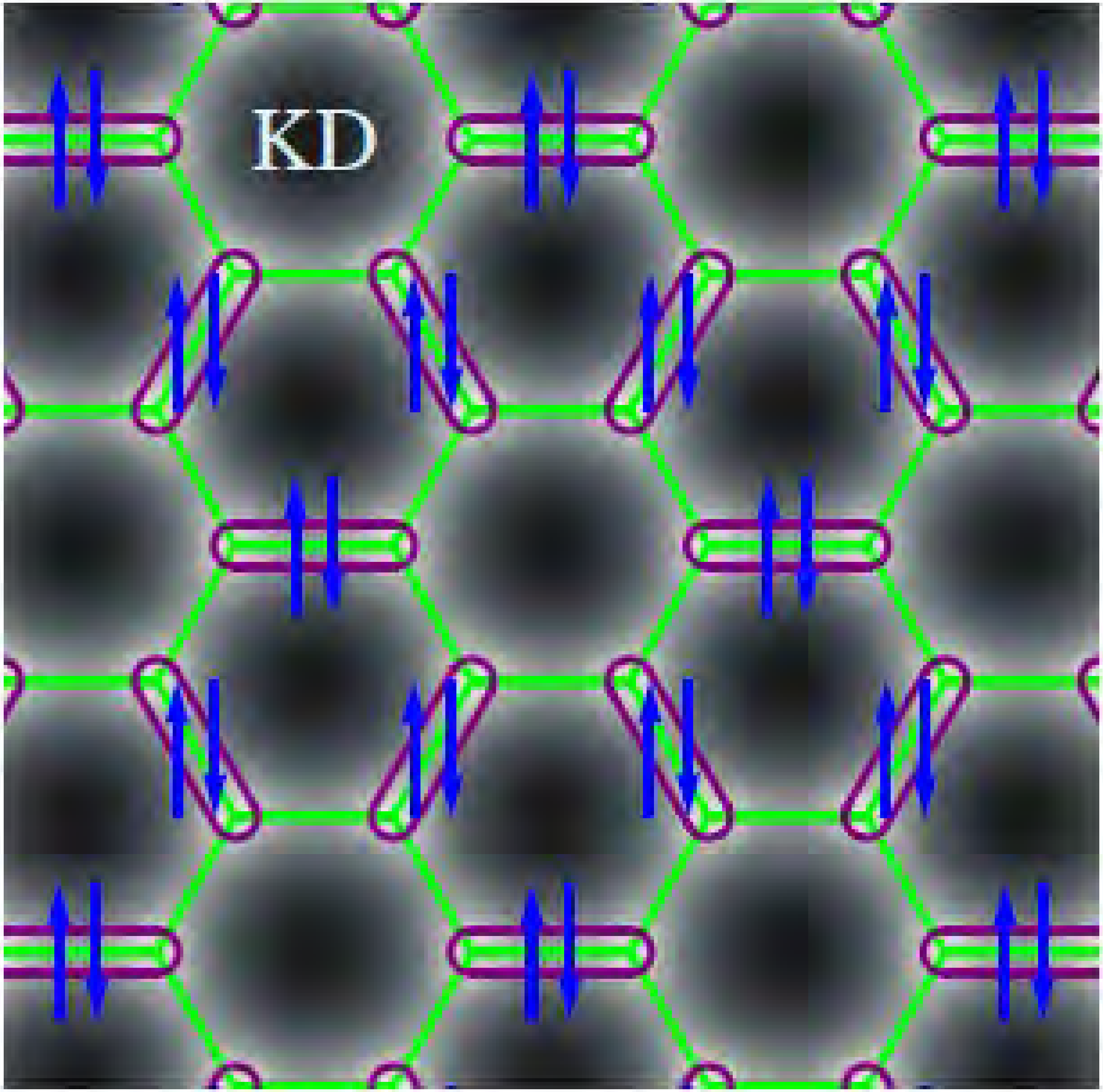
with Zeeman field

$H_v + H_Z$ lifts the SU(4) degeneracy

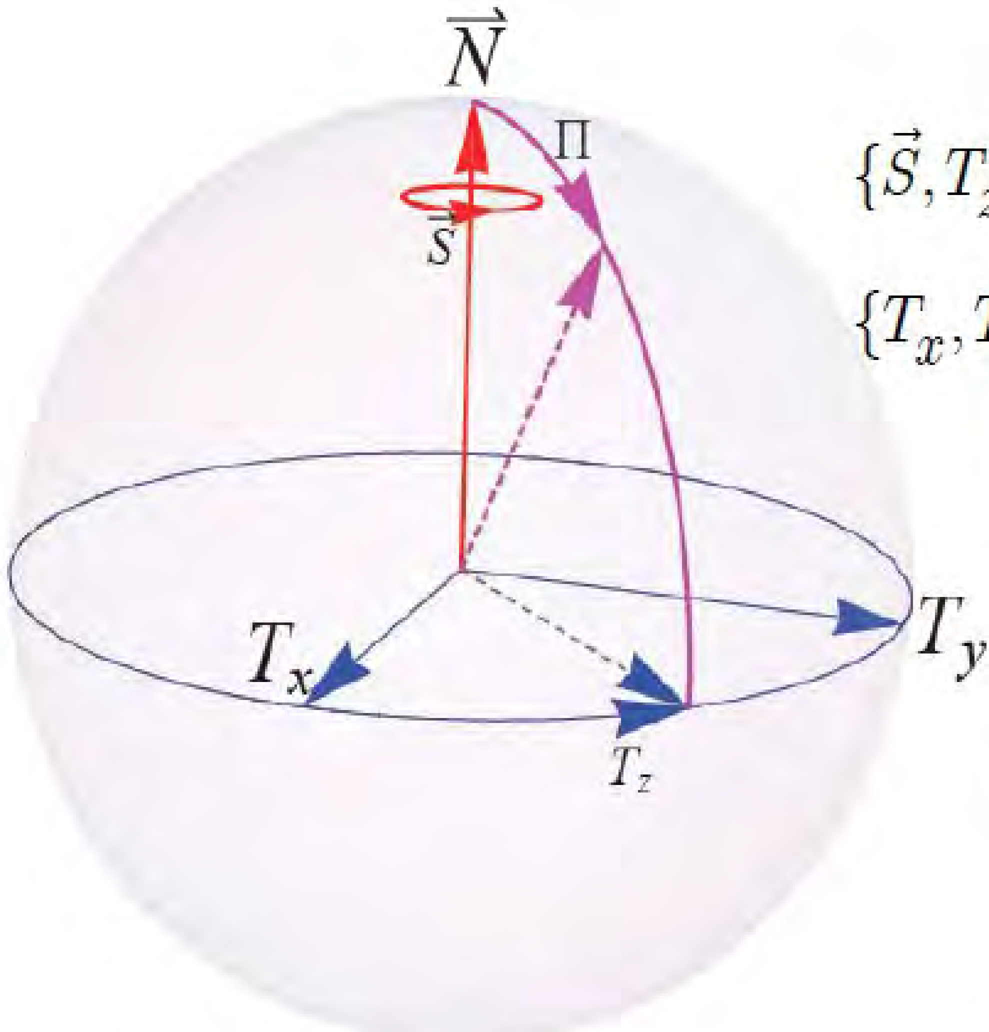

F


AF


CDW


KD

SO(5) group



$\{\vec{S}, T_z, \vec{\Pi}^x, \vec{\Pi}^y\}$ generators of SO(5) group

$\{T_x, T_y, N_x, N_y, N_z\}$ 5D vector

Rotation in 5D space:

$\vec{S} : \{N_x, N_y, N_z\}$

T_z: $\{T_x, T_y\}$

$\Pi : \{N_x, N_y, N_z\} \rightleftarrows \{T_x, T_y\}$

$\{T_x, T_y\} \rightarrow$ Kekule distortion state

$\{N_x, N_y, N_z\} \rightarrow$ Antiferromagnetic state graphene vs *d*-wave superconductor

A Unified Theory Based on *SO*(5) Symmetry of Superconductivity and Antiferromagnetism

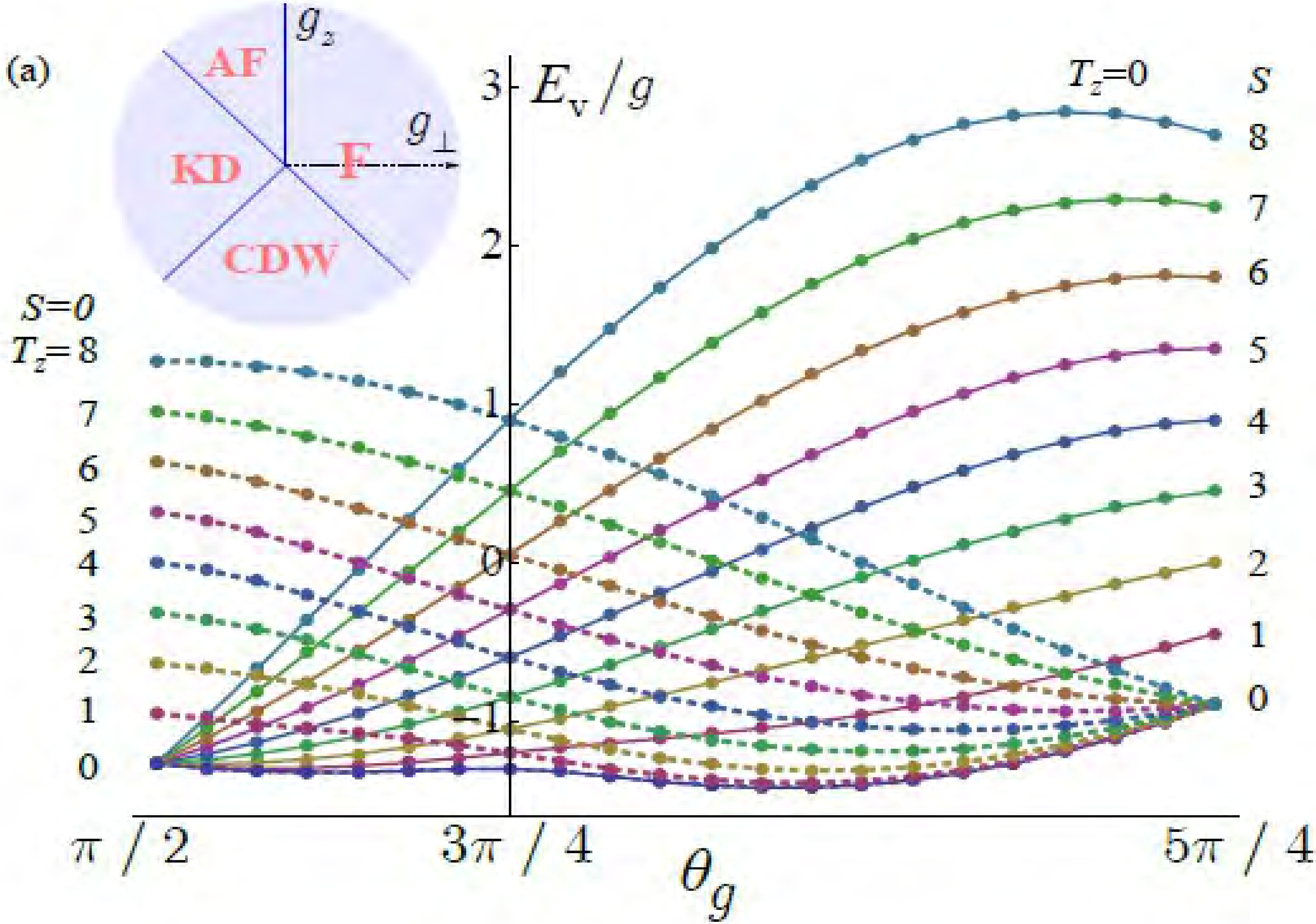
Shou-Cheng Zhang

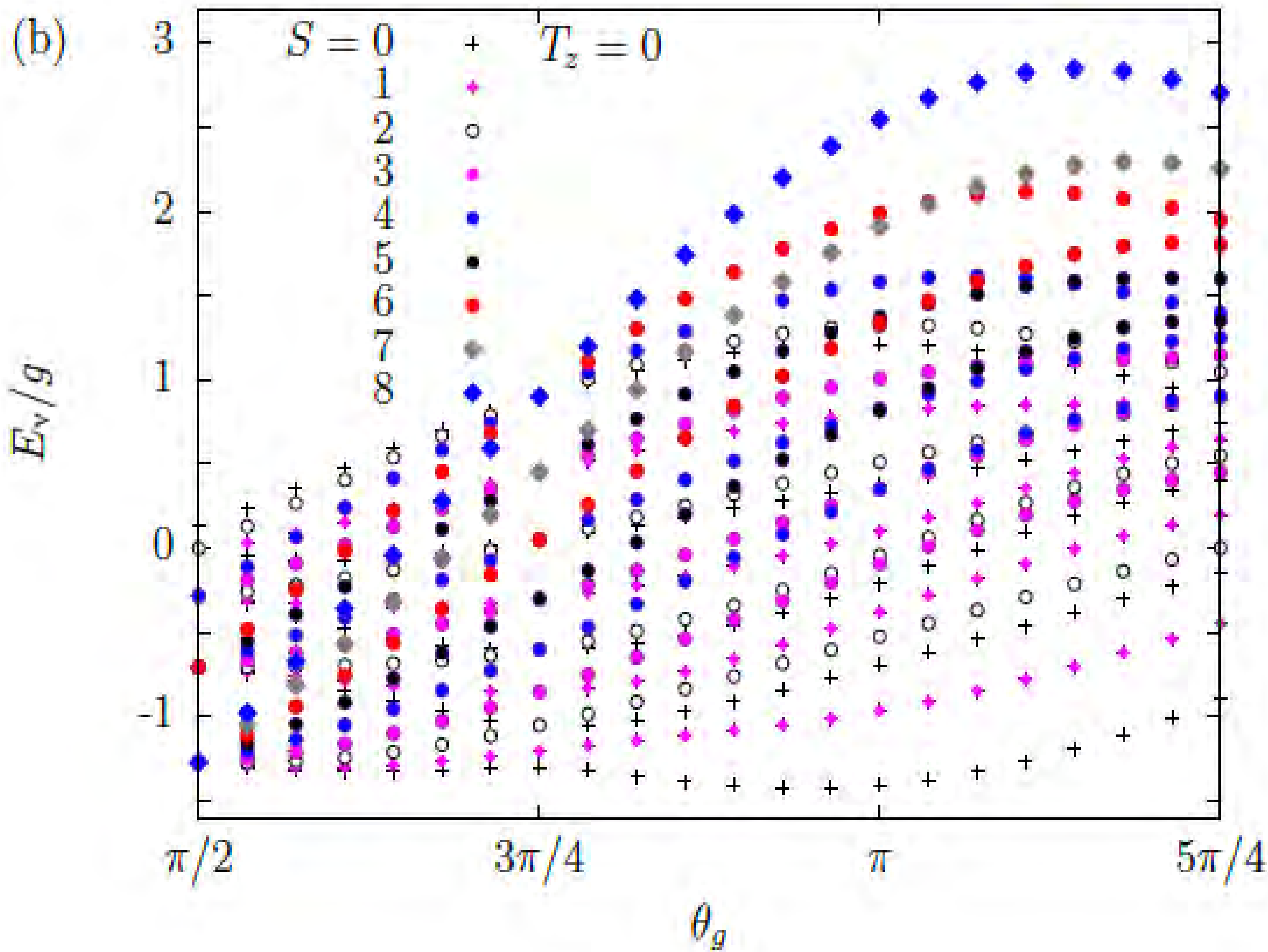
The complex phase diagram of high–critical temperature (T_c) superconductors can be deduced from an *SO*(5) symmetry principle that unifies antiferromagnetism and *d*-wave superconductivity. The approximate *SO*(5) symmetry has been derived from the microscopic Hamiltonian, and it becomes exact under renormalization group flow toward a bicritical point. This symmetry enables the construction of a *SO*(5) quantum nonlinear σ model that describes the phase diagram and the effective low-energy dynamics of the system. This model naturally explains the basic phenomenology of the high-T_c superconductors from the insulating to the underdoped and the optimally doped region.

Shou-Cheng Zhang, Science 275, 1089 (1997).

SO(5) in N=0 LL of graphene		SO(5) in high-Tc superconductor
AFM	⇔	AFM
Kekule-distortion state		d-wave superconductor

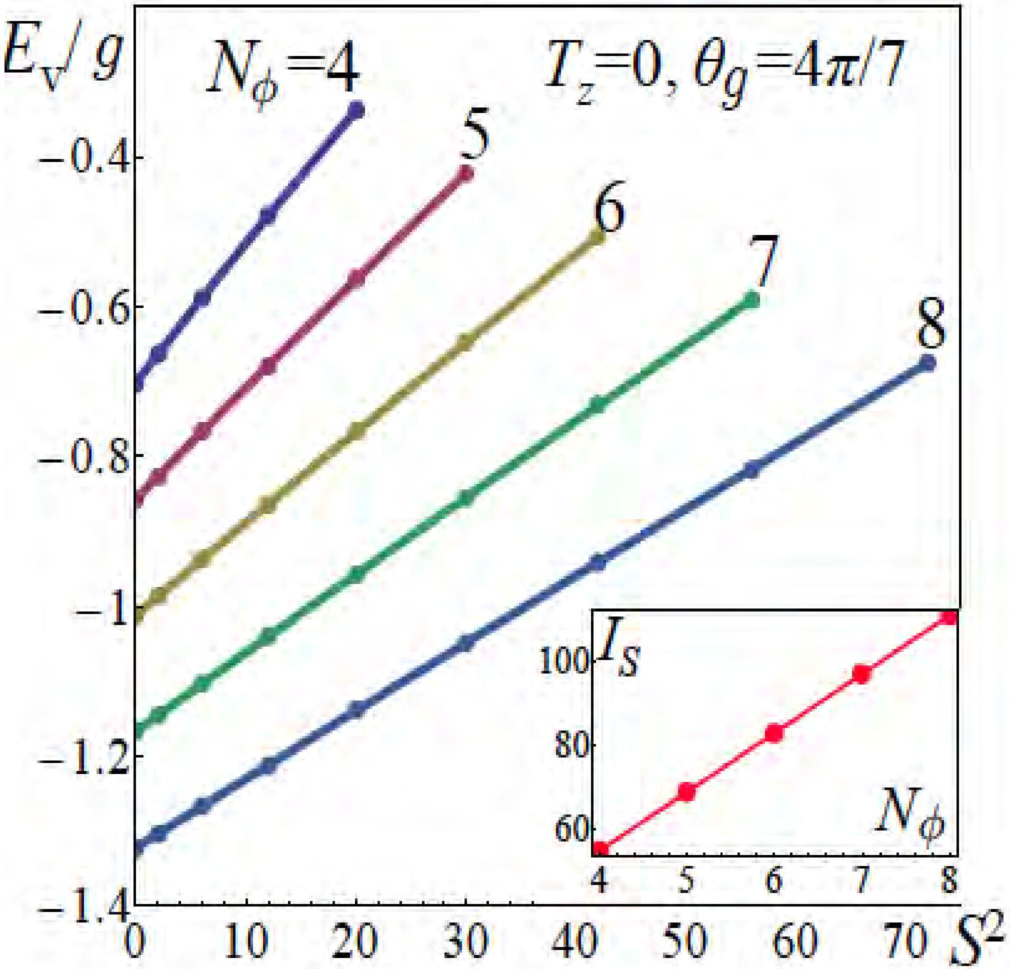
How do we know it is true beyond mean field ??

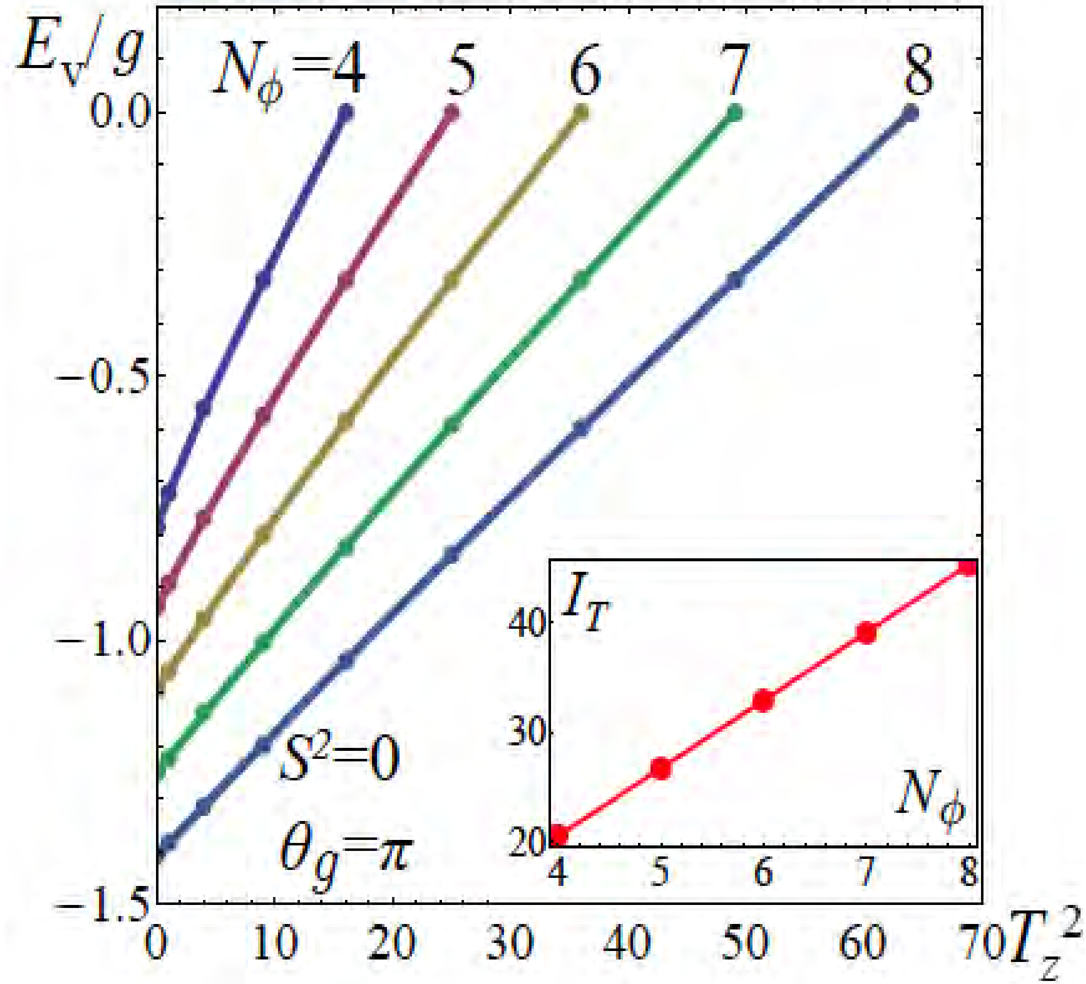


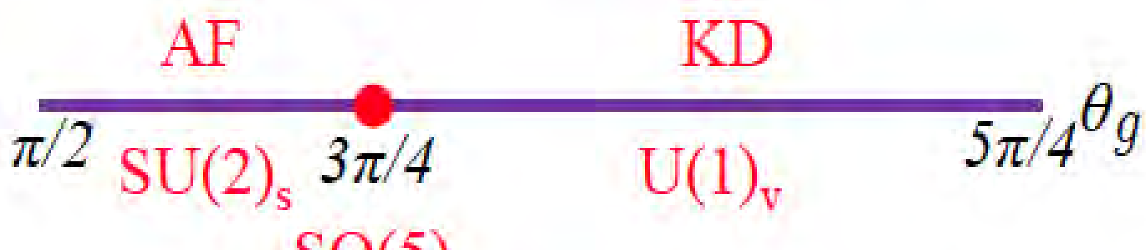


(b)
$S=0$
1
2
3
4
5
6
7
8
$T_z=0$
E_ν/g
3
2
1
0
-1
$\pi/2$
$3\pi/4$
π
$5\pi/4$
θ_g

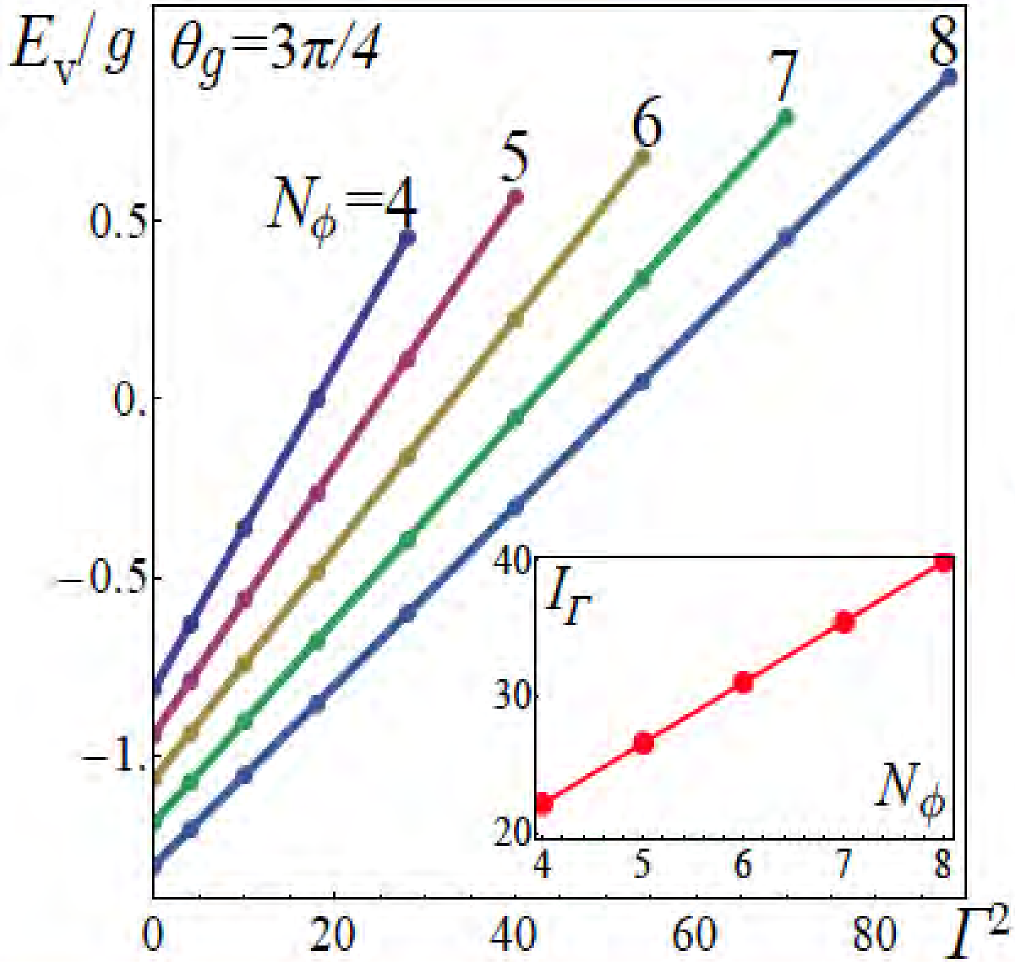
symmetry breaking pattern







Finite size scaling analysis @ SO(5) point



@θ_g=3π/4

$$H_v^{\mathrm{eff}} = u_z\left[\frac{2\Gamma^2}{N_\phi+1} - \frac{N_\phi(N_\phi+5)}{N_\phi+1}\right]$$

Moment of Inertia

- For any finite size system, the ground state is an SO(5) singlet and non-degenerate.
- In the thermodynamic limit, ground states become degenerate, resulting in spontaneous SO(5) symmetry breaking.

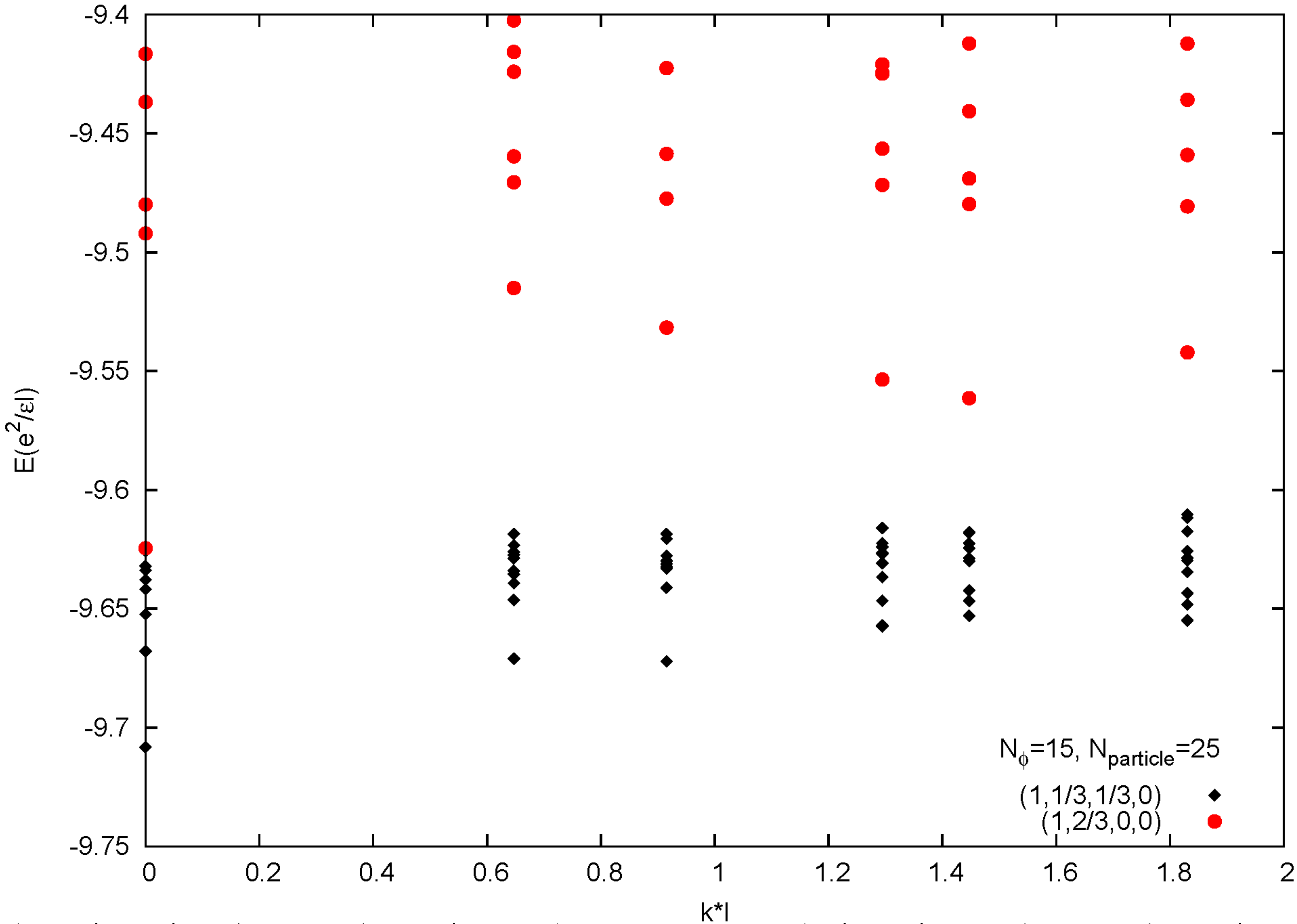
Summary

- Competing phases at graphene neutrality with rich symmetry-breaking pattern
- Anderson's Tower of states signature of symmetry breaking
- SO(5) symmetry relating Kekule and AF states for a realistic Hamiltonian
- MFT is true in this 2+1 system
- All phases CDW, AF, CAF and KD are gapped : not clear yet what is the choice of Nature
- Under way is the study of fractions 5/3 and 4/3

Summary

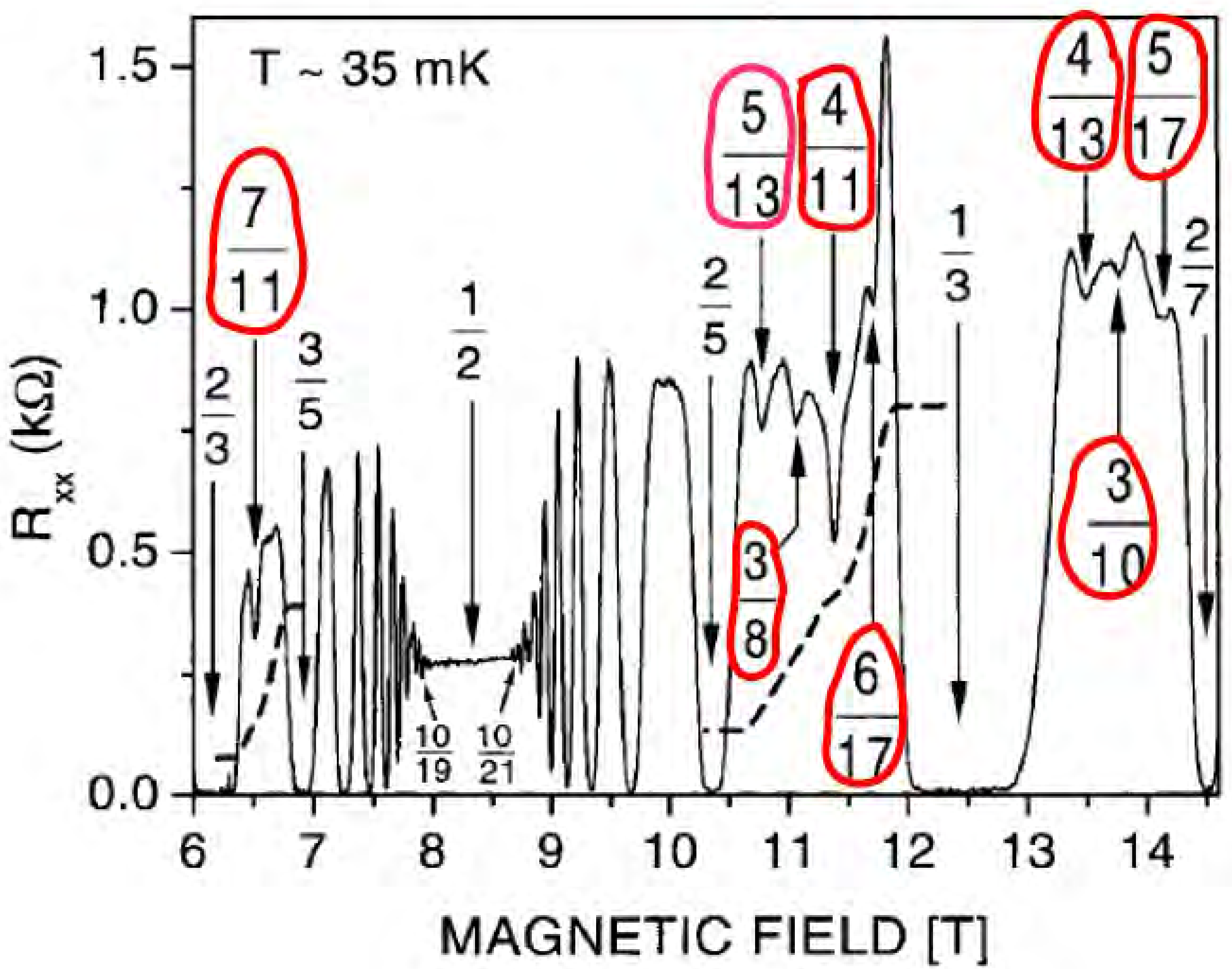
- SO(5) symmetry
- ν=0 quantum hall states
  finite-size effect;
  numerical results agrees with mean-field theory
- fractional filling factors

ν=-1/3 states

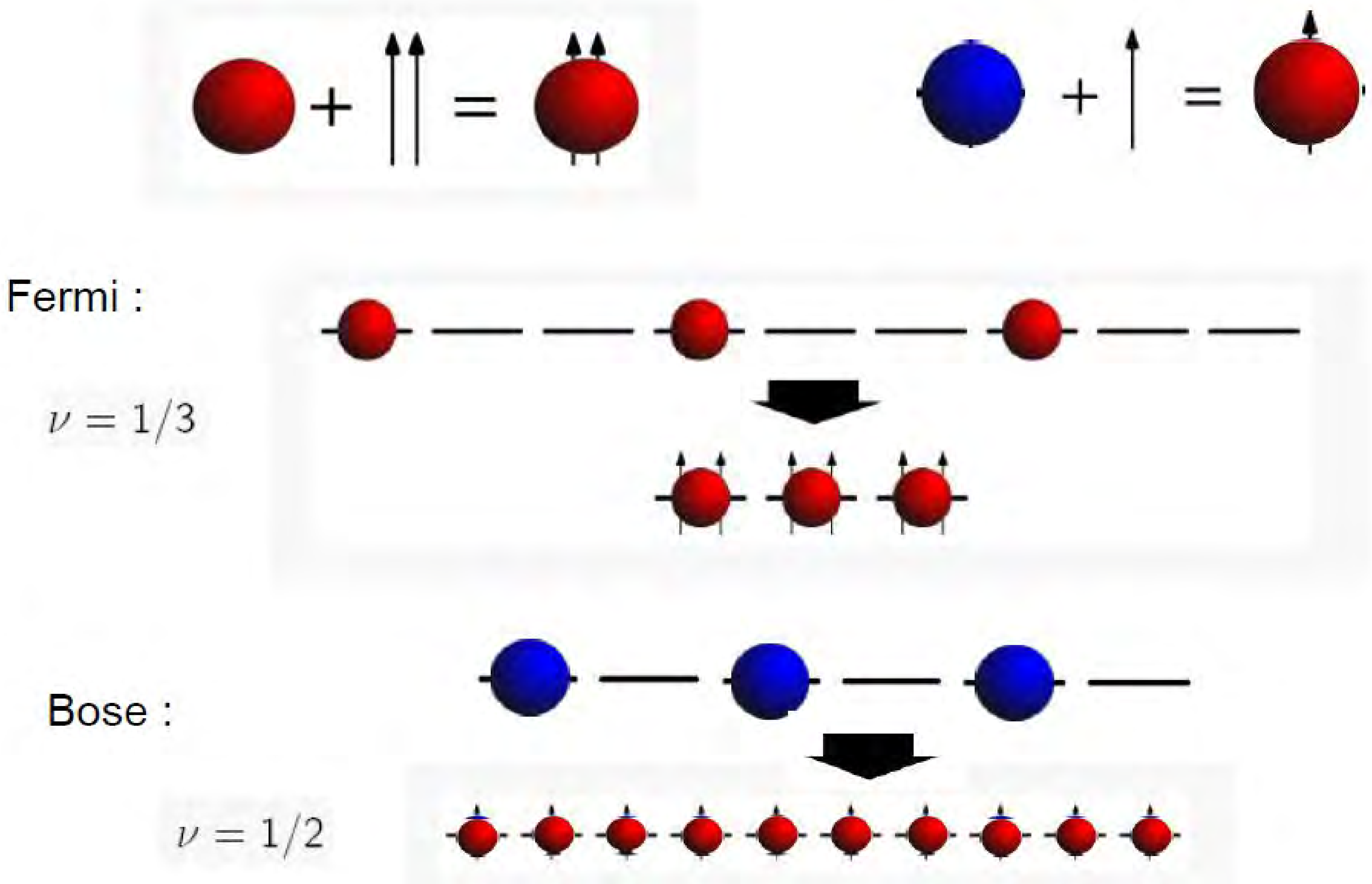


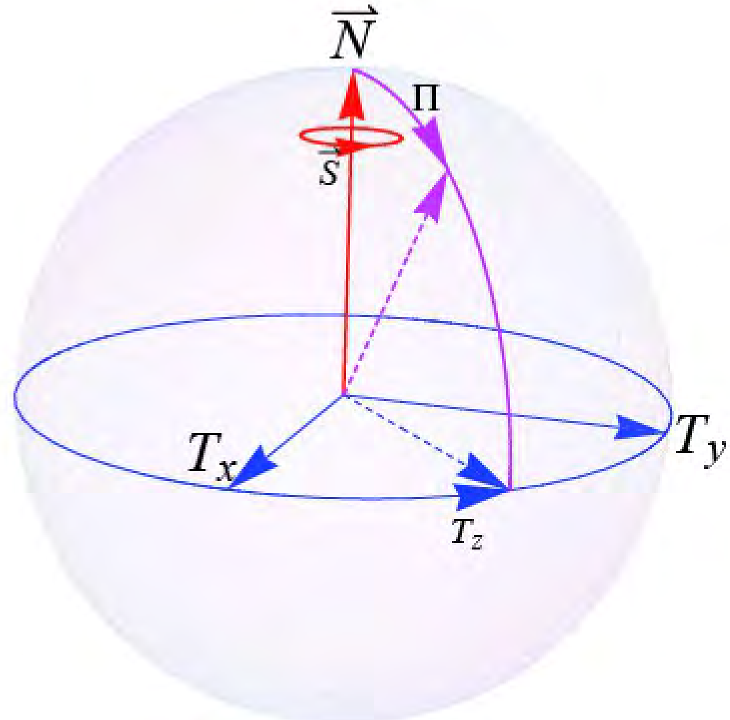
Ec(1,1/3,1/3,0)<Ec (1,2/3,0,0)   Ec(2/3,2/3,0,0)<Ec (1,1/3,0,0)

I. Sodemann and A. H. MacDonald, arXiv:1310.1642.



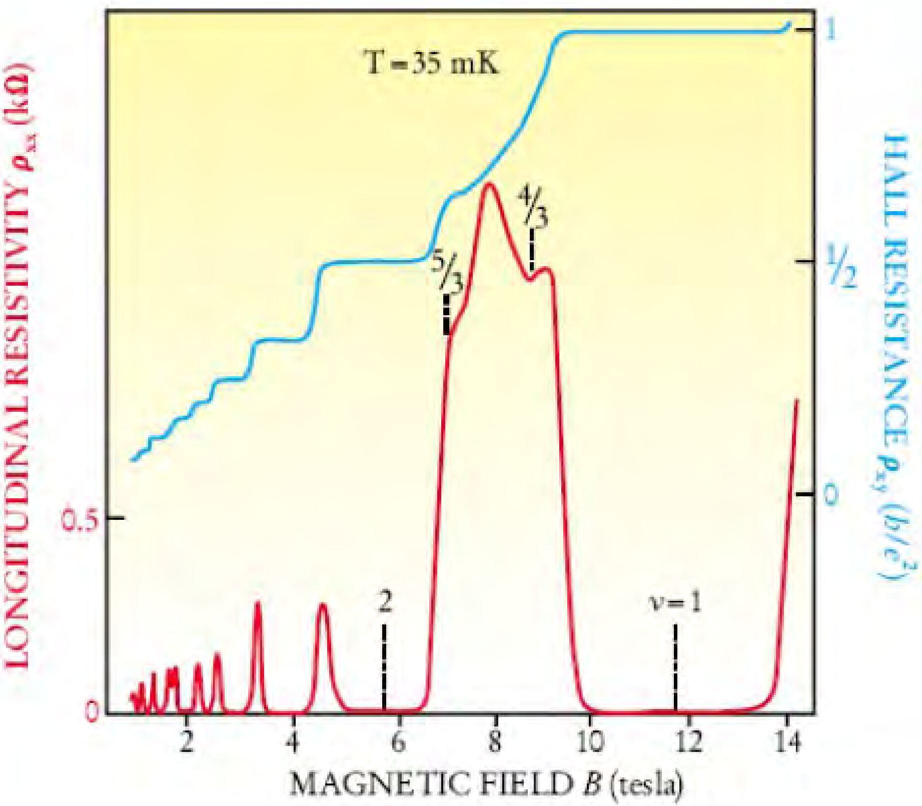
T ~ 35 mK
R_{xx} (kΩ)
MAGNETIC FIELD [T]
2/3
7/11
3/5
1/2
10/19
10/21
2/5
5/13
4/11
3/8
6/17
1/3
4/13
5/17
3/10
2/7


Fermi :
$\nu = 1/3$
Bose :
$\nu = 1/2$

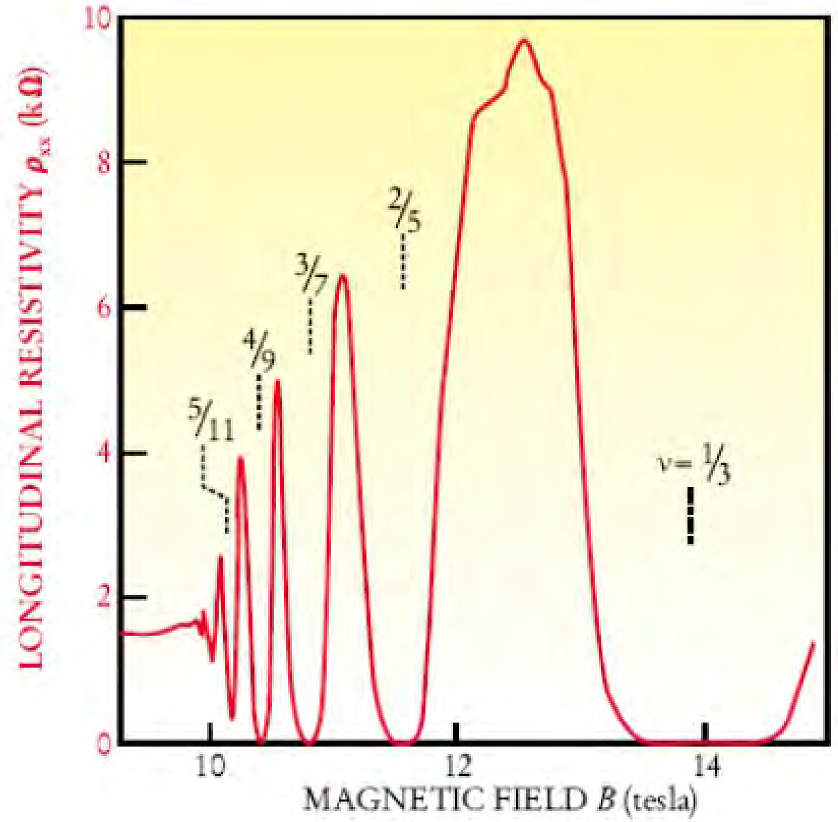


Parameter	Kekulé-distortion state	d-wave state
Order Parameter	(T_x, T_y)	(Δ_x, Δ_y)
U(1) generator	T_z	Charge Q
External Potential	Staggered potential $\epsilon_{\rm v}$	Chemical potential μ

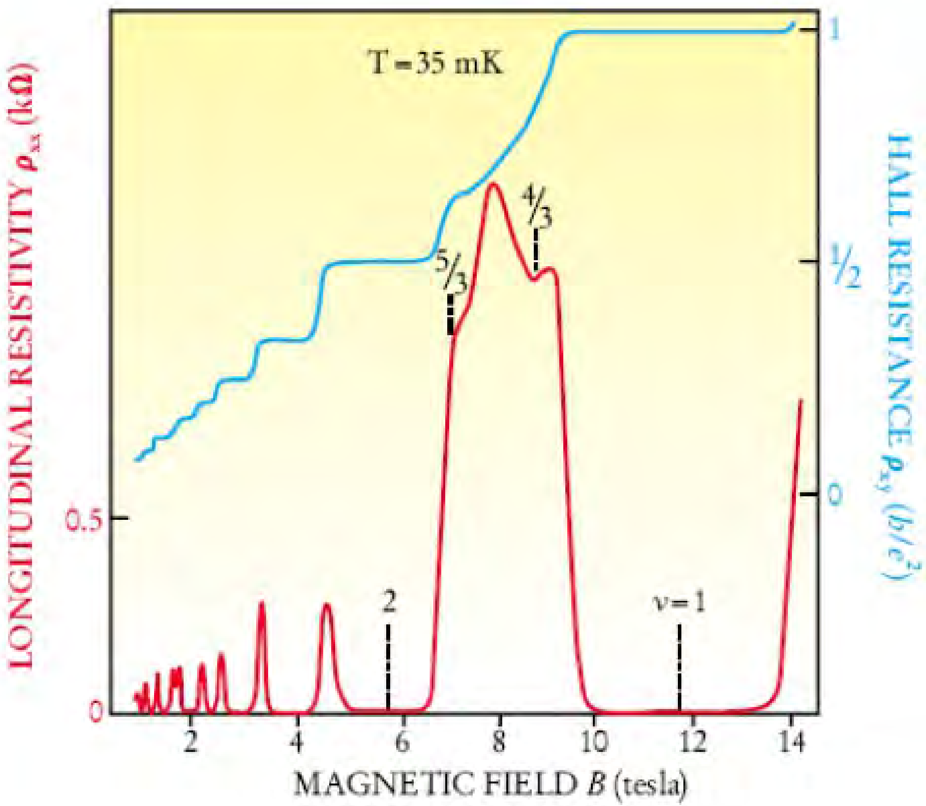
IQHE


T = 35 mK
LONGITUDINAL RESISTIVITY ρ_{xx} (kΩ)
HALL RESISTANCE ρ_{xy} (h/e^2)
MAGNETIC FIELD B (tesla)
0
0.5
1
½
0
2
4
6
8
10
12
14
5/3
4/3
2
ν = 1

FQHE

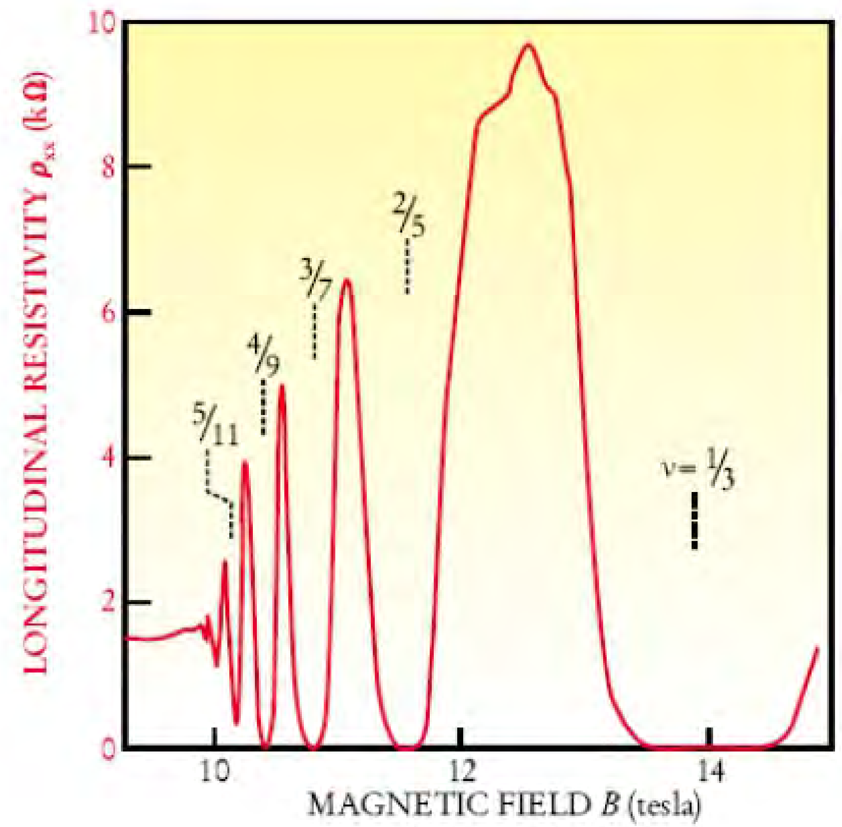

LONGITUDINAL RESISTIVITY ρ_{xx} (kΩ)
MAGNETIC FIELD B (tesla)
0
2
4
6
8
10
10
12
14
5/11
4/9
3/7
2/5
ν = 1/3

IQHE

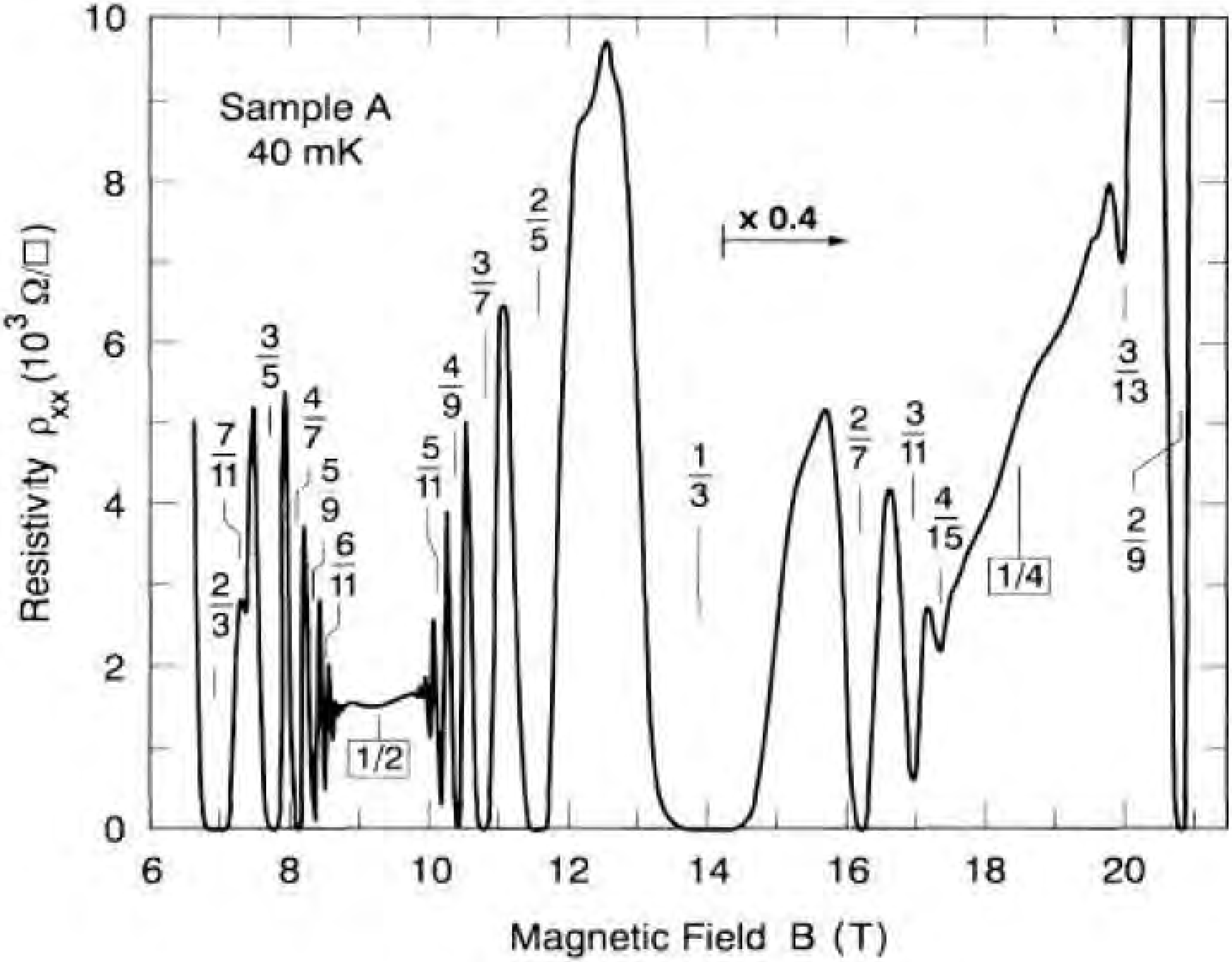


T = 35 mK
LONGITUDINAL RESISTIVITY ρ_{xx} (kΩ)
HALL RESISTANCE ρ_{xy} (h/e^2)
1
½
0
0.5
0
5/3
4/3
2
ν = 1
2
4
6
8
10
12
14
MAGNETIC FIELD B (tesla)

FQHE



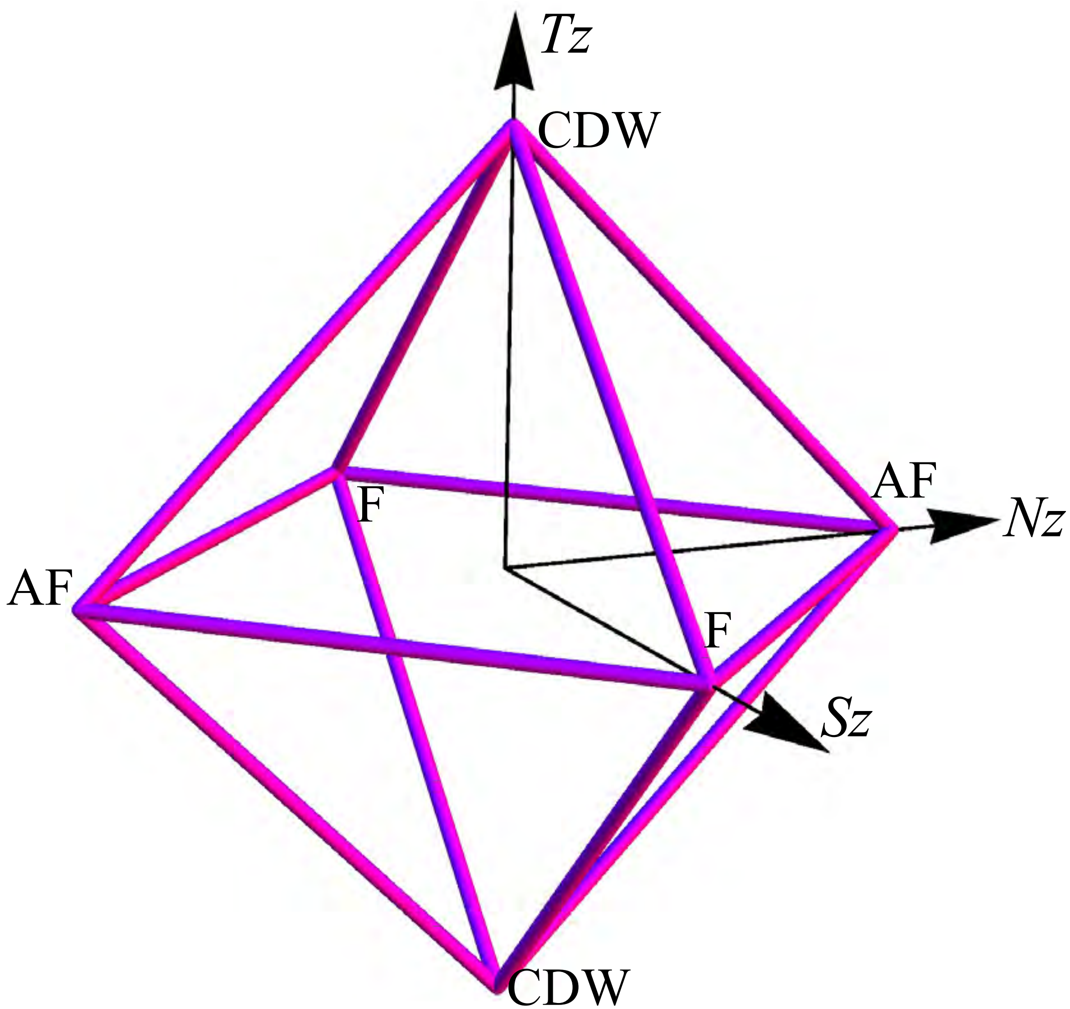
LONGITUDINAL RESISTIVITY ρ_{xx} (kΩ)
10
8
6
4
2
0
2/5
3/7
4/9
5/11
ν = 1/3
10
12
14
MAGNETIC FIELD B (tesla)



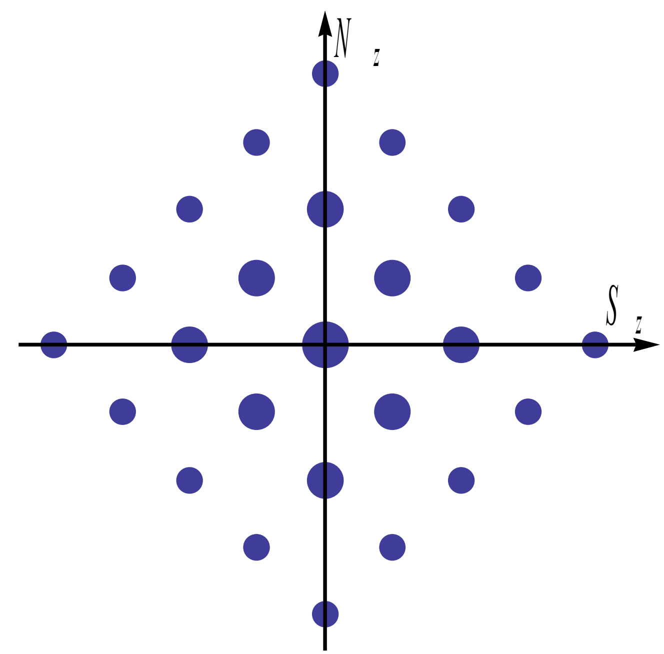
Sample A
40 mK
x 0.4
Resistivity ρ_{xx} ($10^3\,\Omega/\square$)
Magnetic Field B (T)
0
2
4
6
8
10
6
8
10
12
14
16
18
20
2/3
7/11
3/5
4/7
5/9
6/11
1/2
5/11
4/9
3/7
2/5
1/3
2/7
3/11
4/15
1/4
3/13
2/9

ν=0 quantum Hall ferromagnetism

ν=0 Coulomb ground states: 2 spinors are occupied at each LL orbital



Tz-constant plane



SU(4) multiplet structure