Conductivity and Entanglement in holographic superconductors in the large-D limit

UNIVERSITY OF CAMBRIDGE

Cavendish Laboratory

Aurelio Romero-Bermúdez (abr31@cam.ac.uk), Antonio M. García-García

Introduction

Abelian-Higgs coupled to Einstein gravity:

$$S = \int d^{d+1}x\sqrt{-g}\left[R - \frac{1}{4}F^2 - |D_\mu\psi|^2 - V(|\psi|)\right] \quad (1)$$

T=0, [1]

$$V(|\psi|) = -2\Lambda + m^2|\psi|^2 + \frac{u}{2}|\psi|^4 \quad (2)$$

$$ds^2 = e^{2A(r)}\left(-h(r)dt^2 + dx_i^2\right) + \frac{dr^2}{h(r)} \quad (3)$$

T>0, [2,3.4]

$$V(|\psi|) = -2\Lambda + m^2|\psi|^2 \quad (4)$$

$$ds^2 = \frac{1}{z^2}\left(f(z)dt^2 + \frac{1}{f(z)}dz^2 + dx_i^2\right), \;\; f(z) = 1 - z^d \quad (5)$$

Electrical conductivtiy at T=0

Eq. (3) domain wall geometry: $L_{\rm IR}\,,\psi_{\rm IR}\,,L_{\rm UV},\psi_{\rm UV}$

Low frequencies

Following [1], in background (3) solve analytically for $A_x(z,\omega)$.

$$A_x(r) = e^{\frac{-(d-2)r}{2L_{\rm IR}}}H_\alpha^{(1)}(\omega L_{\rm IR}e^{-r/L_{\rm IR}}) \quad (6)$$

$$\alpha = \frac{1}{2}\sqrt{(d-2)^2 + 8\psi_{\rm IR}^2 L_{\rm IR}^2} \quad (7)$$

$$\sigma \propto \frac{-he^{(d-2)A}A_x^*\overleftrightarrow{\partial_r}A_x}{2i\omega|A_0|^2} \quad (8)$$

Low frequency: $r^* \ll r \ll r_{\rm IR}$, $r^* = L_{\rm IR}\log\omega L_{\rm IR}$ and $r_{\rm IR}$

Is the position above which the geometry is significantly deformed from IR. Eq. (6) leads to:

$$A_x(r) \propto (\omega L_{\rm IR})^{-\alpha} \xrightarrow{(8)} \boxed{\mathrm{Re}(\sigma) \propto \omega^{2\alpha-1}} \quad (9)$$

Larger *d* stronger suppression

Large frequencies

Setting $\psi_{\rm IR} = 0$, both domain wall geometries are equal.

$$A_x(r) = e^{\frac{-(d-2)r}{2L}}H^{(1)}_{\frac{d-2}{2}}(\omega Le^{-r/L}) \xrightarrow[r\to\infty]{} C_1\omega^{\frac{2-d}{2}} + C_2\omega^{\frac{d-2}{2}}e^{-\frac{r}{L}(d-2)} \quad (10)$$

Eq. (8) and (10) lead to:

$$\boxed{\mathrm{Re}(\sigma) \propto \omega^{d-3}} \quad (11)$$

Intermediate frequencies

Debye's expansion of $H_\alpha^{(1)}$ valid for:

$$\alpha \simeq \omega L_{\rm IR}e^{-r/L_{\rm IR}} \gg 1, \;\; |\omega L_{\rm IR}e^{-r/L_{\rm IR}} - \alpha| \ll \left(\omega L_{\rm IR}e^{-r/L_{\rm IR}}\right)^{1/3} \quad (12)$$

Large-*d*, from (7) and (12): $\omega L_{\rm IR} \sim \frac{d-2}{2}$.

Taking $r \ll r^* \ll r_{\rm IR}$, Debye's expansion yields:

$$A_x(r) \simeq \tilde{Z}(r,\omega)(\omega L_{\rm IR})^{-\frac{1}{3}} \quad (13)$$

$\tilde{Z}(r,\omega)$ is given in Eq. (10). Eqs. (8) and (13) lead to:

$$\boxed{\mathrm{Re}(\sigma) \propto \omega^{d-3+\frac{2}{3}}} \quad (14)$$

Consistent with the presence of peak

Entanglement Entropy

Stip of width l. Sharp domain wall approximation: $r_{\rm DW}$

T=0 In units of $\frac{2G_N^{d+1}}{a^{d-2}}$

$$S_{\rm UV} = L\left[\frac{e^{(d-2)A_{\rm UV}\frac{r_{\rm UV}}{L}}}{d-2} - \frac{e^{dA_{\rm UV}\frac{2r_*-r_{\rm DW}}{L}}}{d(d-2)} - \frac{e^{(d-2)A_{\rm UV}\frac{r_{\rm DW}}{L}}}{d-2}\right] \quad (15)$$

$$S_{\rm IR} = L_{\rm IR}\left[-\frac{\sqrt{\pi}}{d(d-2)}\frac{\pi}{2}e^{(d-2)\frac{r_*}{L_{\rm IR}}} + \frac{e^{d\frac{2r_*-r_{\rm DW}}{L_{\rm IR}}}}{d(d-2)} + \frac{e^{(d-2)\frac{r_{\rm DW}}{L_{\rm IR}}}}{d-2}\right]$$

At constant l, in the large-*d* limit $r^* \to -\infty$. Also $r_{\rm DW} < 0$, therefore, **finite part of entanglement entropy tends to zero as *d* tends to infinity.**

T≈T$_c$

$$ds^2 = \frac{1}{L^2z^2}\left(-f(z)e^{-\chi(z)}dt^2 + \frac{1}{f(z)}dz^2 + dx_i^2\right) \quad (16)$$

$$f(z) \simeq f_{\rm RN} + \epsilon^2 f_2(z) \qquad \epsilon = \psi_{\rm UV}$$

Entanglement entropy (in untis $\frac{4G_N^{d+1}L^{d-1}}{a^{d-2}}$) in the back-reacted background close to T$_c$:

$$\tilde{s}_A \simeq \frac{2}{(d-2)z_{\rm UV}^{d-2}} - \frac{\pi}{(d-2)(d-1)z_*^{d-2}} + \quad (17)$$

$$+\frac{1}{z_*^{d-2}}\left[\left(\frac{Q^2+1}{2} + \frac{d\epsilon^2\mu_0\delta\mu z_0^2}{\pi}\right)\left(\frac{z_*}{z_0}\right)^d + \mathcal{O}\left(\frac{z_*}{z_0}\right)^{2d-2}\right]$$

Absolute value of EE is smaller than for T>T$_c$.

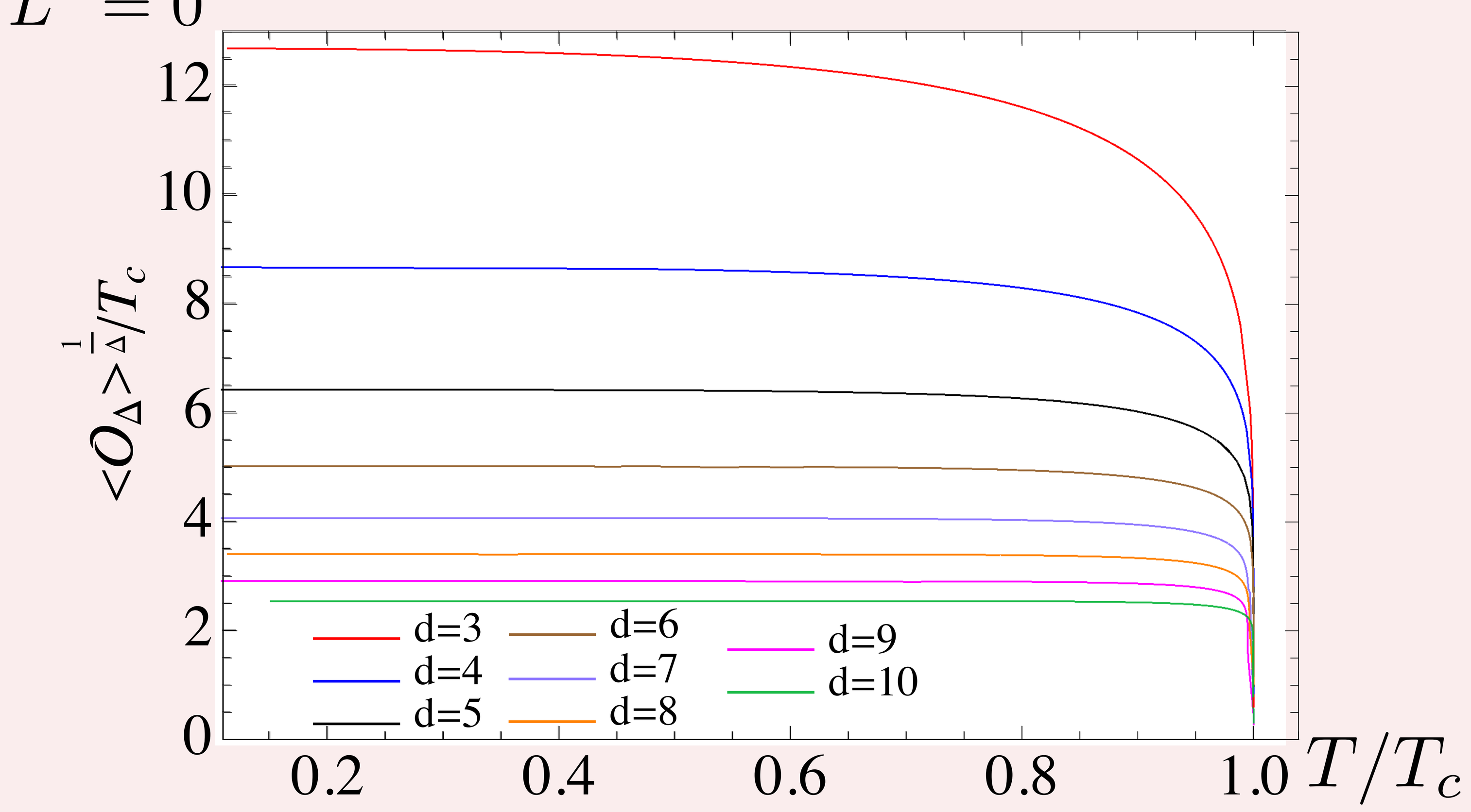
The number of degrees of freedom is smaller at T<T$_c$.

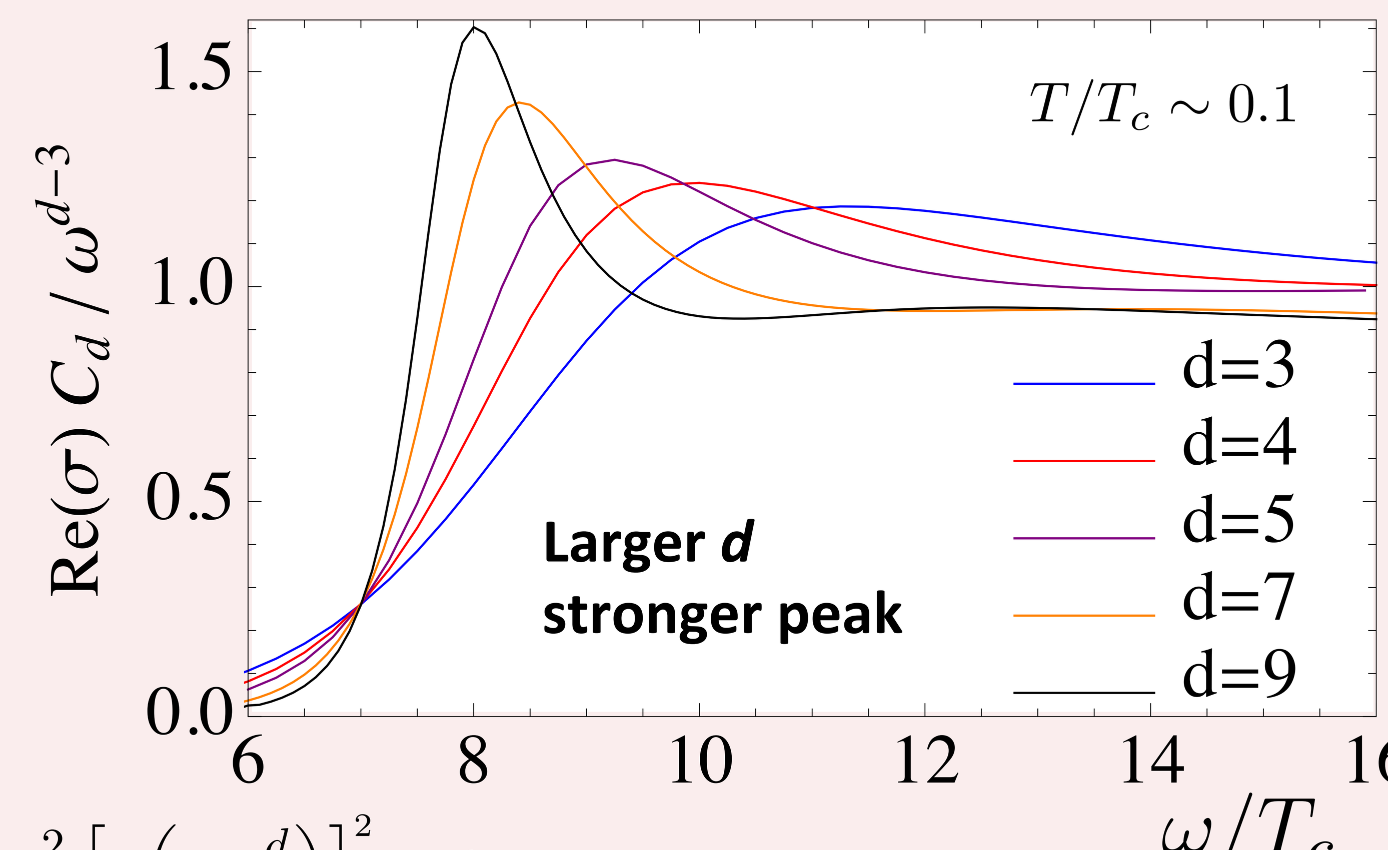
Powerful observable to detect phase transitions.

Numerics

Using model (1) (4) (5). Probe limit. $\langle\mathcal{O}_\Delta\rangle = (2\Delta - d)\psi_{\rm UV}$

$m^2L^2 = 0$





$C_d = \frac{2}{\pi}\left[\Gamma\left(2 - \frac{d}{2}\right)\right]^2 2^{2-d}, \; d$ odd

Conclusions

As $d \to \infty$:

- Higher order power law for Re(σ) at low ω (stronger suppression).
- More pronounced maximum in Re(σ) (smaller decoherence).
- $\langle\mathcal{O}\rangle^{\frac{1}{\Delta}}/T_c \sim 1/d$? Correct parameter rescaling necessary.
- Smaller absolute value of the regularized part of the entanglement entropy (less degrees of freedom).
- The dual field theory becomes more weakly coupled.

References

[1] Gubser, S. and Rocha, F. *Phys. Rev. Lett.* **102** 061601 (2009).

[2] Gubser, S. *Phys. Rev. D.* **78** 065034 (2008).

[3] Hartnoll, S., Herzog, C. and Horowitz, G. *Phys. Rev. Lett.* **101** 031601 (2008).

[4] Emparan, R., Tanabe, K., JHEP **01** (2014) 145