Compressible Fluids in the Membrane Paradigm:
non-AdS Fluid/Gravity correspondences

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Correspondence between black holes and fluids

Two frameworks of the correspondence

1. Membrane paradigm
2. AdS/CFT correspondence (Fluid/Gravity correspondence)

The matters on the surface behaves as a fluid

Surface
- Membrane paradigm: stretched horizon
- AdS/CFT correspondence: boundary of AdS

We consider the energy-momentum tensor on the surface.

Stress-energy tensor of fluid

on $(n+1)$-dim. spacetime (surface)

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

ε: energy density P: pressure u^μ: velocity field

Viscous stress tensor: $\tau^{\mu\nu} = -2\eta\sigma^{\mu\nu} - \zeta\theta\Delta^{\mu\nu}$

Shear: $\sigma^{\mu\nu} = \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{n}g_{\alpha\beta}\nabla_\gamma u^\gamma\right)$

Expansion: $\theta = \nabla_\gamma u^\gamma$

Projection: $\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$

The velocity field is normalized as

$$u^2 = -1$$

Energy-momentum tensor on the surface

- Membrane paradigm

  The energy-momentum tensor is given by the Israel junction condition.

- AdS/CFT correspondence

  The energy-momentum tensor is given by the Brown-York tensor and counter terms.

The energy-momentum tensor is

$$T_{\mu\nu} = \frac{1}{8\pi G}\left(\gamma_{\mu\nu}\mathcal{K} - \mathcal{K}_{\mu\nu}\right) + (\text{additional terms})$$

Extrinsic curvature: $\mathcal{K}_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu n_\nu + \nabla_\nu n_\mu\right)$

$\gamma_{\mu\nu}$: induced metric

Additional terms are

- Contribution from the inside (membrane paradigm)
- Counter terms (AdS/CFT correspondence)

Membrane paradigm

For observers who stay outside the black hole, effects of the black hole can be replaced by those of a fluid on the (stretched) horizon.

The normal vector is tangent to the surface for the horizon

On the horizon, the only causal direction is null.

➡ The normal vector is identified to the velocity field.

$$n^\mu \sim u^\mu$$

Then, the shear and expansion are

$$\sigma^{\mu\nu} = \Delta^{\mu\rho}\Delta^{\nu\sigma}\mathcal{K}_{\rho\sigma} \qquad \theta = \Delta^{\mu\nu}\mathcal{K}_{\mu\nu}$$

The other terms are identified as

$$P = -u^\mu u^\nu \mathcal{K}_{\mu\nu} \qquad \varepsilon = \Delta^{\mu\nu}\mathcal{K}_{\mu\nu}$$

Transport coefficients

$$\eta = \frac{1}{16\pi G} \qquad \zeta = -\frac{1}{8\pi G}\frac{n-1}{n}$$

Negative bulk viscosity

AdS/CFT correspondence

- Surface is placed at the boundary of AdS
- Geometry is (basically) AdS (or D-brane geometries)

Holographic renormalization group

- The surface is placed at arbitrary radius
- Dirichlet boundary condition on the surface

Linear response theory

Identification of the velocity field is not necessary.

Non-linear method [Bhattacharyya-Hubeny-Minwalla-Rangamani '07]

Identification of the velocity field is imposed.

Linear response theory

Einstein equation
- Equation of propagative mode
- Constraints on the surface

Correspondence

Equations for fluids
- Continuity equation
- Navier-Stokes equation

We should impose EOM also in the fluid theory.

In the fluid side, by expanding in Fourier modes,
The velocity fields can be solved as (for sound mode)

$$u^\mu(\omega,k) = \frac{c^{\mu\rho\sigma}(\omega,k)h_{\rho\sigma}(\omega,k)}{\omega^2 - c_s^2 k^2 + \cdots}$$

at linear order in $h_{\mu\nu}$

Then the stress-energy tensor becomes

$$T^{\mu\nu} = G^{\mu\nu\rho\sigma}h_{\rho\sigma}$$

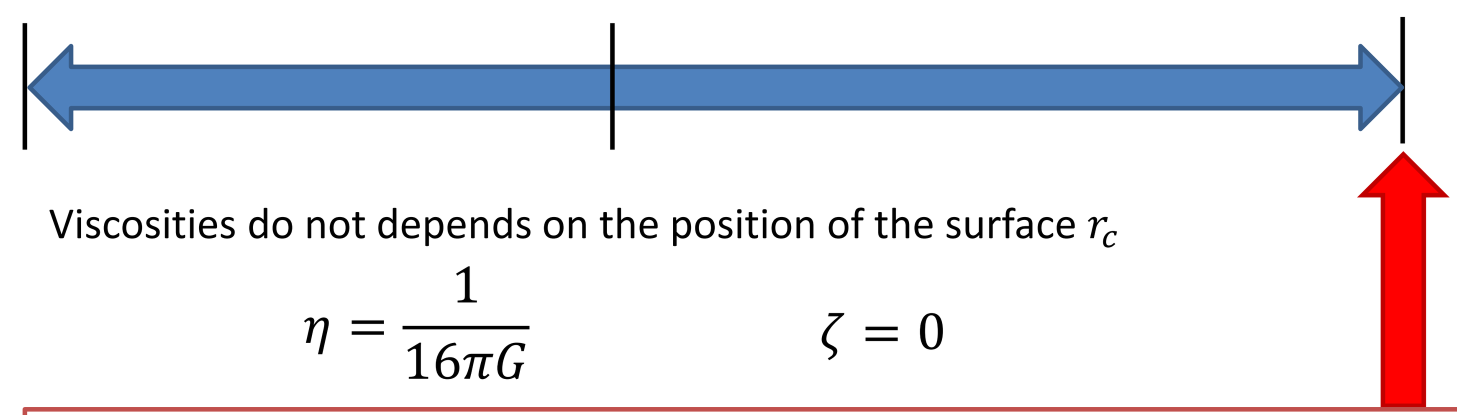
$$G^{\mu\nu\rho\sigma} = \frac{\mathcal{N}^{\mu\nu\rho\sigma}(\omega,k)}{\omega^2 - c_s^2 k^2 + \cdots}$$

A similar expression is obtained from the gravity side.

Transport coefficient can be calculated by comparing $G^{\mu\nu\rho\sigma}$ in both side.

Viscosities for Schwarzschild-AdS

Boundary $r\to\infty$ Surface $r = r_c$ horizon $r = r_H$



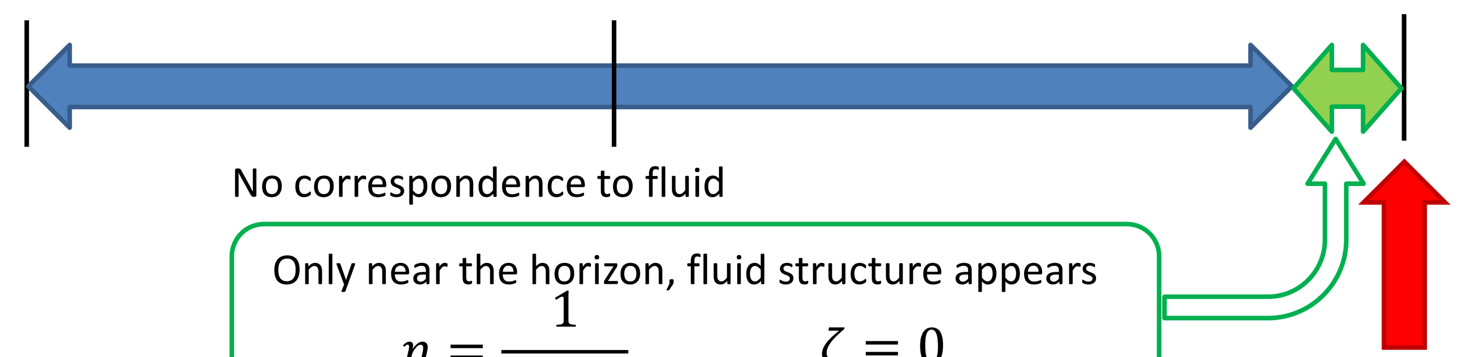
Viscosities do not depends on the position of the surface r_c

$$\eta = \frac{1}{16\pi G} \qquad \zeta = 0$$

For $r_c \to r_H$ (Rindler limit), the fluid becomes incompressible $\theta = 0$.

For non-AdS geometries (asymptotically flat Schwarzschild)

Boundary $r\to\infty$ Surface $r = r_c$ horizon $r = r_H$



No correspondence to fluid

Only near the horizon, fluid structure appears

$$\eta = \frac{1}{16\pi G} \qquad \zeta = 0$$

Incompressible

We considered the near horizon expansion.

Leading order
- Rindler limit
- Compressible mode vanishes

Next-to-leading
- Still corresponds to a fluid
- Compressibility appears

In other word, we considered slightly outside the Rindler limit.

Bulk viscosity for the Schwarzschild black hole

$$\zeta = 0$$

What is the problem of the old membrane paradigm?

- The surface is slightly outside of the horizon (at least as a regularization). Then, the normal vector is no longer equivalent to the velocity field.
- There is also an ambiguity how to distinguish the pressure and expansion.