QFT, String Theory & Cond-Mat Physics

Crete, September 2014

defect CFT and M-theory

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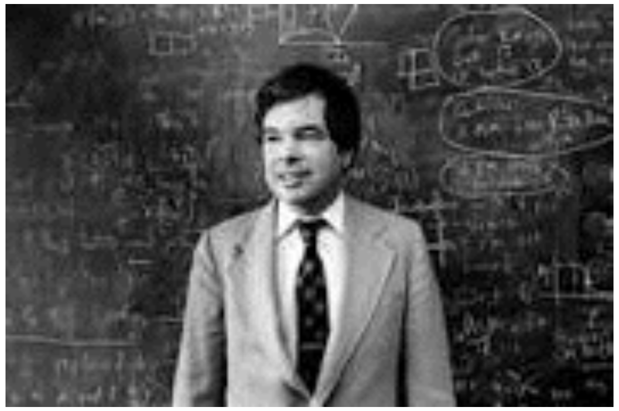
put in context

In the 70's there was a spectacular convergence of interests

of **particle theorists** and **condensed-matter physicists,**

thanks to Quantum Field Theory and the Renormalization group:



– **the Standard Model is a renormalizable QFT**

(after much 'wandering': Fermi theory, quark models, analytic S-matrix …)

– **Universality put some order in the zoo of phenomena of condensed-matter systems**

A new convergence 40 years later ?

Quantum Gravity (String Theory) led to new tools & phenomena:

— black hole evaporation

— dualities, D-branes, holography, non-Lagrangian QFTs

Makes it possible to analyze a wealth of strong-coupling phenomena, in particular in 3 dimensions

But are these realized in condensed-matter systems?

traditional perspectives ....

cond-mat: **top-down**

electron gas

↓

??? IR

hep: **bottom-up**

UV ???

↑

Standard Model

in holography the story is blurred :

$cond-mat: **bottom-up**

UV ???

AdS BHs

Is there a field theory ?
Does it describe real systems ?

Use gravity to learn about strongly-coupled electrons

$hep: **top-down**

string theory

↓

???   IR

Where do (susy gauge) QFTs flow to ?

Use QFT to learn about quantum gravity ?

Shep: somewhere in middle

??? **UV**

M theory

??? **IR**

$$-\mathcal{L}_{(11)} = R + \frac{1}{2 \cdot 4!} F^2 + \frac{1}{6} A \wedge F \wedge F$$

both IR and UV QFTs mysterious

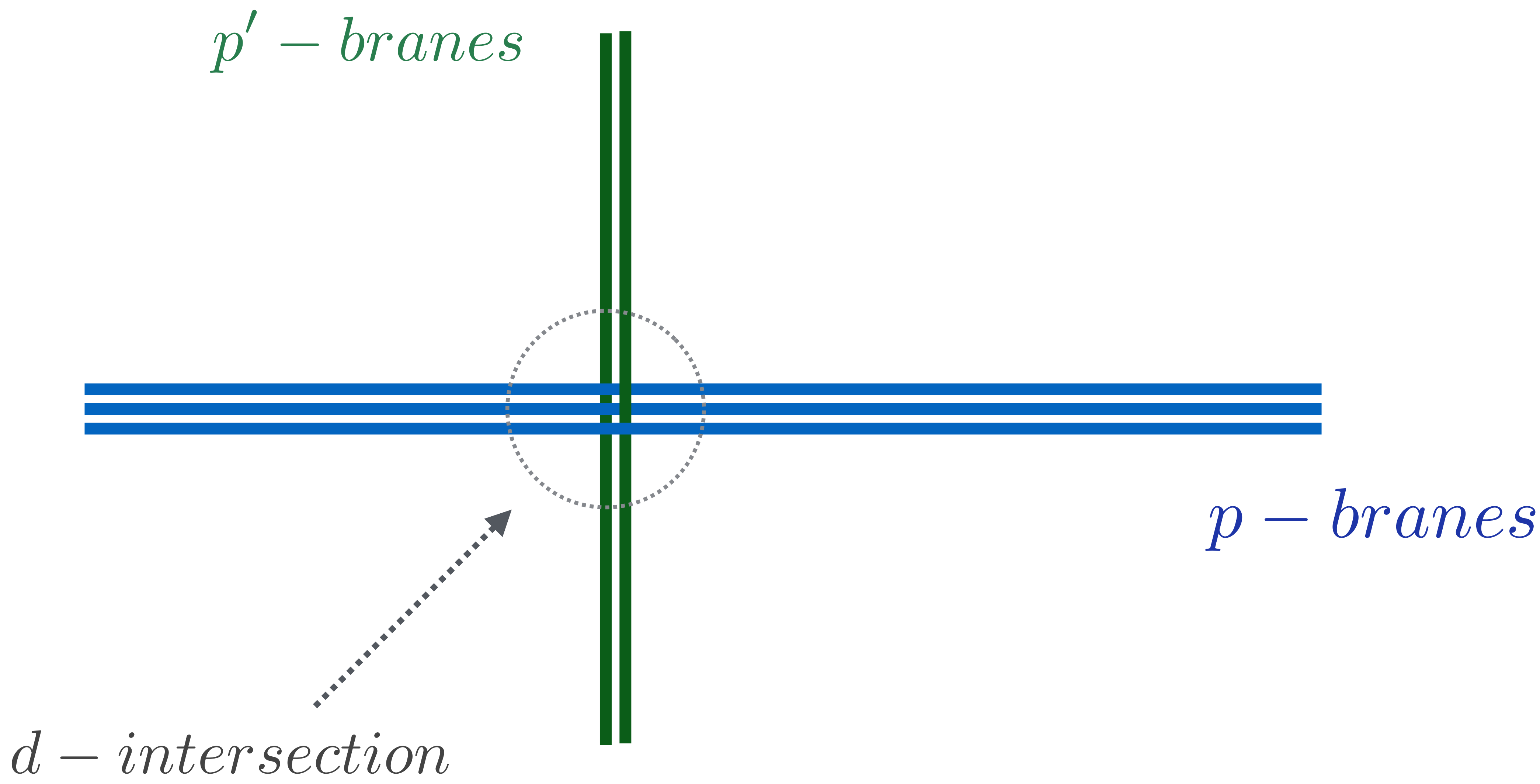
— The QFTs describe collective modes of extended solitons/**branes**

— The near-horizon AdS describes the CFT at the IR fixed point

Basic critical theories with maximal susy:

	d=4, N=4 SYM	$AdS_5 \times S_5$	(IIB) D3-branes
	d=3, N>5 ABJM	$AdS_4 \times S_7/Z_k$	M2-branes
?	d=6, N=(0,2) CFT	$AdS_7 \times S_4$	M5-branes

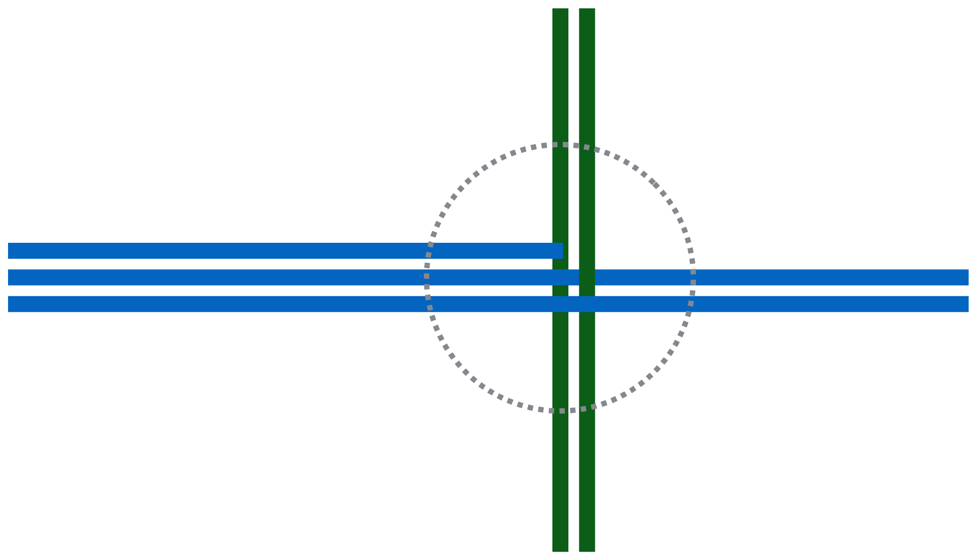
To reduce supersymmetry must consider **intersecting branes:**



$$(d+1) - dim \quad \text{defect} \quad in \ \ (p+1) - dim \ \ CFT$$

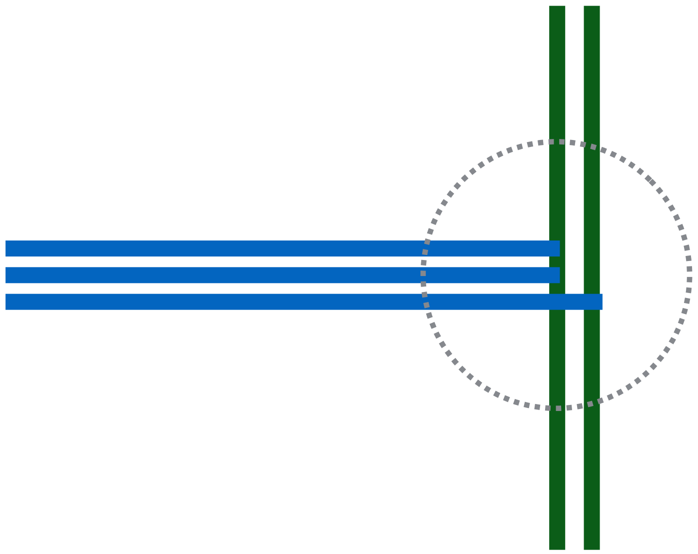
$$\text{or} \quad in \ \ (p'+1) - dim \ \ CFT$$

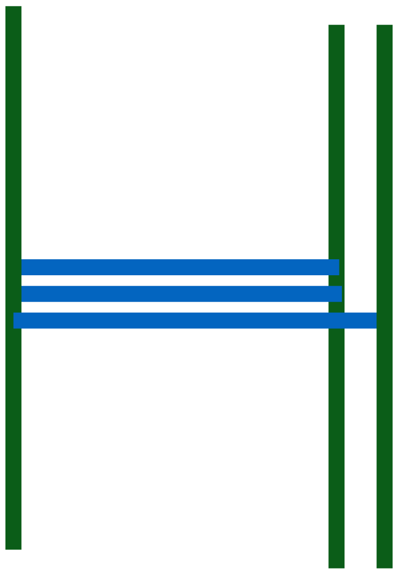
Special case: $p = d+1$, & branes ending on branes:



(1) **'domain-wall' defect**

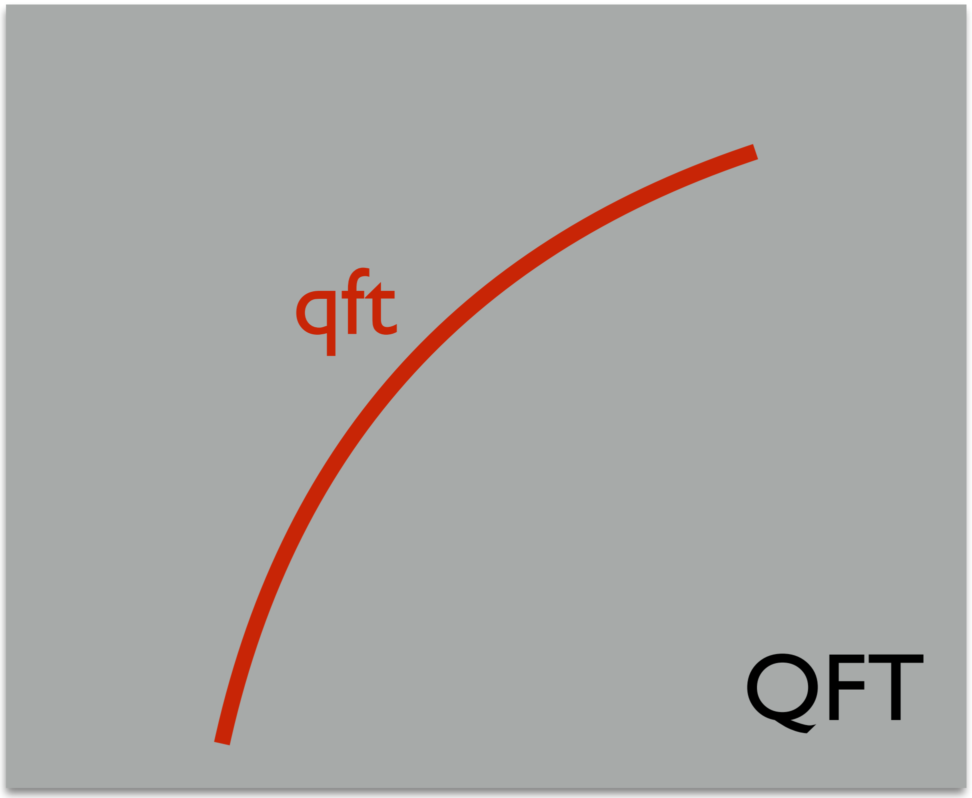
(2) **boundary**





(3) **(d+1)-dim CFT**

Defect QFTs have rich coupled dynamics, RG flows & fixed points.



e.g **Kondo, external quark …..**

Gravity duals often treated approximately:

— **probe approximation** (AdS probe brane in AdS bulk)

— (partial) **smearing** (restore translation symmetry)

In past five years, progress in obtaining fully **back-reacting, localized** solutions, describing strongly-coupled fixed points.

Work contained in hep-th/1103.2800, 1106.4253, 1210.2590 & **hep-th/1312.5477,** on-going

my collaborators:

Benjamin **Assel**

Eric **D'Hoker**

(not sleepy!)
John **Estes**

Jaume **Gomis**

Darya **Krym**

to summarize:

— I will describe some **1/2-maximal susy solutions of 11d** (and 10d, IIB) **supergravity**, dual to critical dCFTs

— **Strongly-coupled 3D critical theories**; but far from any ‘real’ cond-mat systems (though little closer than ABJM or N=4 SYM)

— **Simplest intersections of basic M2 & M5 branes:** learn about most mysterious d=6 SCFT

REST OF THE TALK

1. **General setup**

2. **Interfaces for N=4 SYM**

3. **Conformal limits & holography**

4. **The M2 – M5 – M5' system**

5. **Global solutions & questions**

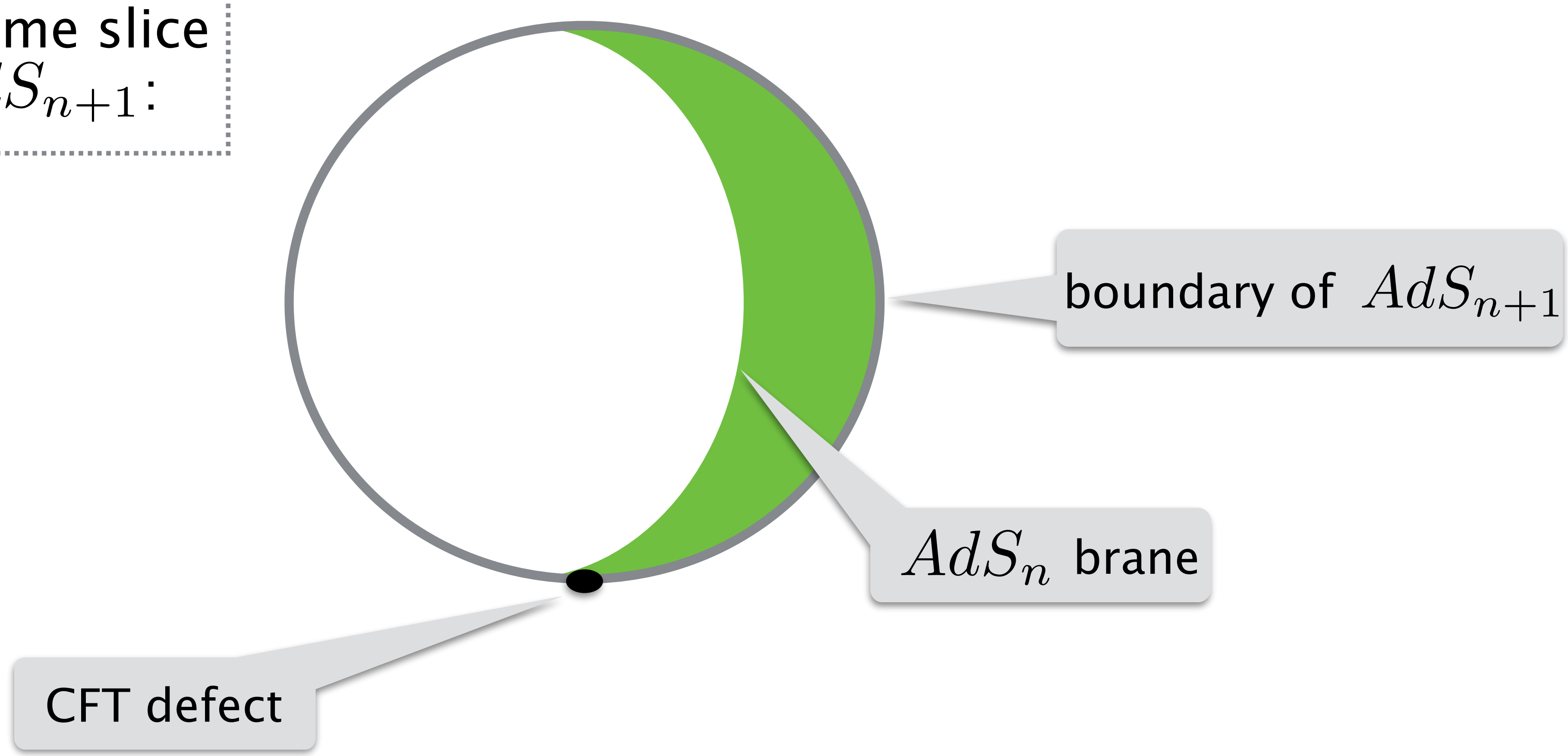
1. General setup

Will (mainly) consider conformal codimension-1 defects

◆ Their holographic duals are AdS_{n+1} branes in AdS_n bulk. In the 'thin-brane' approximation:

Karch-Randall '00

fixed-time slice of AdS_{n+1}:

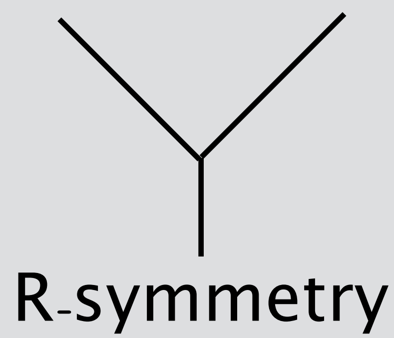


— The boundary CFT is one of the maximally susy theories

— In the full solution, the thin brane is replaced by a smooth configuration, with the same underlying symmetry:

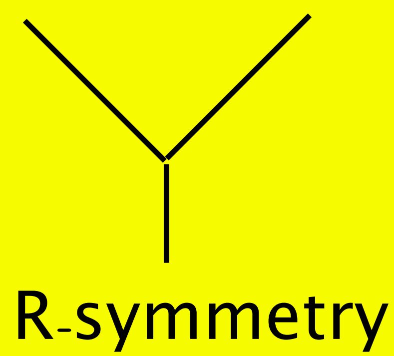
SCFT3 coupled to d=4 SYM

$$SO(2,3) \times SO(3) \times SO(3) \subset OSp(2,2|4)$$



SCFT2 coupled to d=3 ABJM

$$SO(2,2) \times SO(4) \times SO(4) \subset D(2,1;\gamma) \oplus D(2,1;\gamma)$$



This large symmetry is encoded by the ansatz:

d=10 IIB supergravity: $(AdS_4 \times S^2 \times S^2) \times_w \Sigma$

Riemann surface

d=11 supergravity: $(AdS_3 \times S^3 \times S^3) \times_w \Sigma$

Thus the (Killing-spinor) equations reduce to PDEs on a d=2 Riemann surface.

The **general local solution** of these equations was derived

— for **type-IIB sugra**, in a series of beautiful papers by

D'Hoker, Estes & Gutperle '08

(based also on earlier work by Gomis & Romelsberger; Lunin)

Everything depends on two harmonic functions h_1, h_2

— for **11d sugra** by

D'Hoker, Estes, Gutperle & Krym '09

Estes, Feldman & Krym '12

CB, D'Hoker, Estes & Krym '13

Everything depends on a harmonic function h , and a complex function G obeying: $h\,\partial G = \mathrm{Re}(G)\,\partial h$

for example, in type-IIB:

metric :

$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{\mathrm{S}^2_1} + f_2^2 ds^2_{\mathrm{S}^2_2} + 4\rho^2 dz d\bar{z} \ ,$$

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} \ , \quad f_1^8 = 16\, h_1^8 \frac{N_2 W^2}{N_1^3} \ , \quad f_2^8 = 16\, h_2^8 \frac{N_1 W^2}{N_2^3}$$

$$\rho^8 = \frac{N_1 N_2 W^2}{h_1^4 h_2^4}$$

dilaton :

$$e^{4\phi} = \frac{N_2}{N_1}$$

where

$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial\bar{\partial}(h_1 h_2) \ ,$$

$$N_1 = 2h_1 h_2 |\partial h_1|^2 - h_1^2 W \ , \qquad N_2 = 2h_1 h_2 |\partial h_2|^2 - h_2^2 W \ .$$

3-form & 5-forms : ……….

and in 11d supergravity:

$$ds^2 = f_1^2\, ds^2_{AdS_3} + f_2^2\, ds^2_{S_2^3} + f_3^2\, ds^2_{S_3^3} + \rho^2 |dw|^2 \ ,$$

$$f_1^6 = \frac{h^2 W_+ W_-}{c_1^6\,(G\bar{G}-1)^2}\ , \qquad f_2^6 = \frac{h^2 (G\bar{G}-1) W_-}{c_2^3 c_3^3\, W_+^2}\ ,$$

$$\rho^6 = \frac{|\partial_w h|^6}{c_2^3 c_3^3\, h^4}(G\bar{G}-1) W_+ W_-\ , \qquad f_3^6 = \frac{h^2 (G\bar{G}-1) W_+}{c_2^3 c_3^3\, W_-^2}\ ,$$

where

$$W_+ = |G+i|^2 + \gamma (G\bar{G}-1) \qquad W_- = |G-i|^2 + \frac{1}{\gamma}(G\bar{G}-1)$$

$$c_1 + c_2 + c_3 = 0 \qquad \frac{c_2}{c_3} = \gamma$$

There is also an expression for the 4-form field.

The task is then to find:

— The **admissible singularities** on the surface Σ

asymptotic regions,
or coordinate sings

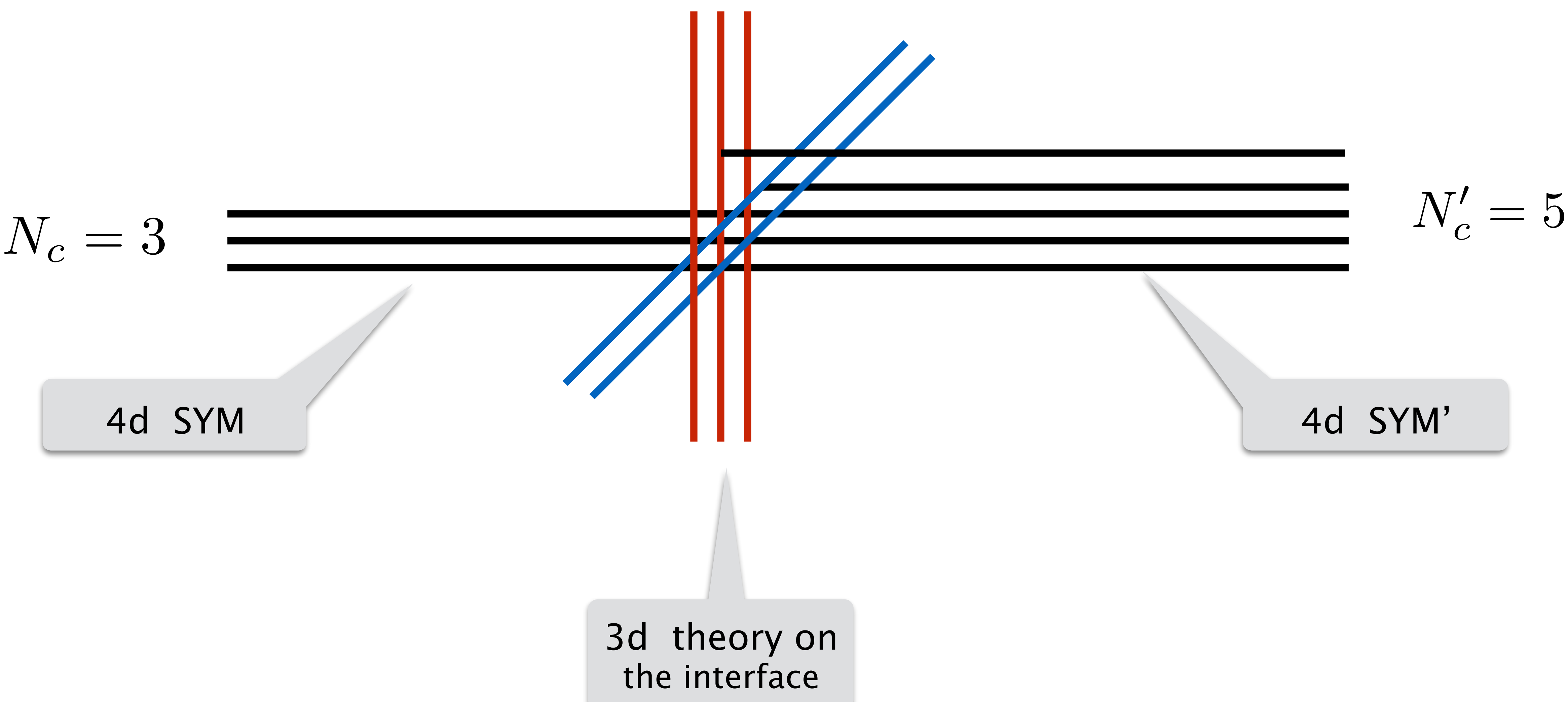
— **Global solutions**

— Their **interpretation in QFT**

2. Interfaces of N=4 SYM

Gaiotto & Witten '08 proposed a classification of non-trivial IR fixed points

Starting point: realize as intersections of **D3**-branes with **D5**, **NS5**-branes



To preserve 1/2 supersymmetry, the (probe) branes must be oriented as follows:

	012	3	456	789
D3				
D5				
NS5				

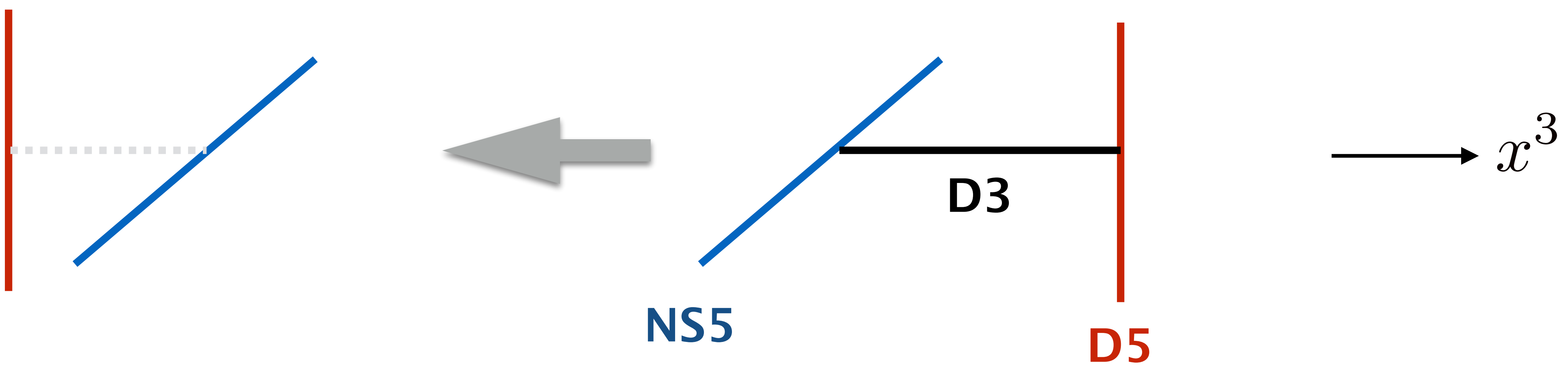
The superconformal symmetry is:

$$PSU(2,2|4) \supset OSp(4|4,R) \supset SO(2,3) \times SO(3) \times SO(3)$$

R-symmetry

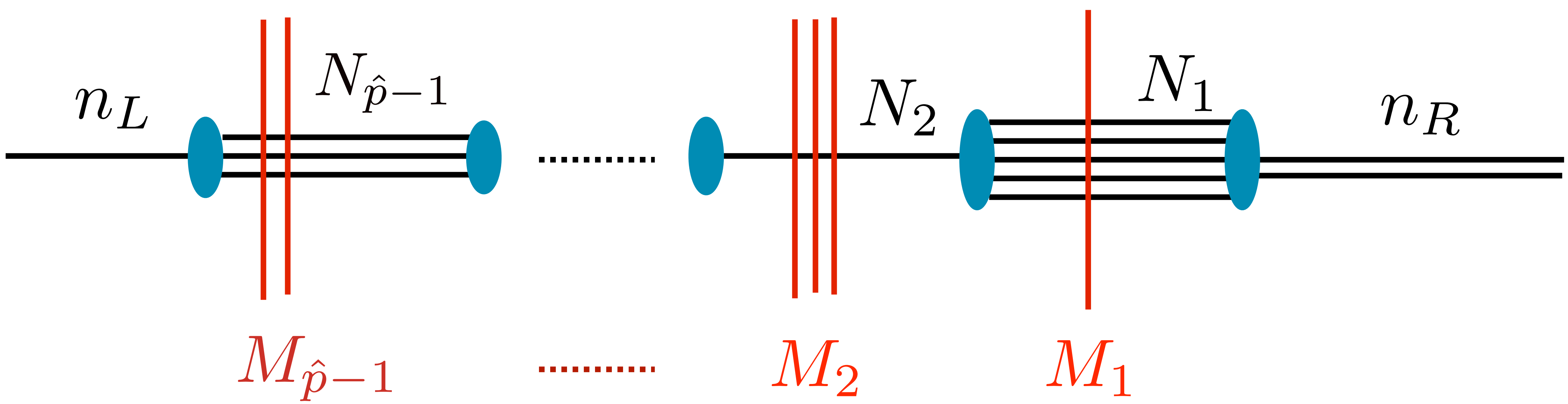
Some standard string-theory technology allows to read off the microscopic (UV) Lagrangian(s). Instrumental for this is the

Hanany-Witten move:



Using such moves, can bring the ('good') configurations to one of 3 equivalent standard forms:

★ D5-branes have no D3-branes ending or emanating from them



'electric'

★ NS5-branes have no D3-branes ending or emanating from them

'magnetic'

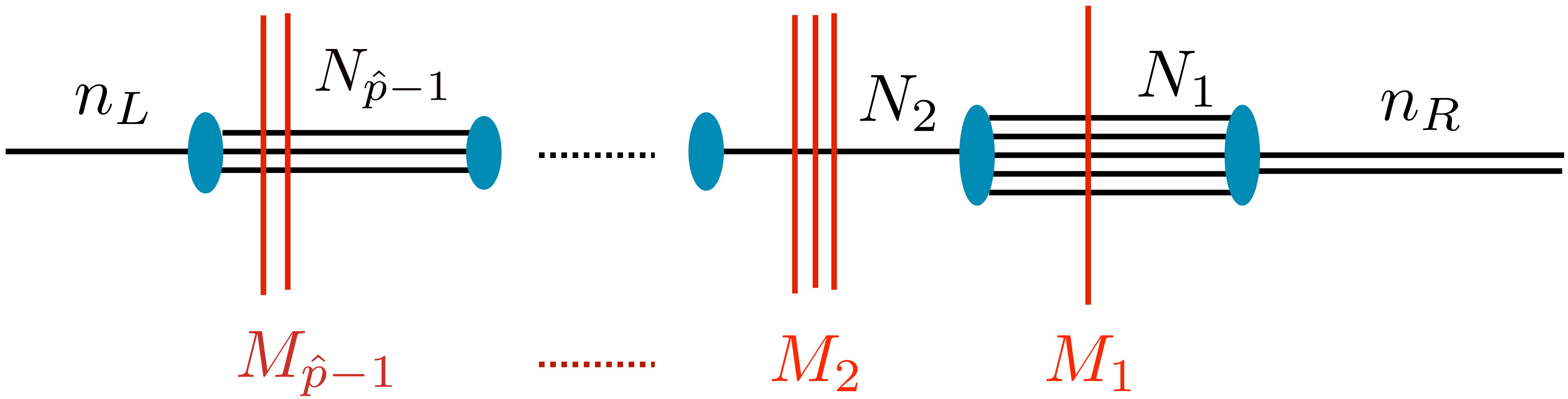
★ All NS5-branes lie to the left of all D5-branes

‘democratic’

Here, the configuration is described by **two partitions**,

ρ of $N - n_L$, and $\hat{\rho}$ of $N - n_R$

The microscopic (UV) field theory can be read off the first two, equivalent (in the IR) pictures, e.g.



A $U(N_a)$ gauge theory for each set of D3-branes

A fundamental 'electric quark' (hypermultiplet) for each intersecting D5

A bi-fundamental $U(N_a) \times U(N_{a+1})$ hyper for each NS5

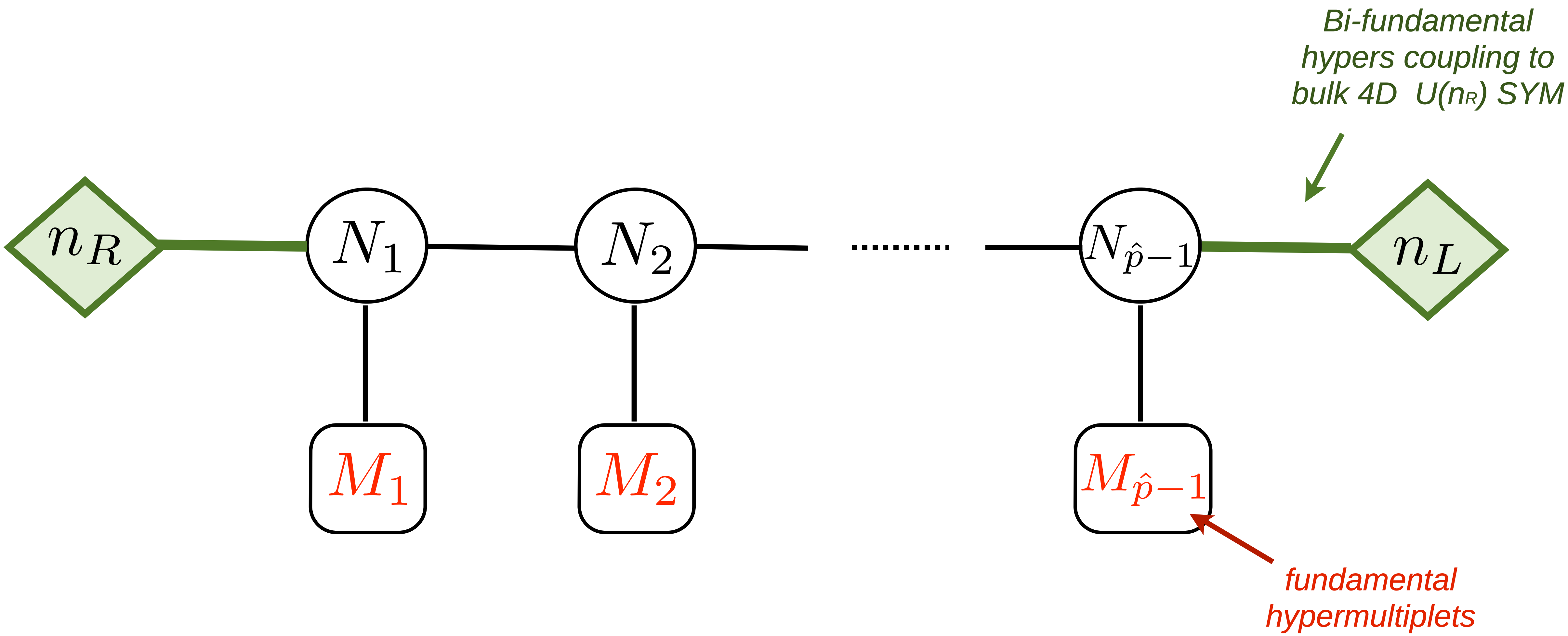
There is a dual ('magnetic') theory with **blue** exchanged with **red.**

The gauge-group ranks and numbers of matter fields are also

determined by ρ and $\hat{\rho}$, and are denoted by hats.

After all the dust has settled, one is left with a rich set of

linear-quiver, (defect) QFTs that live on the branes:



$U(N_1) \times U(N_2) \times \cdots \times U(N_{\hat{p}-1})$	3d gauge group
$U(M_1) \times U(M_2) \times \cdots \times U(M_{\hat{p}-1})$	**manifest** global symmetry
$U(\hat{M}_1) \times U(\hat{M}_2) \times \cdots \times U(\hat{M}_{p-1})$	**hidden** global symmetry

NB: **Setting** $n_L = n_R = 0$ **gives a d=3 QFT**

3. IR fixed-points & holography

Consider for simplicity $n_L = n_R = 0$

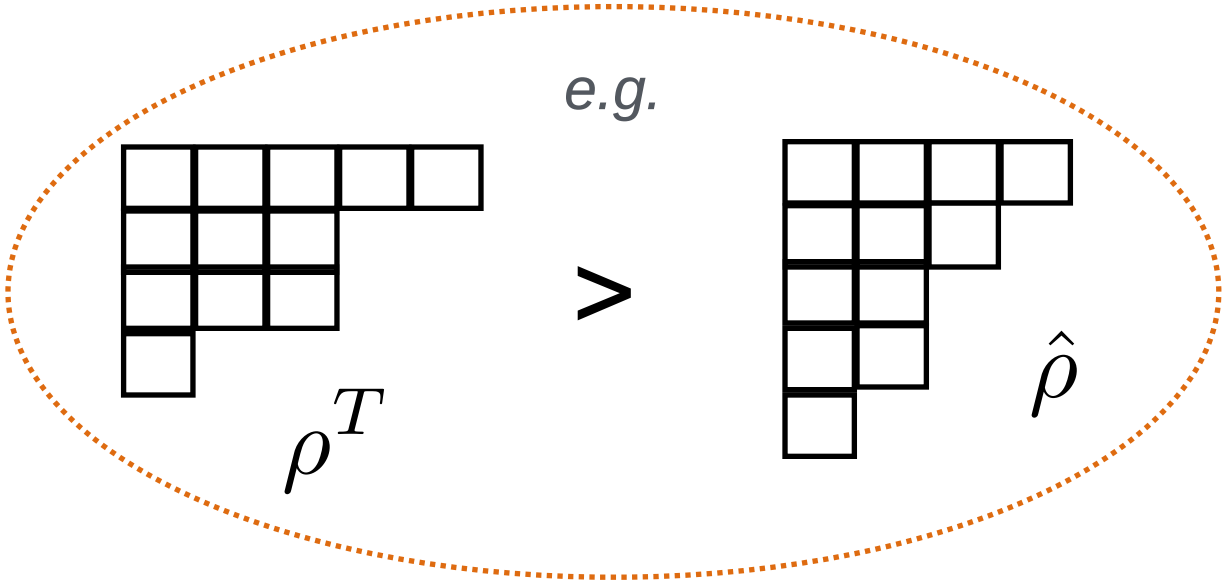
d=3 gauge theories flow in general to strong coupling in the IR

Gaiotto & Witten guessed a simple criterion for the existence of a (non-trivial) superconformal IR theory:

$$\rho^T > \hat{\rho}$$

where the (partial) ordering of Young tableaux is defined by:

(# of boxes in first n columns of ρ) $>$ (# boxes in first n rows of $\hat{\rho}$) $\forall n$



These conjectured **N=4 3d SCFTs** are dubbed $T_\rho^{\hat{\rho}}(SU(N))$

Global supergravity solutions with the required topology are in **1-to-1** correspondence with these putative IR theories

Assel, CB, Estes & Gomis

Aharony, Berdichevsky, Berkooz & Shamir

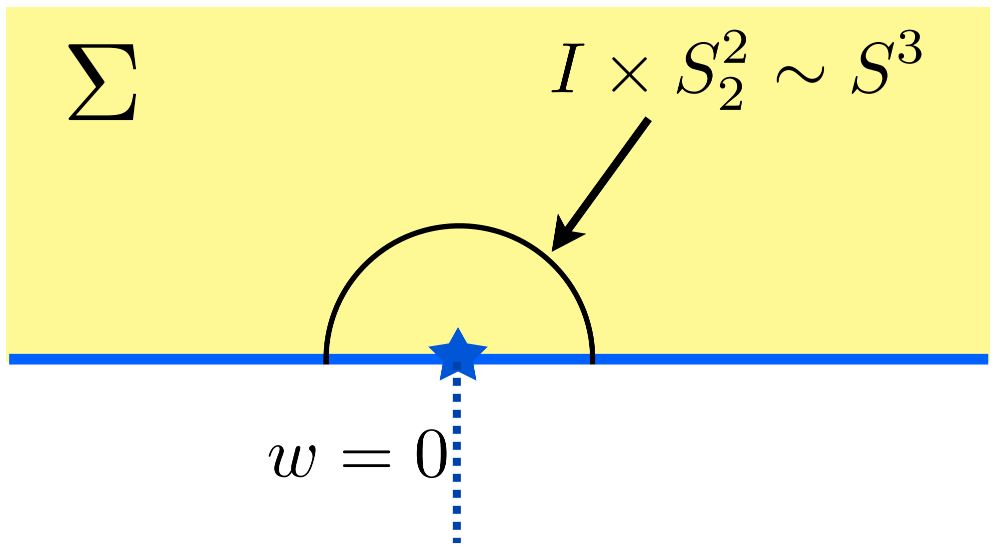
Σ has topology of **disk.** Regularity requires that $h_1, h_2 > 0$ in in the interior, while on $\partial\Sigma$:

h_1 Neuman h_2 Dirichlet	or	h_2 Neuman h_1 Dirichlet	

(one of the two S^2 shrinks to zero)

3 types of admissible singularities on the boundary:

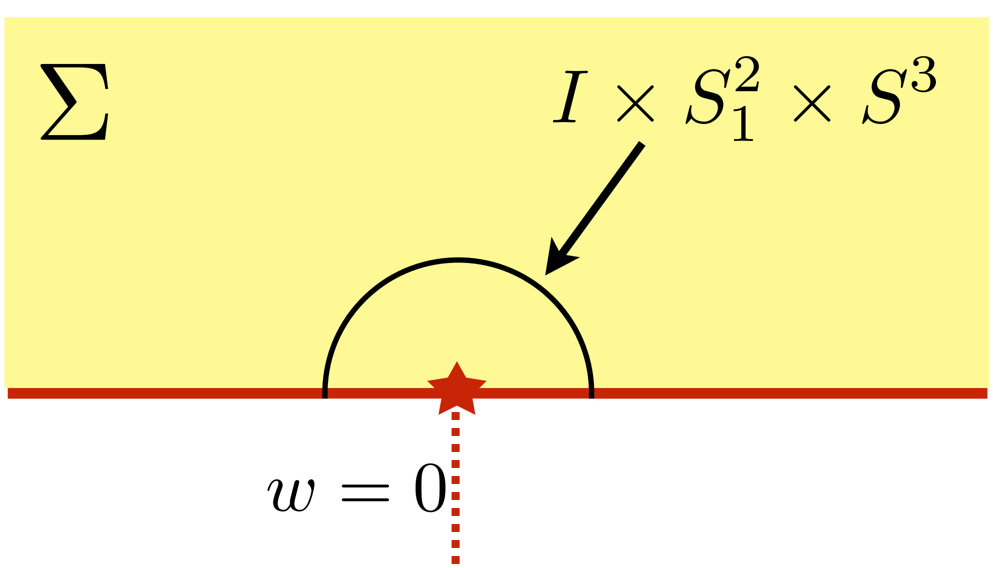
Stack of $\hat{M}$ NS5 with linking # $\hat{\ell}$



$$h_1 \simeq -i\pi \hat{M}\hat{\ell} + \cdots + c.c.$$
$$h_2 \simeq -\hat{M}\log w + \cdots + c.c.$$

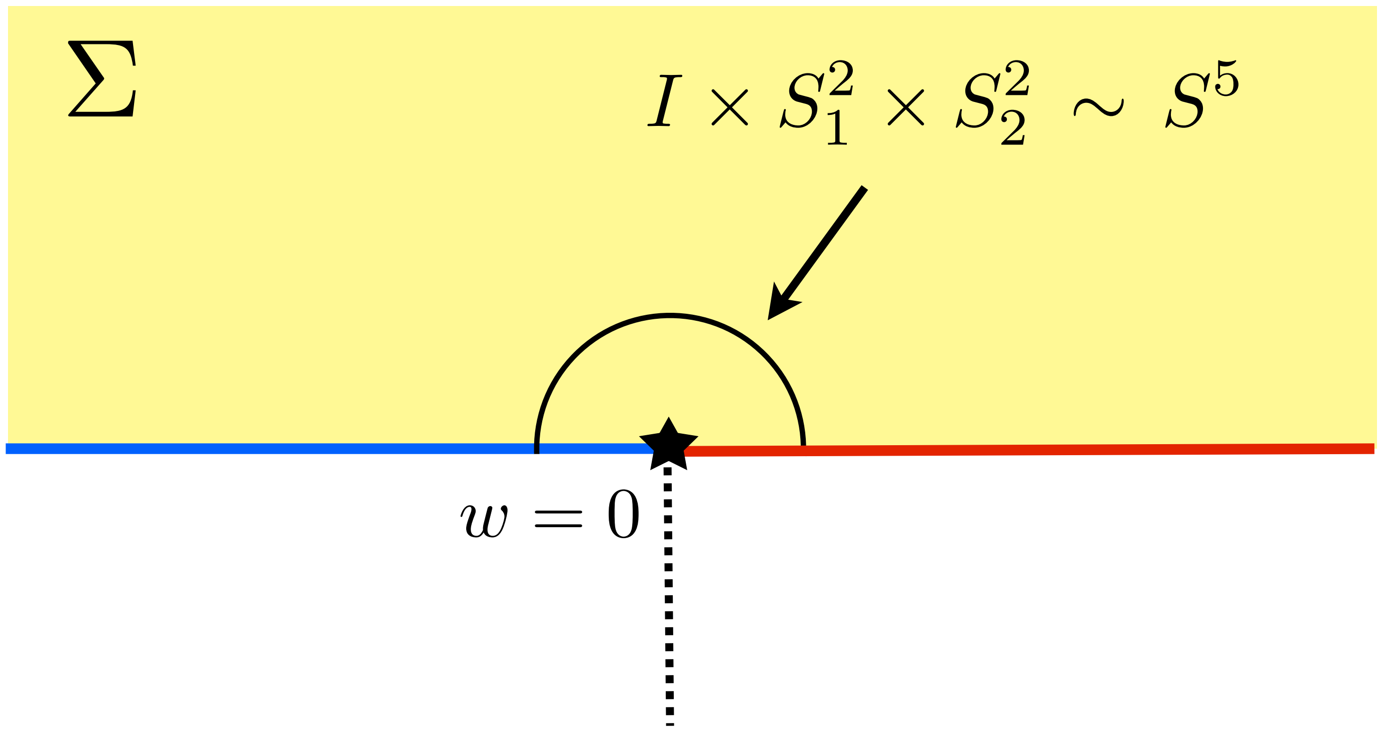
Page charge

Stack of M D5 with linking # ℓ



$$h_1 \simeq -M\log w + \cdots + c.c.$$
$$h_2 \simeq -i\pi M\ell + \cdots + c.c.$$

Stack of $n = \frac{a_2 b_1 - a_1 b_2}{2\pi}$ **semi-infinite D3**



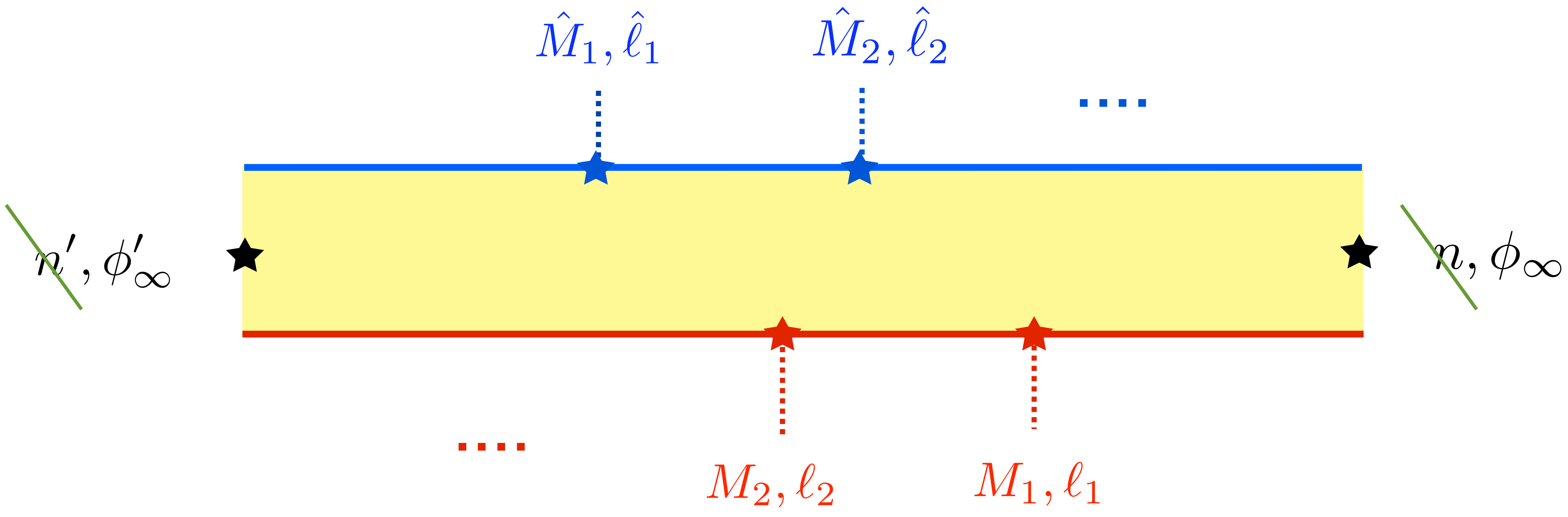
$$h_1 \simeq a_1 w^{-\frac{1}{2}} + b_1 w^{\frac{1}{2}} + \cdots + c.c.$$
$$h_2 \simeq a_2 w^{-\frac{1}{2}} + b_2 w^{\frac{1}{2}} + \cdots + c.c.$$

Important point: $a_1 = a_2 = 0$ gives just a coordinate singularity

The full solution :

$$h_1 = -i\alpha \sinh(2z - \beta) - \sum_{a=1}^{q} M_a \log\left[\tanh\left(\frac{i\pi}{2} - (z - \delta_a)\right)\right]$$

$$h_2 = \hat{\alpha} \cosh(2z - \hat{\beta}) - \sum_{b=1}^{\hat{q}} \hat{M}_b \log\left[\tanh\left(z - \hat{\delta}_b\right)\right]$$



- The linking numbers of the 5-brane stacks are functions of the positions of the **localized** 5-branes: **dimensional transmutation**

- They obey automatically the conditions of Gaiotto & Witten, confirming their conjecture !

  sugra 'knows' about $\rho^T > \hat{\rho}$

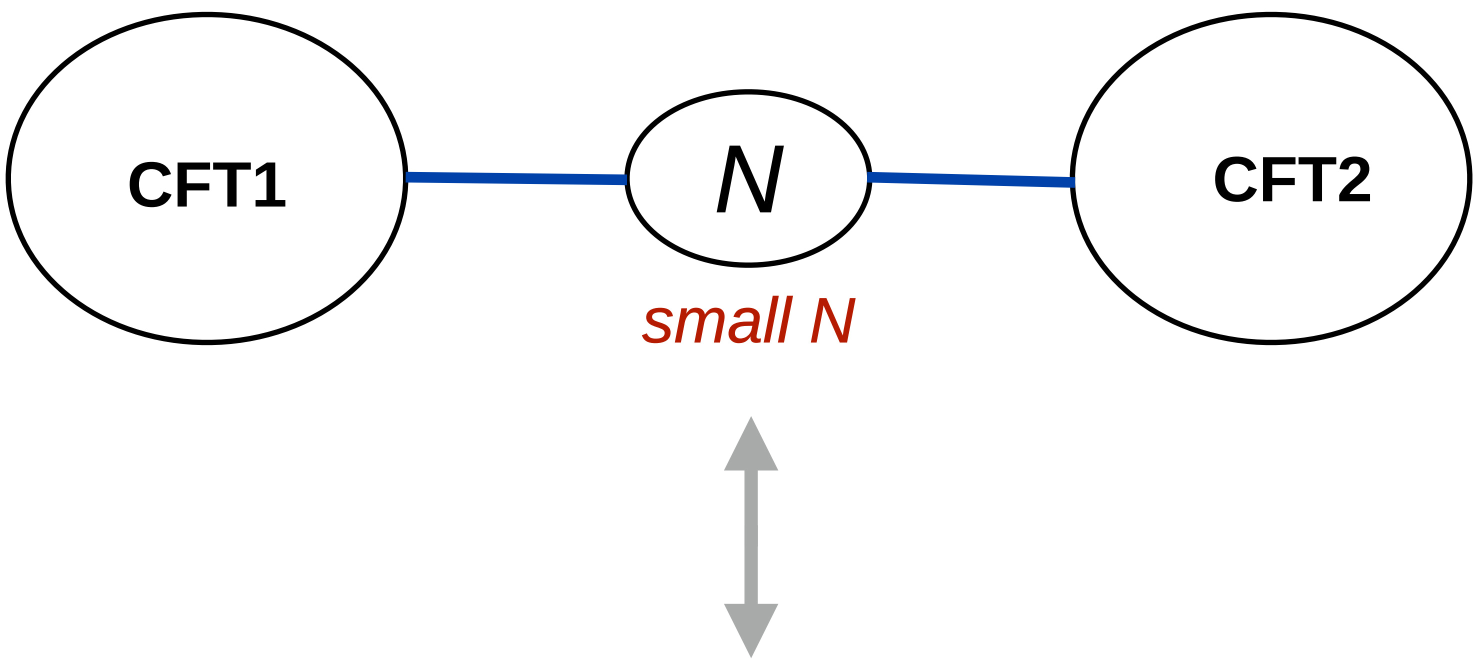
- Global (flavor) symmetries are realized, on the string theory side, as **gauge symmetries** on the 5-brane stacks.

- Can be generalized to **circular-quiver** d=3 SCFTs

— Tests holography for a rich set of (top down)

AdS_4 backgrounds ⟷ N=4 SCFT

— Several open questions, e.g. weak-rank link:



'approximate wormhole' background

ER = EPR ? *Maldacena, Susskind*

4. M2 - M5 - M5'

Consider next the 1/4-BPS configurations of M theory:

	012	3	4567	78910
M2	■	■		
M5	■		■	
M5'	■			■

The dual defect field theory is either (i) a **domain wall of ABJM**, or (ii) a **self-dual string** of the d=6 theory, or (iii) a **d=2 (4,4) SCFT.**

The story looks very similar to type-IIB, but there are notable differences.

First: little is known on the field theory side. In particular, the degrees of freedom on the M2-M5 intersection are not understood.

cf Howe, Lambert & West '97;
Harvey & Basu '04; ....
Niarchos & Siampos '12

Second: the superconformal algebra $D(2,1;\gamma) \oplus D(2,1;\gamma)$ depends on a real parameter γ . The bosonic part is $SO(2,2) \times SO(4) \times SO(4)$;

γ enters only in the fermionic part, as well as in its affine $N = (4,4)$ extension. In this latter $|\gamma| = \frac{k_1}{k_2}$ is the ratio of Kac-Moody levels.

Gunaydin, Sierra, Townsend '86
Sevrin, Troost, Van Proeyen '88

$\gamma \to 1/\gamma$ is a symmetry. Furthermore, for special values of γ $D(2,1;\gamma) \oplus D(2,1;\gamma)$ can be embedded in a bigger superalgebra:

$\gamma = 1$

$$D(2,1;\gamma,0) \oplus D(2,1;\gamma,0) \subset OSp(8|4,R)$$

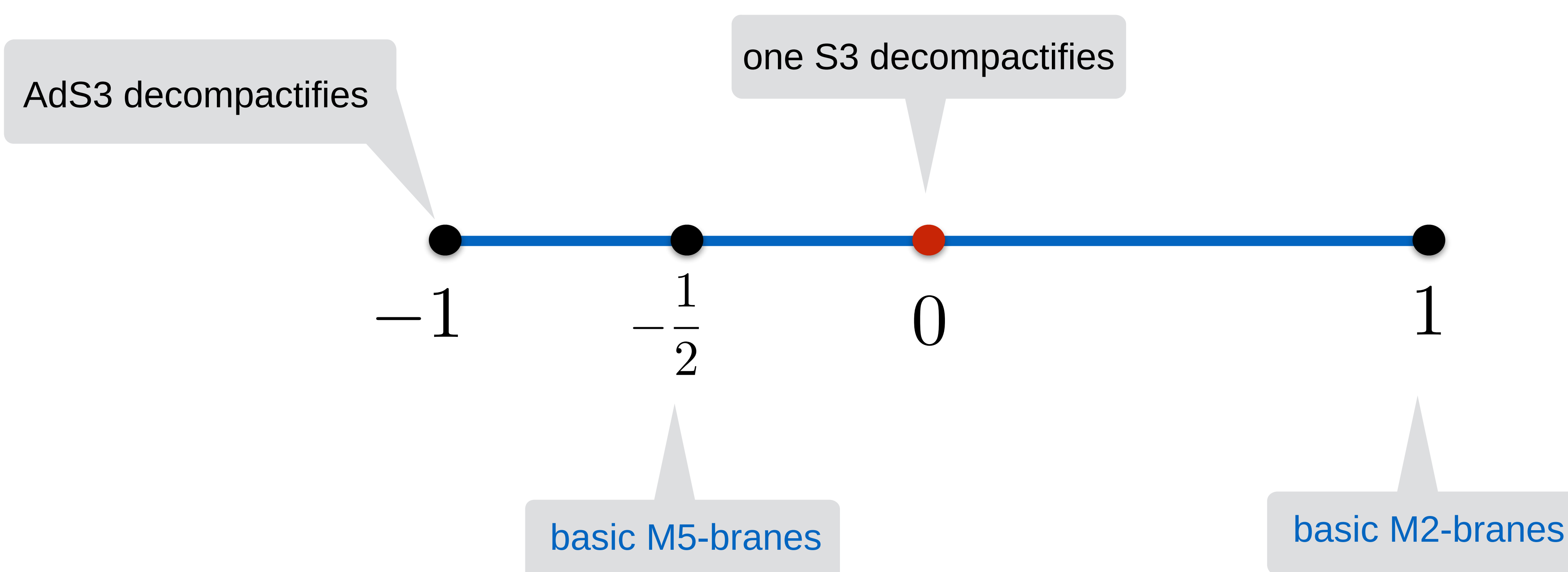
$AdS_4 \times S^7$

$\gamma = -\frac{1}{2}$

$$D(2,1;\gamma,0) \oplus D(2,1;\gamma,0) \subset OSp(8^*|4)$$

$AdS_7 \times S^4$

The γ - moduli space :



Recall: we are now solving the 11d supergravity equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}(F)$$

4-form

$$d \wedge {}^*F = F \wedge F$$

or, more exactly, the **Killing spinor** equations:

$$\nabla_\mu \epsilon + \frac{1}{2^6 3^2} \left[-\gamma_\mu (F \cdot \gamma) + 3 (F \cdot \gamma) \gamma_\mu \right] \epsilon = 0$$

with isometries $SO(2,2) \times SO(4) \times SO(4)$

The general form of the solutions is a fibration of $AdS_3 \times S^3 \times S^3$ over a Riemann surface Σ. The background is determined by the **harmonic** function h and the **complex** function G obeying

$$\partial\bar{\partial}h = 0 \ , \qquad h\partial G = \mathrm{Real}(G)\partial h$$

and the **regularity conditions:**

one of the S^3 shrinks to a point

$$h = 0, \ G = \pm i \quad \text{on} \quad \partial\Sigma$$

$$h > 0 \ , \quad \gamma(|G|^2 - 1) > 0 \quad \text{inside} \ \ \Sigma$$

One immediate <u>corollary</u>:

All solutions come in continuous families parametrized by $|\gamma|$.

This includes the basic AdS4xS7 and AdS7xS4 backgrounds

In solutions with both M5 and M5' charges, changing $|\gamma|$ rescales these charges in opposite directions. γ

The sign of labels **two distinct branches of solutions.**

To proceed, we solved completely the problem locally, and characterized admissible singularities & asymptotics.

There are 4 types of allowed singularity near $\partial\Sigma$:

- $(AdS_4/Z_2) \times S^7$: **semi-infinite M2-brane** asymptotics $\forall\gamma$
- $AdS_7' \times S^4$: **M5-brane** asymptotics $\forall\gamma$
- **M5-brane** wraping $AdS_3 \times S^3$ for $\gamma > 0$

  (**no higher-dim conformal boundary**)
- A **coordinate singularity** ('the cap') $\forall\gamma$

  (n semi-infinite M2 with $n=0$)

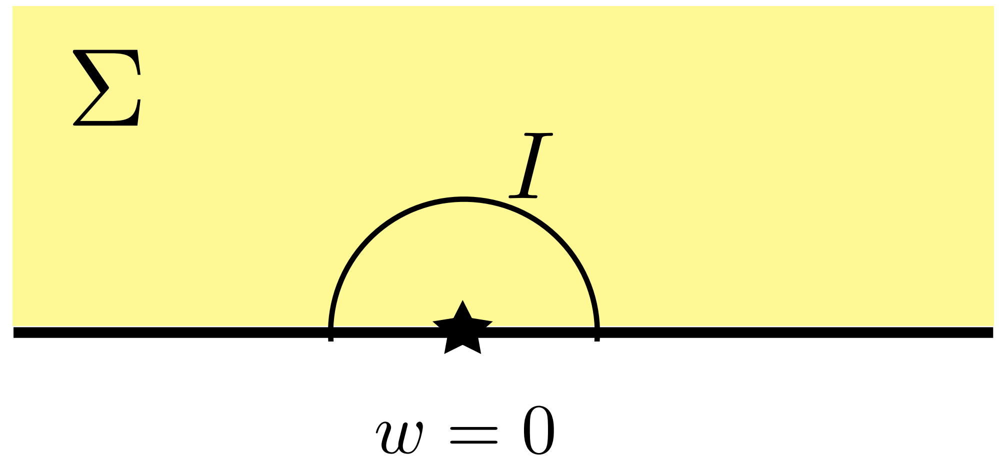
Note: $AdS_n \sim (AdS_3 \times S^{n-4}) \times_w \mathbb{R}^+$

$$(-t_1^2 - t_2^2 + x_1^2 + x_2^2) + (x_3^2 + \cdots x_{n-1}^2) = -1$$

$\uparrow$ $-(r^2+1)$ $\quad\quad$ $\uparrow$ r^2

Thus, in the first two types of singularity (where the conformal boundary is higher-dimensional) the scale factor f_1 must diverge.

The topology of these local solutions is as follows:



$$I \times S^3 \times S^3 \sim \begin{cases} S^7 & \text{if } G \text{ flips from } i \text{ to } -i \text{ on } \partial\Sigma \\ S^3 \times S^4 & \text{if no flip} \end{cases}$$

The wraped M5 has divergent G and a simple zero of h

The other three have finite G and a simple pole of h

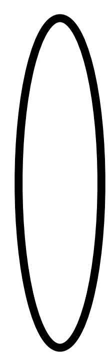
Combine these Lego pieces in global solutions ?

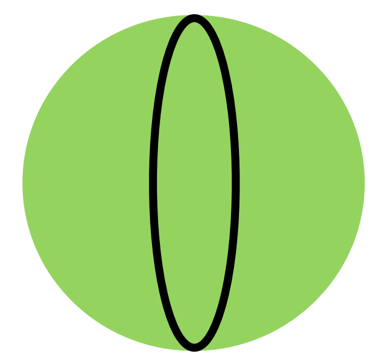
5. Global solutions & questions

… in progress

Possible **conformal** boundaries ($\times$ time):

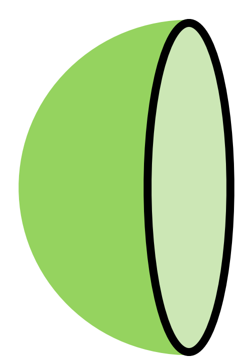
SCFT2 S^1

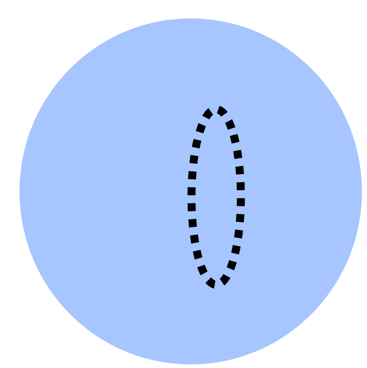




$S^1 \subset S^2$ **ABJM interface**

ABJM on half-space $S^1 = \partial \mathcal{D}^2$





$S^1 \subset S^5$ **self-dual string**

A simple 'theorem': **(Near) Uniqueness of solutions dual to SCFT2**

Proof: From the metric expressions one finds

radii of 3 (pseudo)spheres

$$c_1 c_2 c_3 f_1 f_2 f_3 = \pm h$$

$$(c_1 f_1)^2 \geq (c_2 f_2)^2 + (c_3 f_3)^2$$

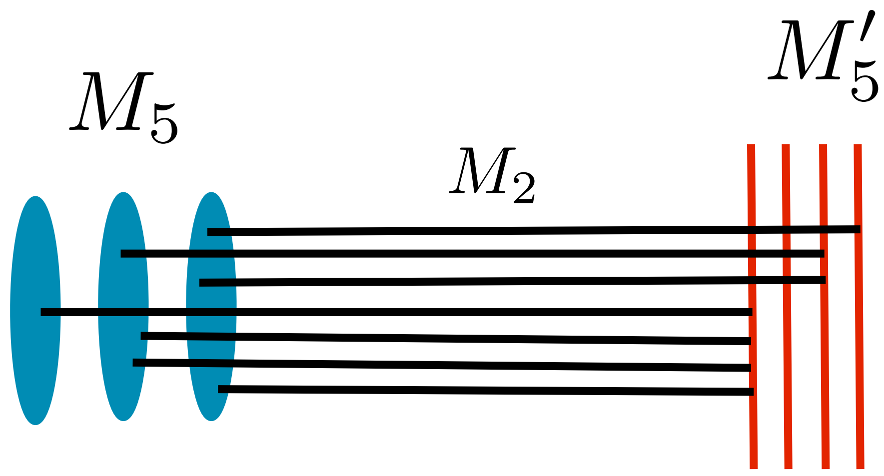
This implies that singularities of h are **loci** where the AdS_3 radius **blows up**. So, if we want the conformal boundary to be 2-dimensional, then h must be everywhere smooth. This means that $h = 0$

or Σ has no boundary and h = constant.

The corresponding solutions are $AdS_3 \times S^3 \times S^3 \times (\mathbb{R}^2/\Gamma)$

(this is the near-horizon geometry of configurations with **5-branes smeared in their common transverse coordinate).**

Boonstra, Peeters, Skenderis '98



All information on the two partitions is lost, unlike IIB

A consequence of the Mermin-Wagner theorem ?

Two more 'general' statements:

No solutions with disconnected boundary ('wormholes').

For $\gamma < 0$ we can prove this; for $\gamma > 0$ have found no solutions.

cf Witten & Yau, hep-th/9910245;

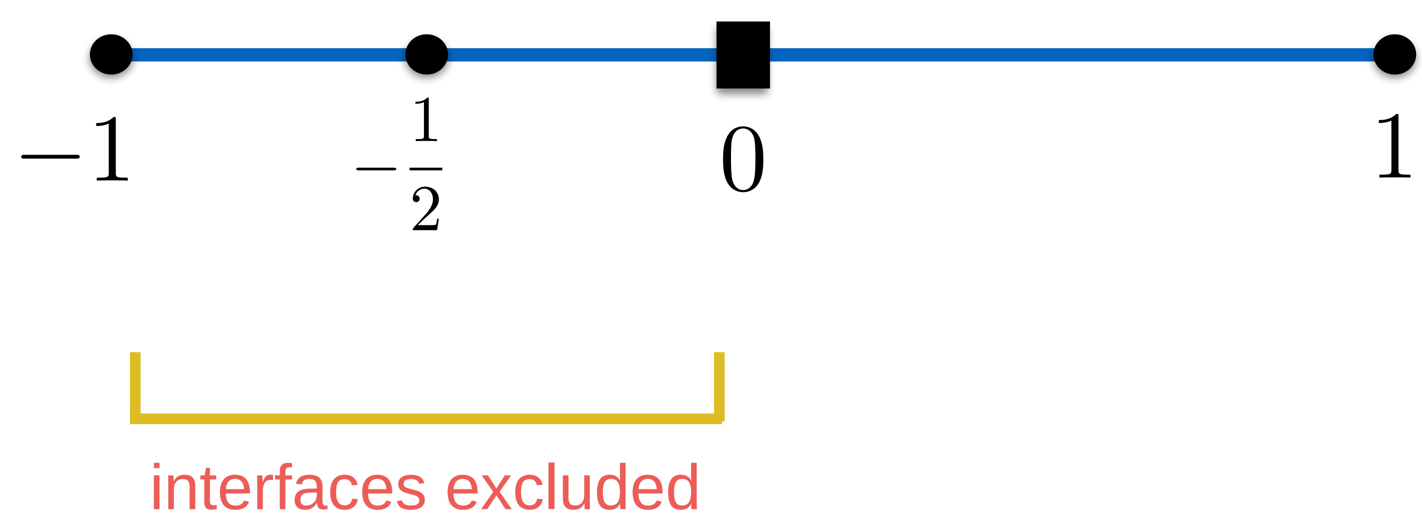
Galloway, Schleich, Witt, Woolgar hep-th/9912119

This excludes also solutions extrapolating between $AdS_7 \times S^4$ and $AdS_4 \times S^7$ (**'rigidity'**)

G finite

For $\gamma < 0$ one can prove a stronger result: h has at most one singularity, so at most one stack of semi-infinite M2-branes.

No **interface CFT3** for negative γ



Solutions with one asymptotic region

'cap', coordinate singularities

$\gamma < 0$

Self-dual strings
$$h = -i(w - \bar{w}), \quad G = -i \left(1 + \sum_{j=1}^{2n+2} (-)^j \frac{w - \xi_j}{|w - \xi_j|} \right)$$

Semi-infinite M2
$$h = -i(w - \bar{w}), \quad G = -i \sum_{j=1}^{2n+1} (-)^j \frac{w - \xi_j}{|w - \xi_j|}$$

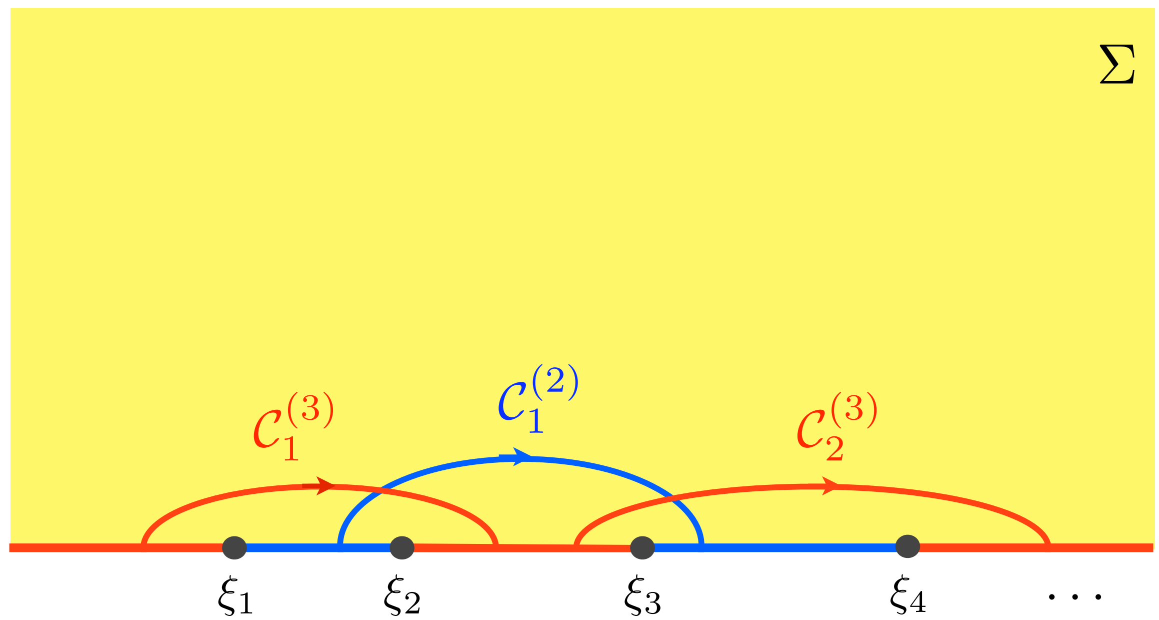
M5-brane singularities

$\gamma > 0$

Self-dual strings
$$h = -iw + c.c. \, , \qquad \pm G = i + \sum_{a=1}^{n+1} \frac{\zeta_a \, \mathrm{Im}(w)}{(\bar{w} - x_a)|w - x_a|}$$

Semi-infinite M2
$$h = -iw + c.c. \, , \qquad \pm G = iw/|w| + \sum_{a=1}^{n+1} \frac{\zeta_a \, \mathrm{Im}(w)}{(\bar{w} - x_a)|w - x_a|}$$

$M5$ or semi $-$ infinite $M2$ at ∞

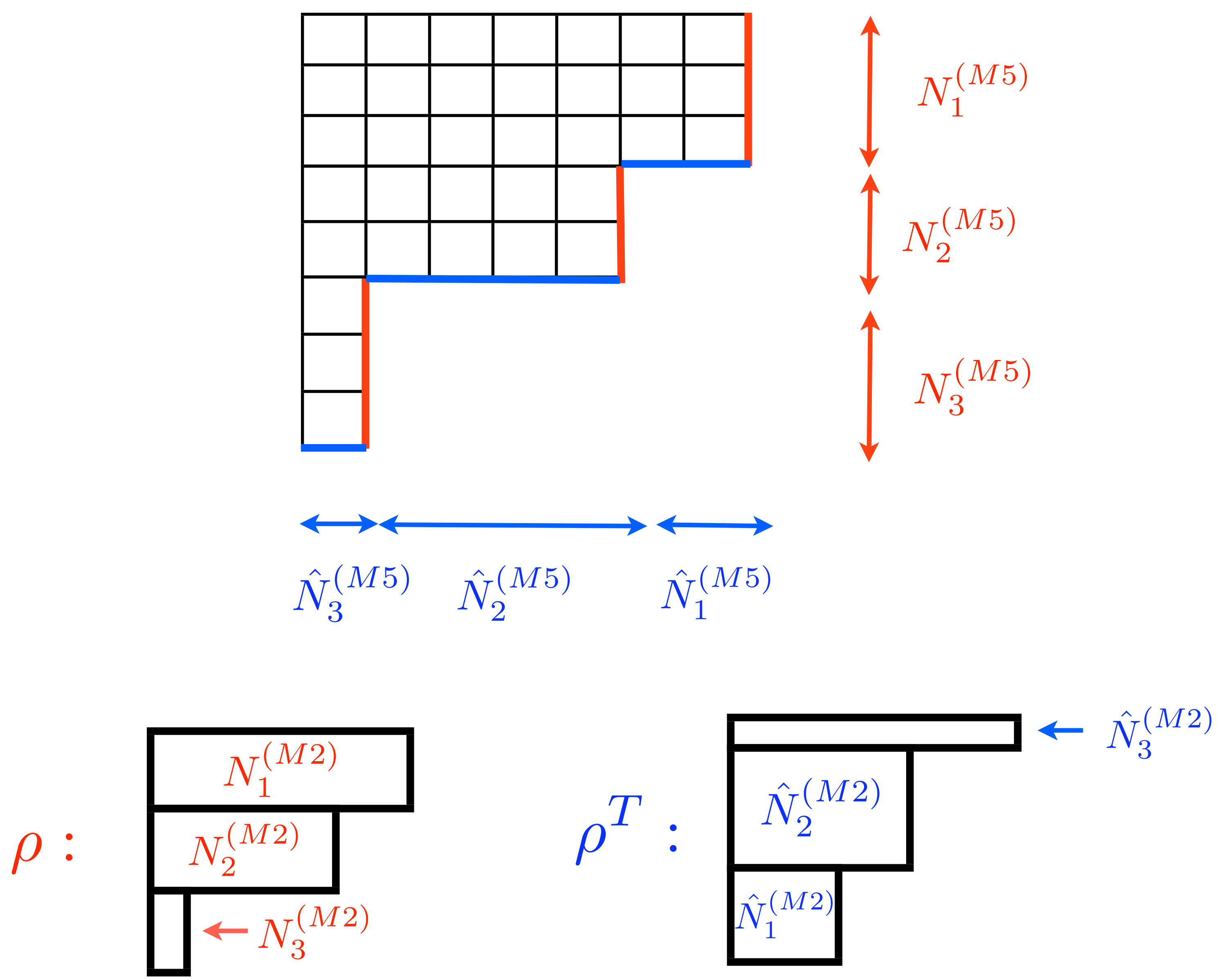


Can calculate the charges of these solutions. For **all of them**,

they seem to be in one-to-one correspondence with **Young tableaux**;

reminiscent of the study of Wilson lines in N=4 SYM

Yamaguchi '06;
Gomis & Passerini '06;
Okuda & Trancanelli '08;
D'Hoker, Estes, Gutperle '07



$N_1^{(M5)}$
$N_2^{(M5)}$
$N_3^{(M5)}$
$\hat{N}_3^{(M5)}$
$\hat{N}_2^{(M5)}$
$\hat{N}_1^{(M5)}$
ρ :
$N_1^{(M2)}$
$N_2^{(M2)}$
$N_3^{(M2)}$
ρ^T :
$\hat{N}_3^{(M2)}$
$\hat{N}_2^{(M2)}$
$\hat{N}_1^{(M2)}$

These solutions describe the long-sought IR limit of

localized M2-branes ending on **M5-branes.**

- They exist for all γ , both from the M2-brane perspective **(ABJM boundaries)** and from the M5-brane perspective **(self-dual strings in 6d)**

- There is a $\gamma = 0$ transition at which the coordinate singularities become wraped M5-brane singularities.

- Have not found general intersecting M2-M5 solutions (ABJM interfaces)
  Only a smooth, Janus solution with no 5-brane charge

D'Hoker, Estes, Gutperle, Krym '09
Bobev, Pilch, Warner '13

Much remains to be done :

- Finish the task of finding all solutions ?
- Count degrees of freedom, put at finite T
- Understand these dCFTs from the QFT side
- 6d - 3d defect duality?

What do we learn about M theory ? Can these be used for CMT ?

!! many thanks to the local organizers !!