

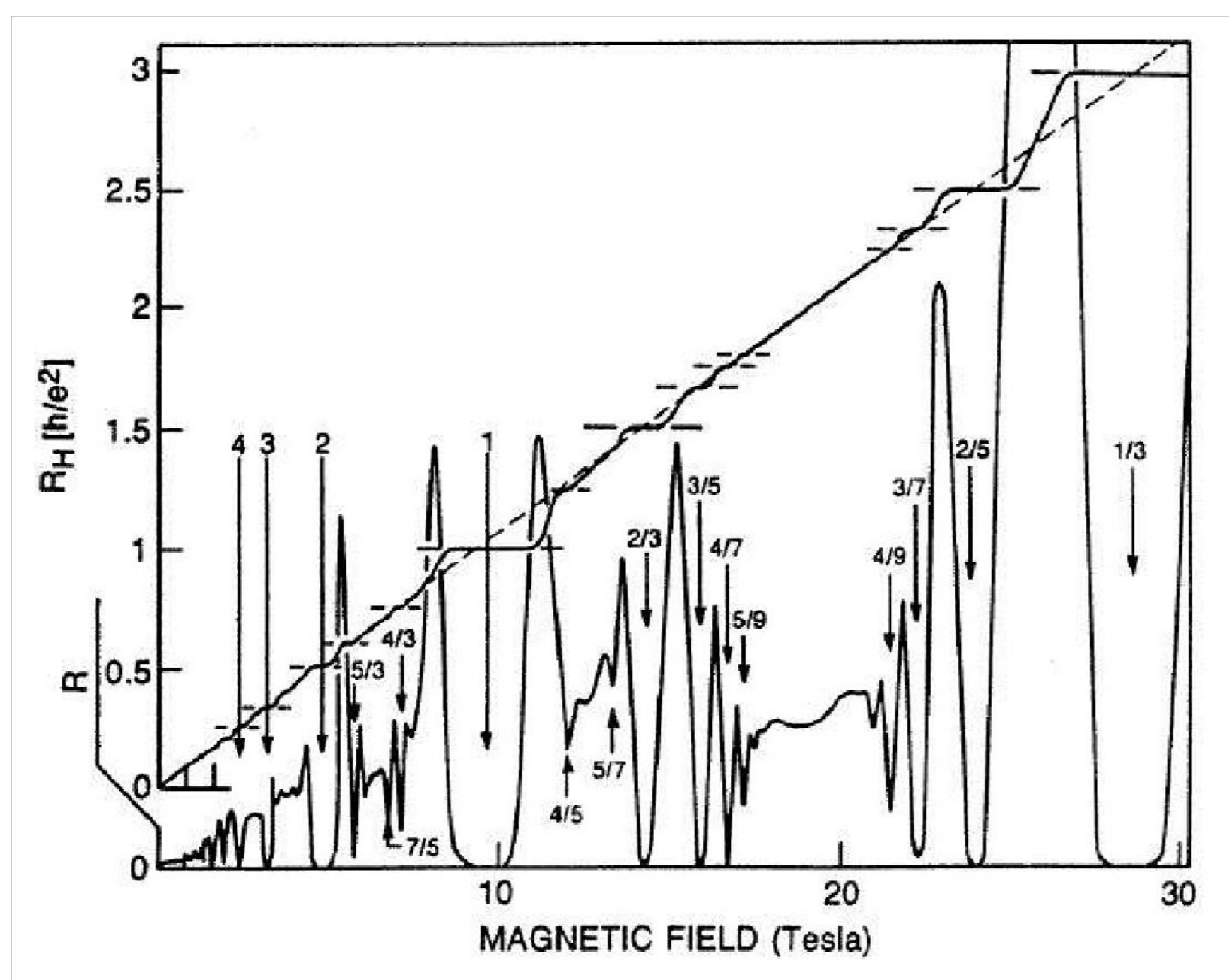
## Abstract

We consider the holography of 1+1-dimensional defects, described by D7 probe branes in an AdS<sub>5</sub> soliton background. When the probe branes are allowed to reach the AdS boundary they become dual to domain walls in Yang-Mills-Chern-Simons theory across which the Chern-Simons level varies, resulting in a chiral current algebra living on each defect that interacts in an interesting manner with irrelevant operators in the system. We examine the holography of the system in detail, which shows many similarities to the fractional quantum Hall effect, as well as 2D QCD.

## 1. Introduction

### • Transport in 2D Electron Gases

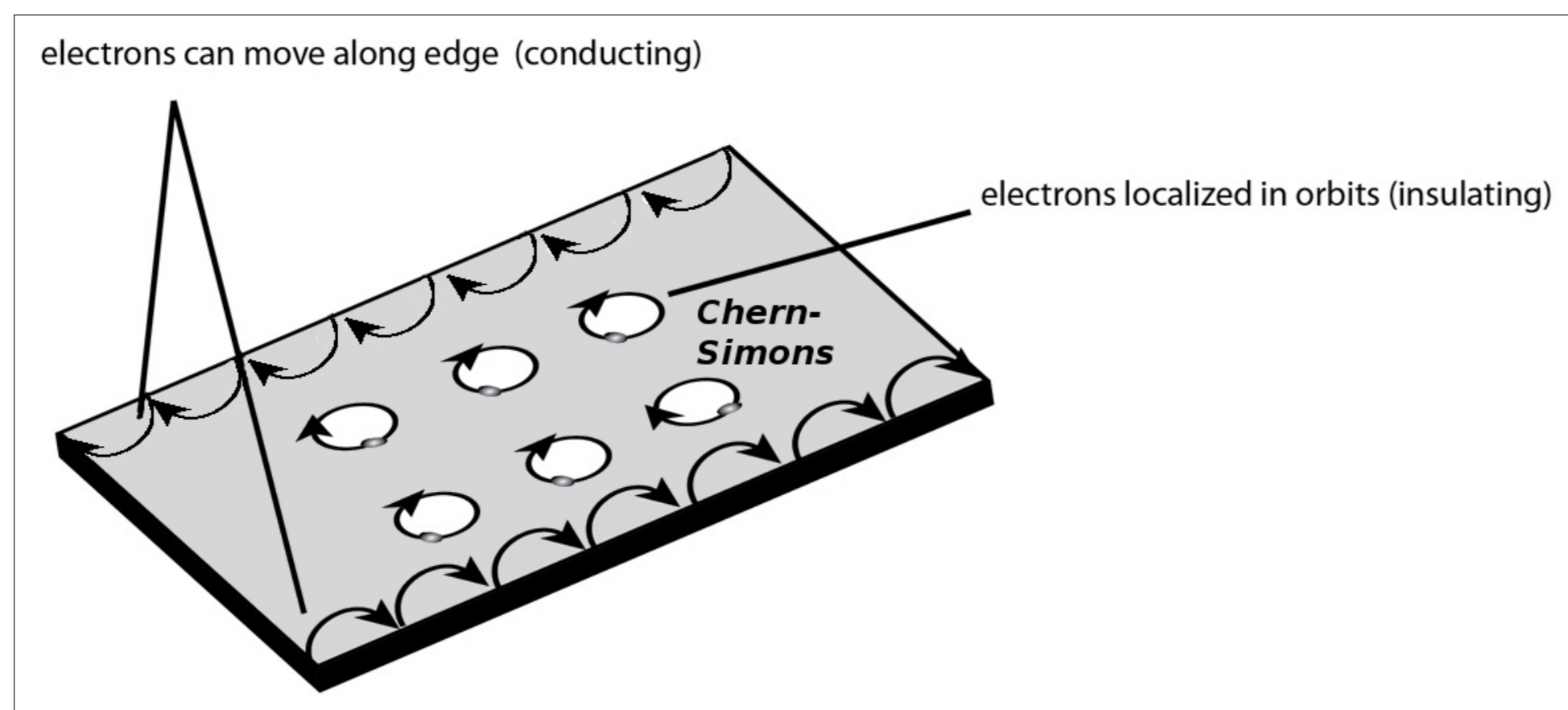
(very low temperatures, high magnetic fields, clean samples)



### • FQH plateaux are gapped with quantized Hall conductivity

$$\sigma_{xy} = \frac{p}{q} \left( \frac{e^2}{h} \right), \quad p, q \in \mathbb{Z} \quad (1)$$

### • FQH Sample: Bulk Chern-Simons Theory + Chiral Edge Modes



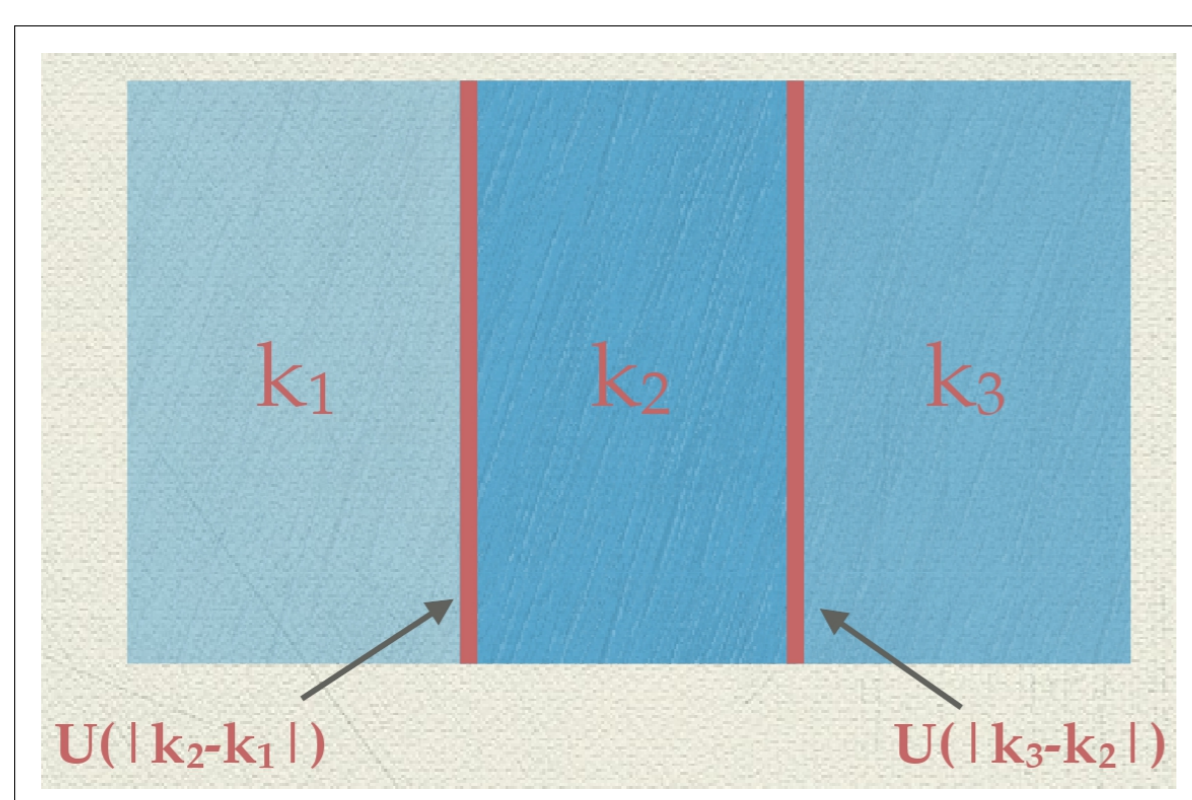
### • FQH samples as Domain Walls of Chern-Simons Theory

$$S_{CS} = \frac{k}{4\pi} \int_{M_3} \omega_3(A), \quad \omega_3(A) = \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \quad (2)$$

### • Chern-Simons Term only gauge invariant up to a total derivative:

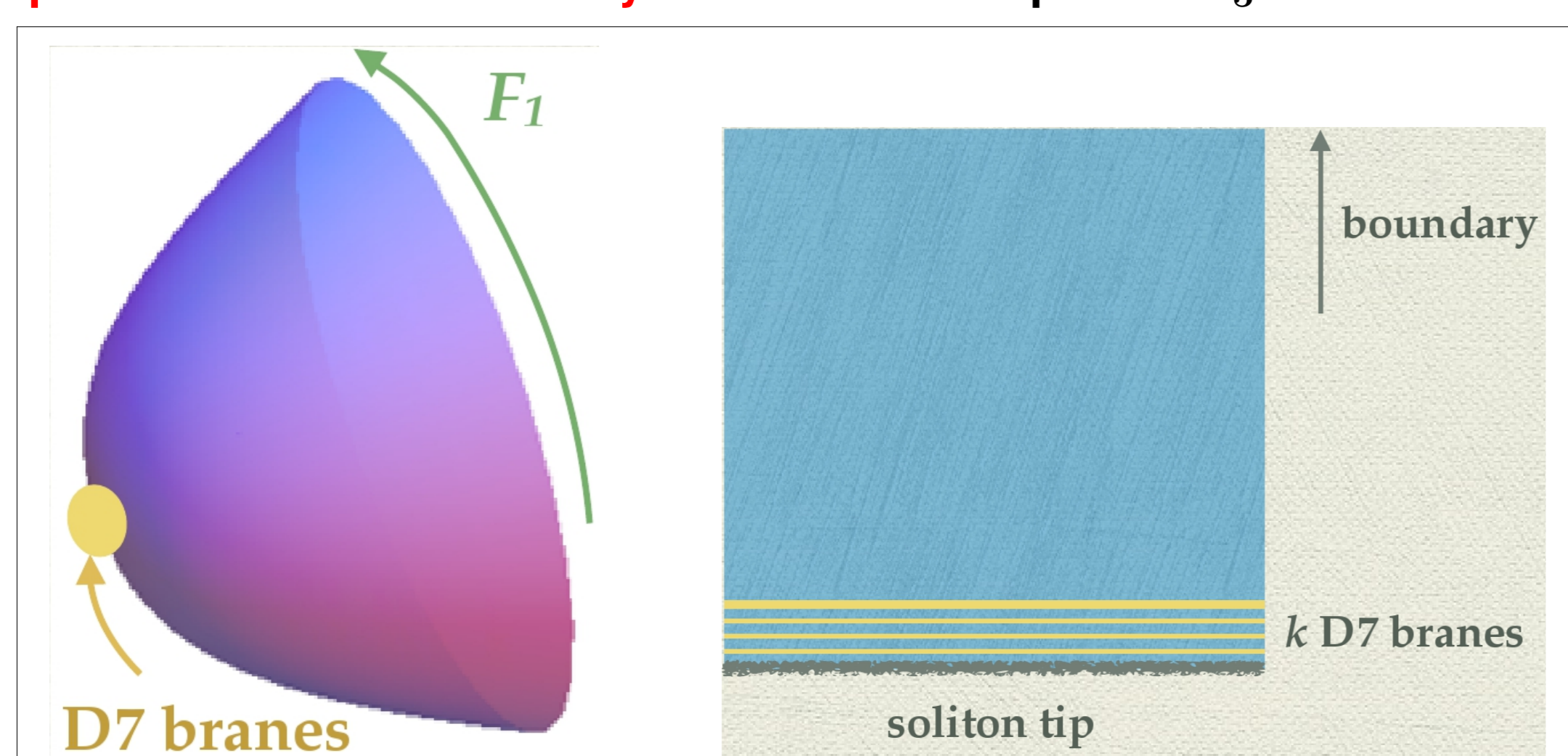
$$\delta_A S_{CS} = \frac{k}{4\pi} \int_{M_3} d \text{Tr}(AF) = \frac{k}{4\pi} \int_{\partial M_3} \text{Tr}(AF) \quad (3)$$

### • Domain walls separating phases at different levels must carry additional edge modes transforming under a $U(|\Delta k|)$ global symmetry



## 2. Domain Walls in $U(N)$ Yang-Mills-Chern-Simons Holography

### • Holographic Dual of YM-CS Theory: D7 Branes at tip of AdS<sub>5</sub>-Soliton

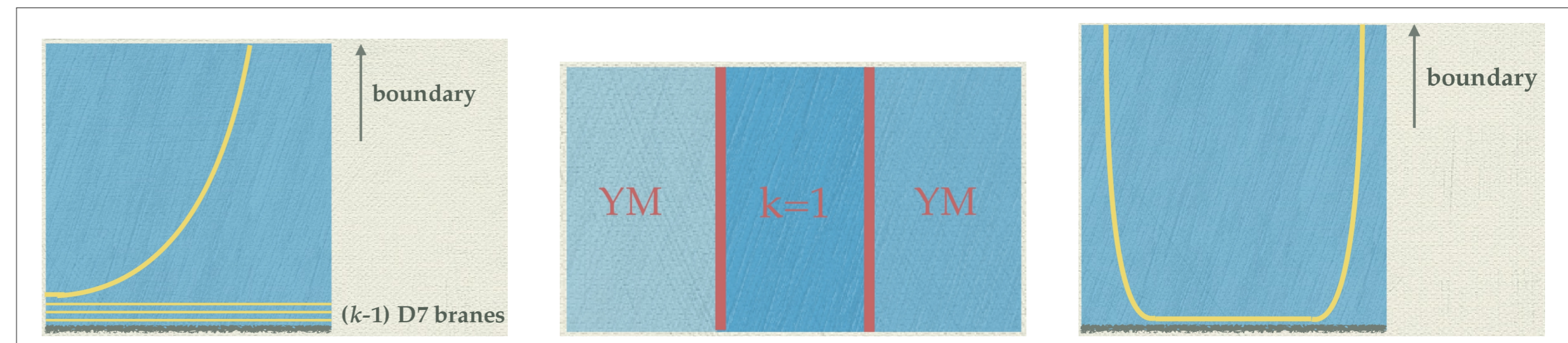


- N=4 SYM on circle, AP b.c.s for fermions  $\Rightarrow$  3D YM Theory in gapped phase
- D7 Branes source k units of C<sub>0</sub> flux on circle, induces Chern-Simons term

$$\Rightarrow \frac{1}{8\pi^2} \int_{M_4} \text{Tr}(F \wedge F) C_0 = \frac{1}{8\pi^2} \int_{M_4} \omega_3(A) \wedge (kd\theta) = \frac{k}{4\pi} \int_{M_3} \omega_3(A) \quad (4)$$

$\Rightarrow$  Dual low energy effective theory: Yang-Mills-Chern-Simons in confined phase

### • Domain Walls: D7 Brane Defects at asymptotic boundary



### • FQH Sample: Special case with $k_1 = k_3 = 0$ . Will mostly consider $k_2 = k = 1$ .

## 3. Properties of Defect Field Theory Dual

### • Level-Rank Duality: $\int_{D7} P[C_4] \wedge \text{Tr}(F_7 \wedge F_7) = N \int_{D7} \omega_3(A_7)$

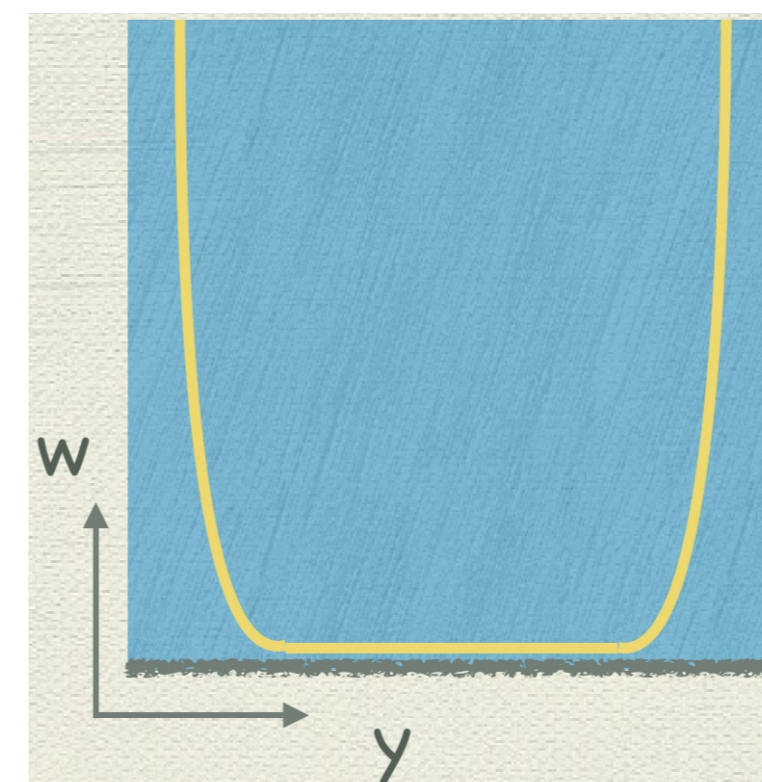
[3]

### • Chiral Fermions: $S_{3-7} = \frac{1}{2\pi} \int_I d^2x q_R^\dagger (i\partial_+ + A_+) q_R$

[4]

### • Field-Operator Dictionary:

[4]



Field	Asymptotics	J	VEV	$\Delta$ [4]	Operator [4]
y	$y_0 - \frac{Rw^3}{4w^4} + \dots$	y <sub>0</sub>		3	$\bar{q}\gamma_- F_{+y} q$
A <sub>-</sub>	$a_-^{(4)} w^4 + \dots + A_-^{(0)} + \dots$	a <sub>-</sub> <sup>(4)</sup>	a <sub>-</sub> <sup>(-4)</sup>	5	$\bar{q}\gamma_- F_{+y}^2 q$
A <sub>+</sub>	$\dots + A_+^{(0)} + a_+^{(-4)} w^{-4} + \dots$	A <sub>+</sub> <sup>(0)</sup>	A <sub>+</sub> <sup>(-4)</sup>	1	$\bar{q}\gamma_- q$

### • Edge Modes from Holography (RHS chiralities, LHS flipped)

Conserved current sector from boundary values of flat connections:  $A^{\text{flat}} = d\varphi$  [5]

Source:  $A_+^{(0)} = \lim_{r \rightarrow \infty} A_+^{\text{flat}}$ , VEV:  $\langle j_- \rangle = \lim_{r \rightarrow \infty} A_-^{\text{flat}}$ , Flatness Cond.:  $\partial_+ A_-^{\text{flat}} = \partial_- A_+^{\text{flat}}$

In the absence of sources:  $\partial_+ \langle j_- \rangle = 0 \Rightarrow$  Chiral Edge Current

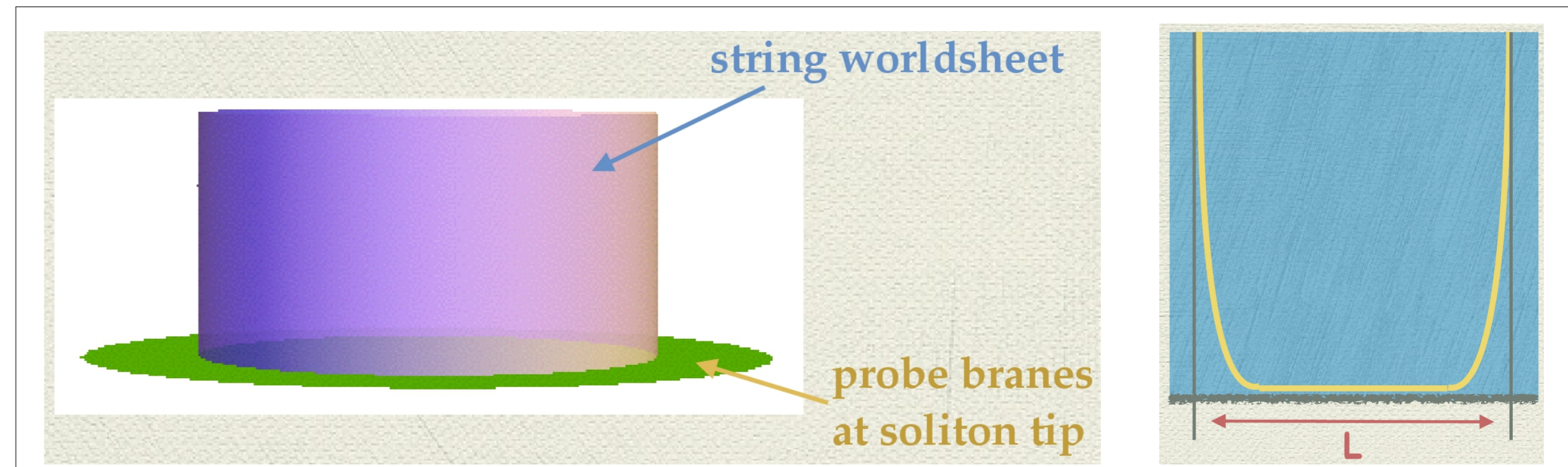
### • Anomaly in external electric field: $\partial_\mu j^\mu = -\partial_+ j_- \propto \partial_- A_+^{(0)} = E_{\text{ext}}$

[6, 5]

### • Chiral Symmetry Breaking $U(|\Delta k|)_L \times U(|\Delta k|)_R \rightarrow U(|\Delta k|)_V$

Nambu-Goldstone Boson:  $\varphi_+ - \varphi_- = \lim_{r \rightarrow \infty} \varphi(r, x^\pm) - \lim_{r \rightarrow \infty} \varphi(r, x^\pm)$

### • Domain Wall Tension & Confinement Free Energy: $F_{DW} \sim L$ as $L \rightarrow \infty$



### • Edge correlations: Source at right boundary leads to response on left boundary and vice-versa. Correlations seem nonlocal, mediated by fields at KK scale $\ell^{-1}$ :

$$\langle V_-^{\text{left}} V_+^{\text{right}} \rangle \sim e^{-2\pi L/\ell}. \quad (5)$$

## 4. Conclusions & Outlook

### • Conclusions:

Holographic dual to level-changing defects in YM-CS  
Top-down string theory construction similar to actual FQH samples  
Similarities to holographic QCD, Surprisingly rich structure: anomalies, chiral WZW models and edge states, unusual correlation properties, relationship with confinement, symmetry breaking, phase transitions...

### • Open Issues - Work in Progress:

Wilson line in some gauges ill-defined:  $\pi = \int A_r dr = \varphi_+ - \varphi_- + (\text{mesons}) \rightarrow \infty$  ???  
Proper Holographic Renormalization of YM-CS or DBI-CS

### • Outlook:

Introduction of external electric and magnetic background SUGRA fields:  $B_2 \sim F_{\text{ext}}$   
Calculation of Hall conductivity  
Extension to other brane intersections - Topological Insulators  
Results for nonabelian Edge modes? New insights into nonabelian CS theory?

## References

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