The thermoelectric properties of inhomogeneous holographic lattices

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Outline

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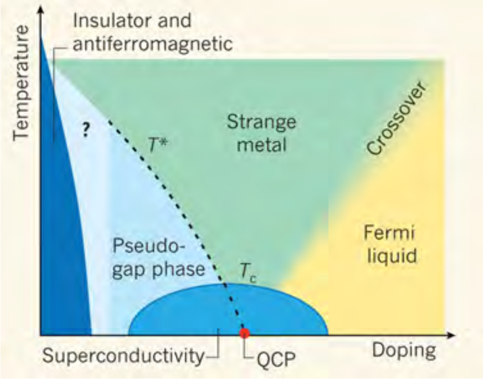
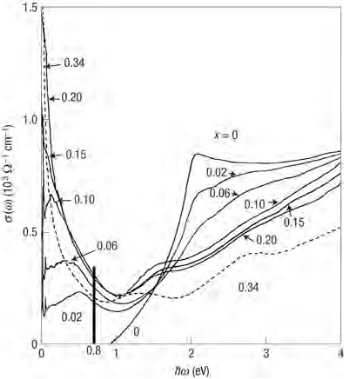
1. Introduction/Motivation

2. Inhomogeneous Lattice in Einstein-Maxwell
   - Background black holes
   - AC/DC Transport

3. Summary

The Cuprates





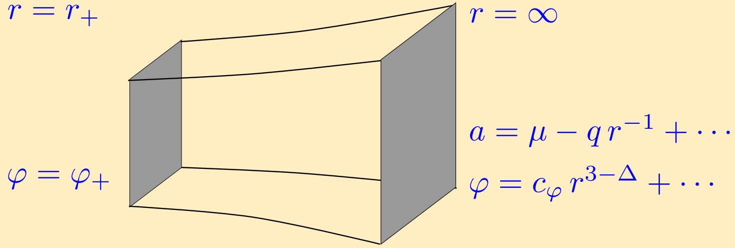
Strong coupling in the Cuprates leads to:

- Interesting phase diagram
- Peculiar transport properties
- Use holographic methods!

Perfect Holographic Conductor

Do it in $D = 4$ Einstein-Maxwell-Dilaton with AdS asymptotics:

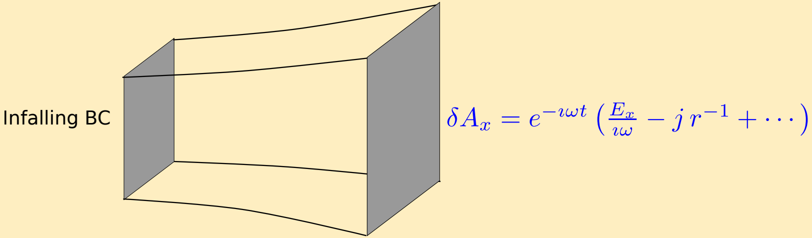
$$\mathcal{L}_{EMD} = R - \frac{Z(\varphi)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial\varphi)^2 - W(\varphi)$$
$$ds_4^2 = -U\, dt^2 + U^{-1}\, dr^2 + e^{2V} \left(dx_1^2 + dx_2^2\right)$$
$$A = a\, dt, \quad \varphi = \varphi(r)$$



Background black hole has temperature T , energy E, pressure P, entropy s and charge q.

Perfect Holographic Conductor

- To calculate conductivity need to source $\delta A_x = -e^{-\imath\omega t}\frac{E_x}{\imath\omega}$ on the boundary
- Momentum (δg_{tx}) couples because of background charge



$$\omega << T \Rightarrow \sigma = j/E_x = \frac{\imath}{\omega}\frac{q^2}{E+P} + \frac{(T\,s)^2}{(E+P)^2}\,Z_+$$

Generalisation of RN result [Hartnoll, Herzog]

Minimal Gauged SUGRAs

There is two interesting gauged SUGRAs with bosonic sectors coming from dim. reductions of string/M-theory [Gauntlett, Varela]

- $N = 2$, $D = 4$ just Einstein-Maxwell

$$\mathcal{L} = \sqrt{-g}\,\left(R + 6 - \frac{1}{4}\,F^2\right)$$

- $N = 1$, $D = 5$ just Einstein-Maxwell with CS coupling

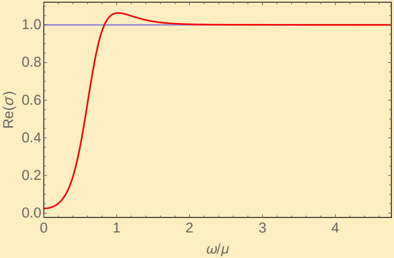
$$\mathcal{L} = *R + *12 - \frac{1}{2}\,F \wedge *F - \frac{1}{3^{\frac{3}{2}}}\,A \wedge F \wedge F$$

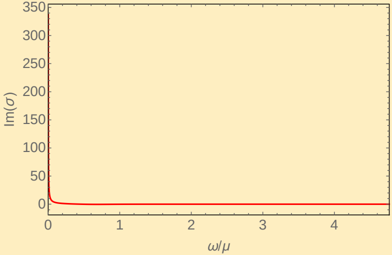
- Chemical potential from $U(1)_R$ charge

Stick with $D = 4$ Einstein-Maxwell for now. CS term will make a difference [AD, Hartnoll]

AdS-RN black hole

AC conductivity around RN black hole





[Hartnoll, Herzog]

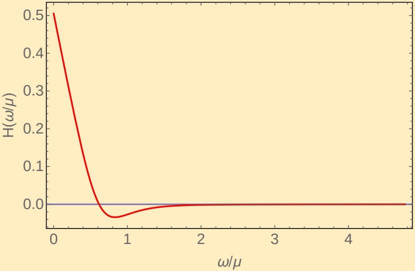
- Recover analytic expression for $\omega << \mu$
- For $\omega >> \mu$ asymptotes to $\sigma_{CFT} = 1$

Sum Rules

- Conductivity is $\sigma = \frac{G(\omega)}{\imath\omega}$
- Causality implies G analytic for $\mathrm{Im}\,\omega \geq 0$
- Charge redistributes spectral weight

$$\mathrm{Re}\,G\,(0) - \mathrm{Re}\,G_{CFT}\,(0) = -\frac{2\,\mu}{\pi}\int_{0^+}^{\infty}\mathrm{Re}\,(\sigma - \sigma_{CFT})$$

$$H(y) = \int_0^y \left[\frac{\pi}{\mu}\,\frac{q^2}{E+T^{xx}}\,\delta\left(\frac{\omega}{\mu}\right) + \mathrm{Re}\sigma - \mathrm{Re}\sigma_{CFT}\right]$$



Holographic Lattice

To add momentum dissipation introduce a UV benign lattice:

- UV relevant deformation $\mathcal{O}(x)$ with period L
- To be "Drude" at low T the IR operator should be irrelevant
- Solve the PDEs and show that the IR remained the same
- Charge density is a universal relevant operator $\Rightarrow$ Impose $A_t = \mu(x) - J^t(x)\, r^{-1} + \cdots$
  [Hartnoll, Hofman][Horowitz, Santos, Tong]

$$\mu(x) = \mu_0 + A(x), \quad \langle A \rangle_L = 0$$

- $\mu_0 \Rightarrow$ chemical potential, $A'(x) \Rightarrow$ periodic electric field

Inhomogeneity

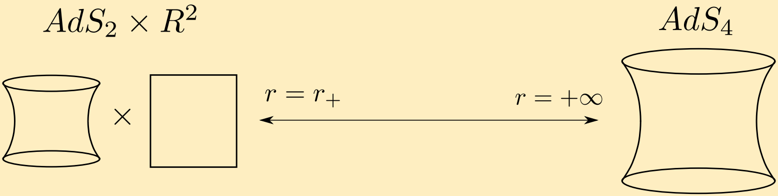
Why bother with inhomogeneous lattices/impurities?:

- Deform the theory by a single mode $\phi(x) \sim \lambda\, A(r)\, \cos(k\, x)$ of a relevant operator
- Higher harmonics sourced due to non-linearities but suppressed
- Q-lattices (also helical) fine-tuned situation where higher modes consistently drop out
  - $\rightarrow$ Momentum dissipation
  - $\rightarrow$ Metal - Insulator transitions [Iizuka, Kachru, Kundu, Narayan, Sircar, Trivedi] [AD, Hartnoll], [AD,Gauntlett], [Andrade, Withers], [Gouteraux], [AD, Gouteraux, Kiritsis], [AD, Blake]
- Sourcing higher modes does have impact on mid-IR physics.
- Necessary in disordered systems [Arean, Farahi, Pando Zayas, Landea, Scardicchio] [Lucas, Sachdev, Schalm] [Hartnoll, Santos]

Outline

The (perturbative) ionic lattice

At $T = 0$ the solution is a domain wall



Add a lattice perturbation on top:

$$\delta g^{\mu}{}_{\nu}(r, x) = \varepsilon\, \delta h^{\mu}{}_{\nu}(r) \cos(k\, x)\,, \quad (\mu, \nu) = \{tt,\, xx,\, yy\}$$
$$\delta A_t(r, x) = \varepsilon\, \delta a_t(r) \cos(k\, x)\,, \quad \varepsilon << \mu$$

- Horizon + Infinity are singular points for the ODEs

$\Rightarrow$ Find converging modes close to the singular points

! Back reacting small perturbations leads to Floppy Horizons [Hartnoll, Santos]

The (non-perturbative) ionic lattice

$$ds_4^2 = -U\, H_{tt}\, dt^2 + \frac{H_{rr}}{U}\, dr^2 + \Sigma\, \left[e^{B}\, (dx + W\, dr)^2 + e^{-B}\, dy^2\right]$$

$$A = a_t\, dt$$

- Plan is to use DeTurck's method
  [Headrick, Kitchen, Wiseman][Figueras, Lucietti, Wiseman]
- Place the horizon at $r = 0$. Regularity + Killing implies

$$U\,(r) = 4\pi\, T\, r + \ldots, \quad a_t(r,x) = a_t^{(0)}\,(x)\; r + \ldots$$

$$H_{tt}(r,x) = H_{tt}^{(0)}\,(x) + \ldots, \quad H_{rr}(r,x) = H_{tt}^{(0)}\,(x) + \ldots$$

$$\Sigma(r,x) = \Sigma^{(0)}\,(x) + \ldots, \quad B(r,x) = B^{(0)}\,(x) + \ldots$$

AC/DC Transport properties

Need to introduce sources to study transport

- AC transport:
- Electric Field: $\delta A_x \sim -\imath\, E\, \omega^{-1}\, e^{-\imath\omega\, t}$
  [Horowitz, Santos, Tong]

$$\delta g_{tt},\, \delta g_{tx},\, \delta g_{xx},\, \delta g_{yy},\, \delta A_t,\, \delta A_x$$

$$\sigma\,(\omega) = (\imath\,\omega)^{-1}\; G_{x,x}\,(\omega)$$

- DC transport:
  [Iqbal, Liu][Davison][Blake, Tong][Blake, Tong, Vegh][Andrade, Withers]
- Electric field: $\delta A_x \sim -E\, t$
- Temperature gradient: $\delta g_{tx} \sim -r^2\, \zeta\, t$, $\delta A_x \sim \mu\,(x)\; \zeta\, t$
  where $\zeta = \nabla_x T/T$

DC Transport

Introduce time dependent perturbation

$$\delta ds^2 = \delta g_{\mu\nu}(r,x)\, dx^\mu\, dx^\nu$$
$$\delta A = \delta a_\mu(r,x)\, dx^\mu - Et\, dx$$

There is a Killing vector $k = \partial_t$

$$\partial_\mu\left(\sqrt{-g}\, F^{\mu\nu}\right) = 0$$
$$\partial_\mu\left(\sqrt{-g}\, G^{\mu\nu}\right) = 3\, k^\nu$$

for

$$G^{\mu\nu} = \nabla^\mu k^\nu + \frac{1}{2} k^{[\mu} F^{\nu]\sigma} A_\sigma + \frac{1}{4}(\psi - 2\theta) F^{\mu\nu}$$
$$L_k A = d\psi, \quad i_k F = d\theta$$

DC Transport

Define

$$J = \sqrt{-g}\, F^{rx}, \quad Q = 2\sqrt{-g}\, G^{rx}$$

- Use previous to show $\partial_\mu J = 0$, $\partial_\mu Q = 0$
- Fall offs at infinity give J is the electric current and

  $$Q = T^{tx} - \mu\, J$$

  is the heat current.
- Constant in x is Ward identities
- Constant in r is used to relate J, Q to E

The same quantities constant when introducing a T gradient on the boundary

DC Transport

In the end:

$$\sigma = \frac{1}{C} + \frac{M^2}{X\,C}, \quad \bar{\alpha} = \alpha = 4\pi\frac{M}{X},$$
$$\bar{\kappa} = \frac{(4\pi)^2 T\,C}{X}, \quad \kappa = \frac{(4\pi)^2 T\,C}{X + M^2}$$

where

$$X = \left(\int e^{B^{(0)}}\right)\left(\int e^{B^{(0)}}\left(\frac{a_t^{(0)}}{H_{tt}^{(0)}}\right)^2\right) - \left(\int e^{B^{(0)}}\frac{a_t^{(0)}}{H_{tt}^{(0)}}\right)^2$$
$$+\,C\Upsilon$$
$$C = \int e^{B^{(0)}}, \quad M = \int e^{B^{(0)}}\frac{a_t^{(0)}}{H_{tt}^{(0)}}, \quad \Upsilon = \int \frac{1}{\Sigma^{(0)}}\left[\partial_x \ln\frac{e^{B^{(0)}}}{\Sigma^{(0)}}\right]^2$$

Similar structure with homogeneous lattices but not quite the same

DC Transport

- At $T >> \mu$

$$\sigma = 1 + \frac{(\int \mu)^2}{\int \mu^2 - (\int \mu)^2}\,, \qquad \alpha = \frac{(4\pi)^2}{3} \frac{\int \mu}{\int \mu^2 - (\int \mu)^2}\, T$$

$$\bar{\kappa} = \frac{(4\pi)^4}{9} \frac{1}{\int \mu^2 - (\int \mu)^2}\, T^3, \qquad \kappa = \frac{(4\pi)^4}{9} \frac{1}{\int \mu^2}\, T^3$$

- At $T << \mu$ and assuming AdS_2

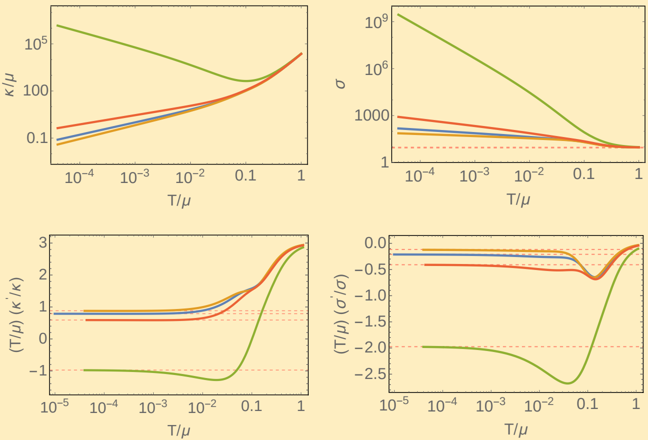
$$\sigma \sim T^{-2\delta_-(k/\lambda)}, \quad \alpha \sim T^{-2\delta_-(k/\lambda)}, \quad \bar{\kappa} \sim T^{1-2\delta_-(k/\lambda)}$$

$$\kappa \sim T$$

Agrees with memory matrix formalism
[Hartnoll, Hofman]

$$\delta_-(k) = \frac{1}{2}\left(-1 + \sqrt{5 + 8\,\frac{k^2}{\mu^2} - \sqrt{1 + 4\,\frac{k^2}{\mu^2}}}\right)$$

DC Transport



- Fixed $\mu(x)/\mu = 1 + \frac{1}{2}\cos(k\,x)$
- Different $k/\left(\sqrt{2}\,\mu\right) = 1/3,\ 2/5,\ 1/2, 1$
- Agrees with AdS_2 scalings at $T << \mu$. Tension with Floppy horizons [Hartnoll, Santos]

DC Transport

Examine Wiedemann-Franz type of laws

- Form the Lorentz ratio $\bar{L} = \frac{\bar{\kappa}}{\sigma T}$. At low temps or small lattices

$$\bar{L} \to \frac{s^2}{q^2}$$

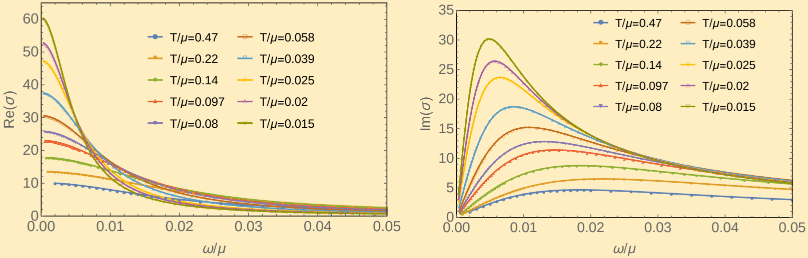
  Similar relation from memory matrix formalism [Mahajan, Barkeshli, Hartnoll]

- Interesting to also examine

$$\frac{\bar{\kappa}}{\alpha} = \frac{4\pi T \int e^{B^{(0)}}}{\int e^{B^{(0)}} \frac{a_t^{(0)}}{H_{tt}^{(0)}}}$$

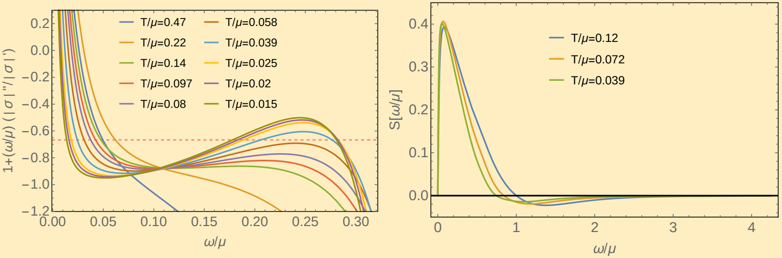
- Different from homogeneous lattices but the same for small temperature or lattice $\frac{Ts}{q}$

AC Transport



- Fix $\mu(x)/\mu = 1 + \frac{1}{2}\cos(k\,x)$ with $k/\mu = 2^{-1/2}$
- At low frequencies $\omega < T$ there is now a "Drude" peak [Horowitz, Santos, Tong]

AC Transport



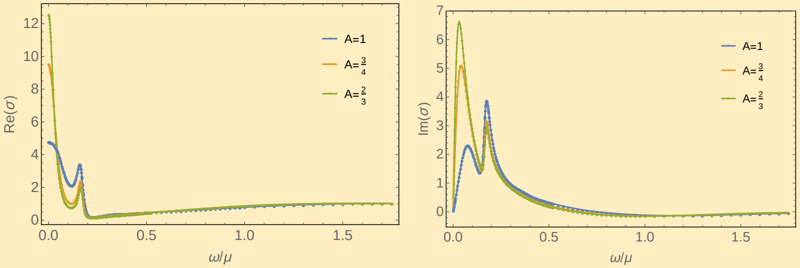
- Plot $1 + \frac{\omega}{\mu}\frac{\sigma''}{\sigma'}$ but no clear sign of scaling laws
- Sum rule seems to work

$$S(y) = \int_0^y (\mathrm{Re}\sigma - \mathrm{Re}\sigma_{CFT})$$

Lattice resolves the delta function at the origin

AC Transport

- Deform by $\mu(x)/\mu = 1 + A\cos(k\,x)$ for fixed $k/\mu = \left(3\sqrt{2}\right)^{-1}$



- CFT result for $\omega >> k, T$
- Large lattices shift more spectral weight to mid-infrared
- Mid-infrared peak

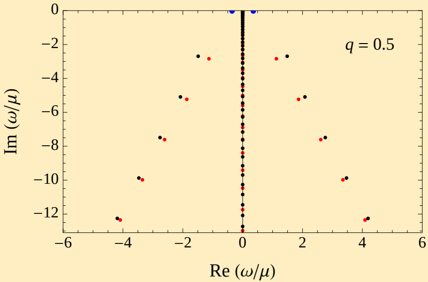
  [Horowitz, Santos, Tong]

AC Transport

- At linearised level, background lattice couples current to longitudinal modes of fluctuations of undeformed RN bh

$$\delta g_{tt},\ \delta g_{tx},\ \delta g_{xx},\ \delta g_{yy},\ \delta A_t,\ \delta A_x$$

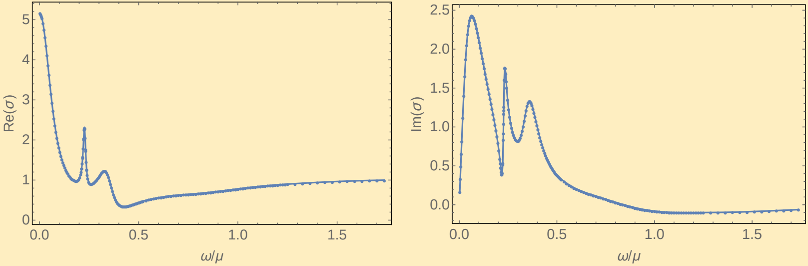
- Sound mode of RN bh lies in this sector
[Policastro, Son, Starinets][Edalati, Jottar, Leigh] [Davison, Kaplis]



- Sound mode has $\mathrm{Re}(\omega) \sim \frac{1}{\sqrt{2}}\,k \to$ Peak in σ at $\omega \sim k/\sqrt{2}$

AC Transport

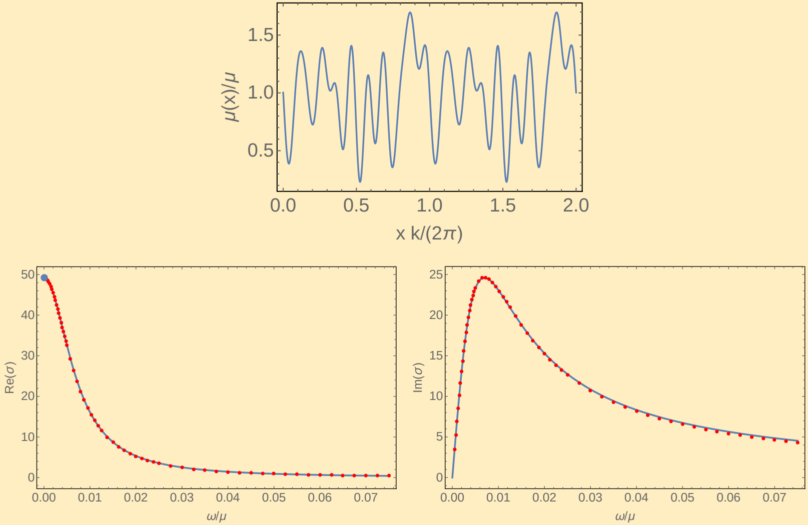
- Deform by higher harmonics
  $\mu(x)/\mu = 1 + A\,\cos(k\,x) + B\,\cos(2\,k\,x)$



- Second peak appears

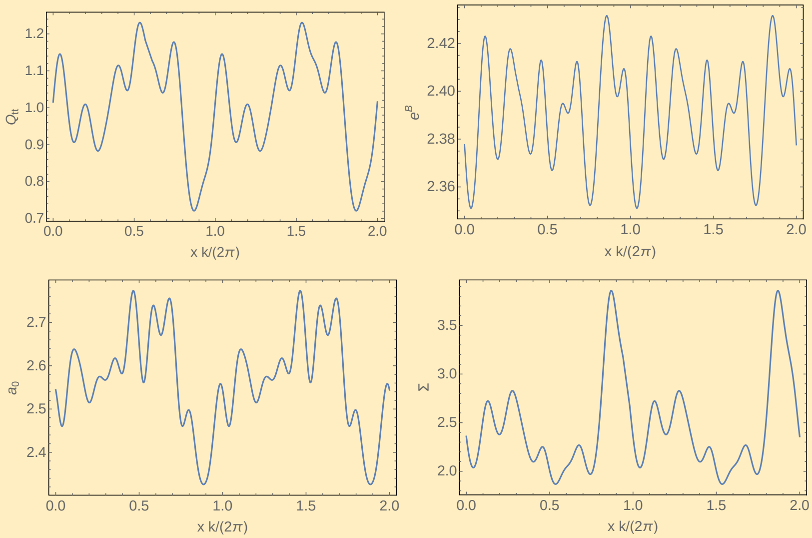
$\Rightarrow$ Sourcing higher modes affects the mid-infrared. How?

AC Transport



- Consider a "dirty lattice"
- Small frequency regime $\omega < T < k, A$ gives a "Drude" peak

AC Transport



- Used horizon data to find σ_{DC}

Outline

Summary

- Examined low temperature behaviour of lattice deformed RN black hole
- Showed analytic expressions of transport coefficients in terms of bh horizon data
- Revisited intermediate scaling laws in the optical conductivity
- Connection between sourced lattice modes and optical conductivity peaks