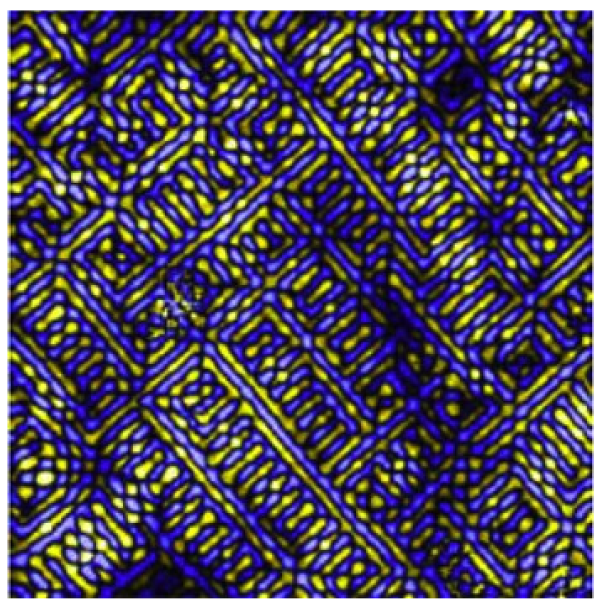
NORDITA Nordic Institute for Theoretical Physics

Charge density wave instability In holographic D-wave superconductor

Alexander Krikun, *JHEP 1404 (2014) 135*, arXiv:1312.1588

Abstract

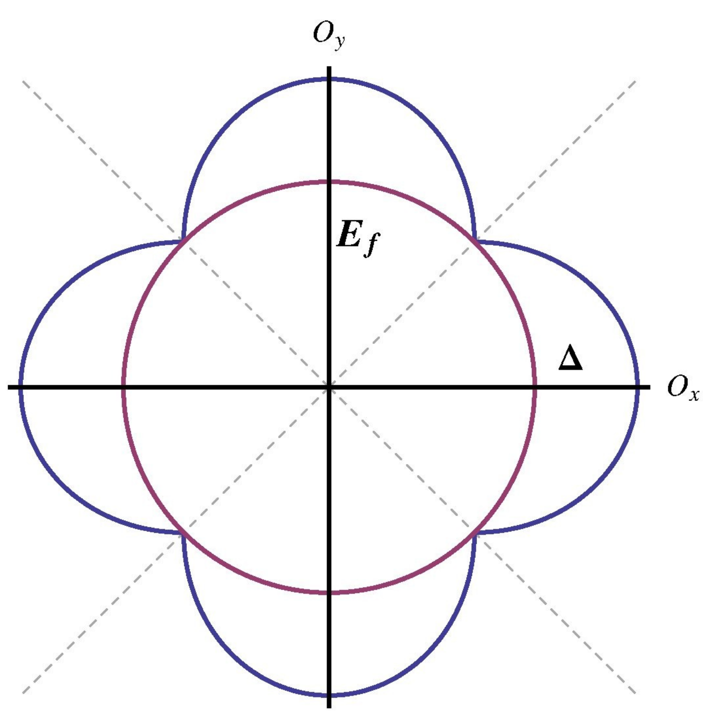
We study the stability of the holographic model of D-wave superconductor and find the **spatially modulated** static mode in the spectrum of fluctuations around the condensed phase. The mode involves the time component of the gauge field that is related to the **charge density wave** in the dual superconductor.



Holographic model of D-wave superconductor

The holographic model includes the spin 2 field propagating on the static backround of the charged black hole in 4D AdS.
F. Benini, C. P. Herzog, R. Rahman and A. Yarom, JHEP ***1011****, 137 (2010)*

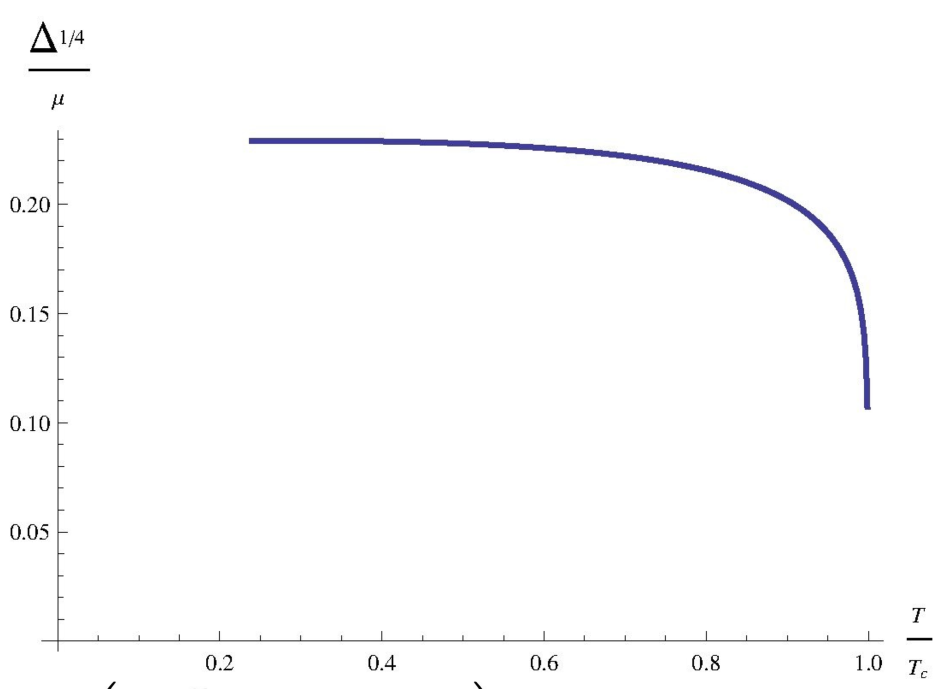
$$L = -|D_\rho \phi_{\mu\nu}|^2 + 2|\phi^\mu|^2 - 2\left(\phi^\mu (D_\mu\phi)^* + c.c.\right) + |D_\mu \phi|^2$$
$$- m^2(|\phi^{\mu\nu}|^2 - |\phi|^2) + 2R_{\alpha\beta\gamma\delta}\phi^{\alpha\gamma}\phi^{*\beta\delta} - \frac{R}{d+1}|\phi|^2$$
$$- iqF_{\mu\nu}\phi^{\mu\rho}\phi^{*\nu}_{\rho} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



with metric

$$ds^2 = \frac{L^2}{z^2}\left(-f dt^2 + \frac{dz^2}{f} + dx^2 + dy^2\right),$$
$$f(z) = 1 - \left(\frac{4\pi}{3}T\right)^3 z^3$$

At low temperature the zero-th order equations of motion have the solution with profile of the tensor field. This corresponds to the finite superconducting condensate with D-symmtry.

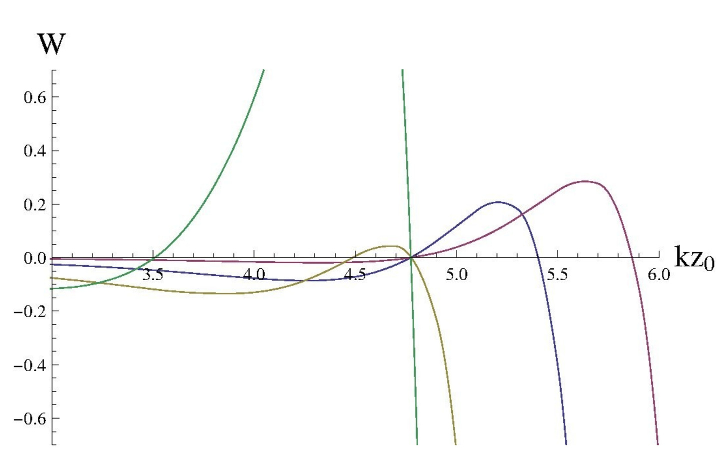


$$0 = \partial_z^2 \tilde{A}_t(z) - \frac{q^2}{z^2 f(z)}\tilde{\psi}^2 \tilde{A}_t,$$
$$0 = \partial_z^2 \tilde{\psi}(z) + \left(\frac{f'(z)}{f(z)} - \frac{2}{z}\right)\partial_z\tilde{\psi}(z) + \left(\frac{q^2\tilde{A}_t^2}{f(z)^2} - \frac{m^2}{z^2 f(z)}\right)\tilde{\psi}(z).$$

Numerical search for nontrivial static modes

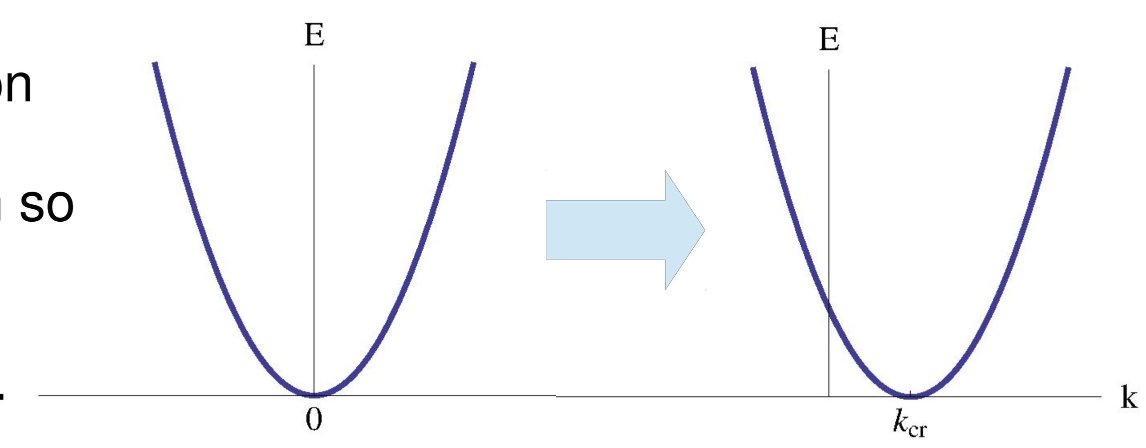
We construct two families of solutions by shooting from the opposite ends of the interval of integration and check, that the Wronskian vanishes independently of the connection point.

$$W(z) = \begin{vmatrix} \xi_1^1(z) & \dots & \xi_1^3(z) & \eta_1^1(z) & \dots & \eta_1^3(z) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \xi_3^1(z) & \dots & \xi_3^3(z) & \eta_3^1(z) & \dots & \eta_3^3(z) \\ \partial_z\xi_1^1(z) & \dots & \partial_z\xi_1^3(z) & \partial_z\eta_1^1(z) & \dots & \partial_z\eta_1^3(z) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \partial_z\xi_3^1(z) & \dots & \partial_z\xi_3^3(z) & \partial_z\eta_3^1(z) & \dots & \partial_z\eta_3^3(z) \end{vmatrix}$$



Spontaneous breaking of translation symmetry

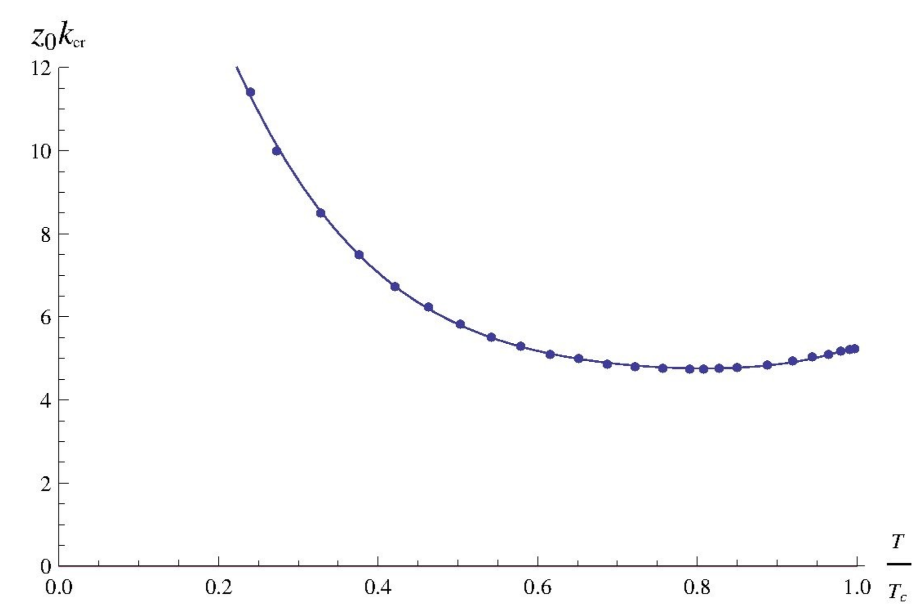
The mixing of modes in the equations of motion can lead to the shift of the dispersion relation so that the ground state corresponds to **nonzero momentum**.
A.Donos and J. P. Gauntlett, JHEP ***1108****, 140 (2011)*



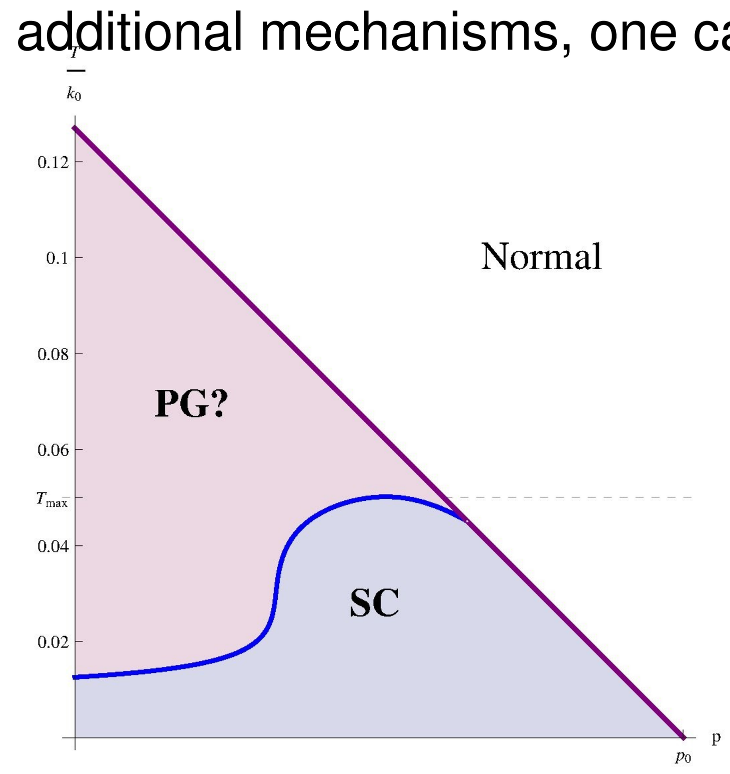
In the holographic model the equations for the excitations on the background of the **condensed phase** exhibit the mixing proportional to momentum

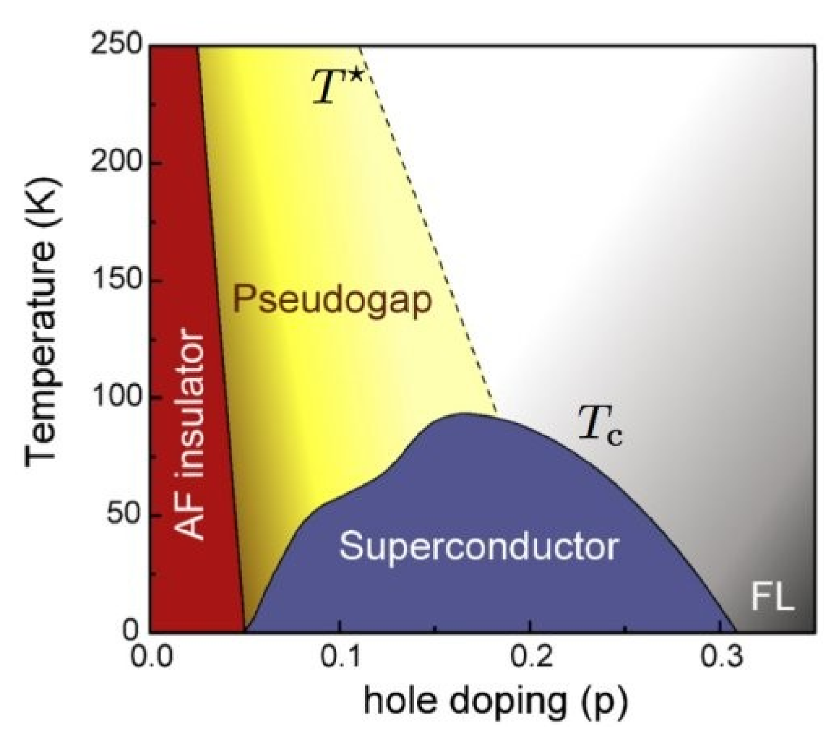
$$0 = \left[\partial_z^2 + \left(\frac{f'(z)}{f(z)} - \frac{2}{z}\right)\partial_z - \frac{m^2}{z^2 f(z)} + \frac{(\tilde{A}_t)^2}{f(z)^2}\right]\psi^1_{xy}$$
$$+ k_y 2z^2\left[\partial_z + \frac{f'(z)}{f(z)}\right]\psi^1_{zx} - k_y\frac{\tilde{A}_t}{f(z)^2}\psi^2_{tx} + 2\frac{\tilde{A}_t\tilde{\psi}}{f(z)^2}A_t$$
$$0 = \left[\partial_z^2 - \frac{2}{z}\partial_z - \frac{m^2 + z^2 k_y{}^2}{z^2 f(z)}\right]\psi^2_{tx} + k_y\frac{\tilde{A}_t}{f(z)}\psi^1_{xy}$$
$$+ 2z^2\left[\tilde{A}_t\partial_z + \frac{1}{2}\partial_z\tilde{A}_t\right]\psi^1_{zx} + k_y\frac{\tilde{\psi}}{2f(z)}A_t$$
$$0 = \left[-m^2 + \frac{z^2}{f(z)}(\tilde{A}_t)^2 - z^2 k_y{}^2\right]\psi^1_{zx} + \frac{1}{2f(z)}\left[\tilde{A}_t\partial_z + \frac{1}{2}\partial_z\tilde{A}_t\right]\psi^2_{tx} - \frac{k_y}{2}\partial_z\psi^1_{xy}$$
$$0 = \left[\partial_z^2 - \frac{k_y{}^2}{f(z)} - \frac{(\tilde{\psi})^2}{z^2 f(z)}\right]A_t - 2\frac{\tilde{A}_t\tilde{\psi}}{z^2 f(z)}\psi^1_{xy} + k_y\frac{\tilde{\psi}}{2z^2 f(z)}\psi^2_{tx}$$

We find the **static** nontrivial solutions to the equations of motion at nonzero k.
This signals the onset of the **spatially modulated phase** with corresponding momentum.



If one demands, that the the momentum of this phase is fixed by additional mechanisms, one can plot a phase diagram for the system





Surprisingly, the estimate for the maximum superconducting temperature, based on the data of cuprates, gives **reasonable value**.

$$T_{max} = 0.05\frac{\hbar c}{K_b}k_0 \sim 140\ ^\circ\mathrm{K}$$