Towards Collisions of Inhomogeneous Shockwaves in AdS

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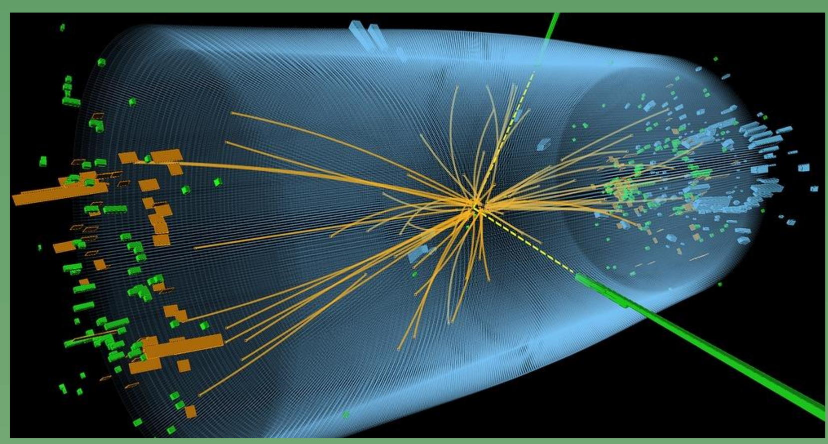
Holographic Model of a Heavy Ion Collision

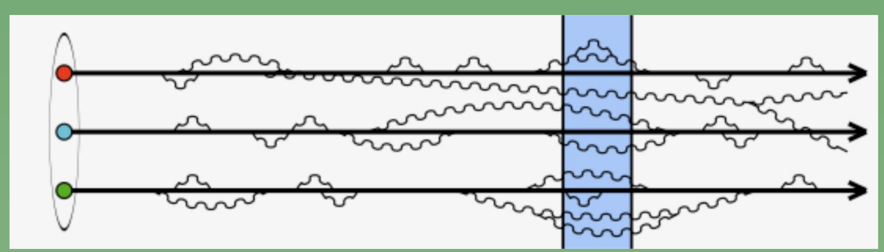
(Disclaimer: $\mathcal{N}=4$ SYM theory)
(Strong coupling)
(Large N_c)

RHIC: Au-Au ($Z=79$)
LHC: Pb-Pb ($Z=82$)

1.36 TeV per nucleon
Lorentz factor >1000

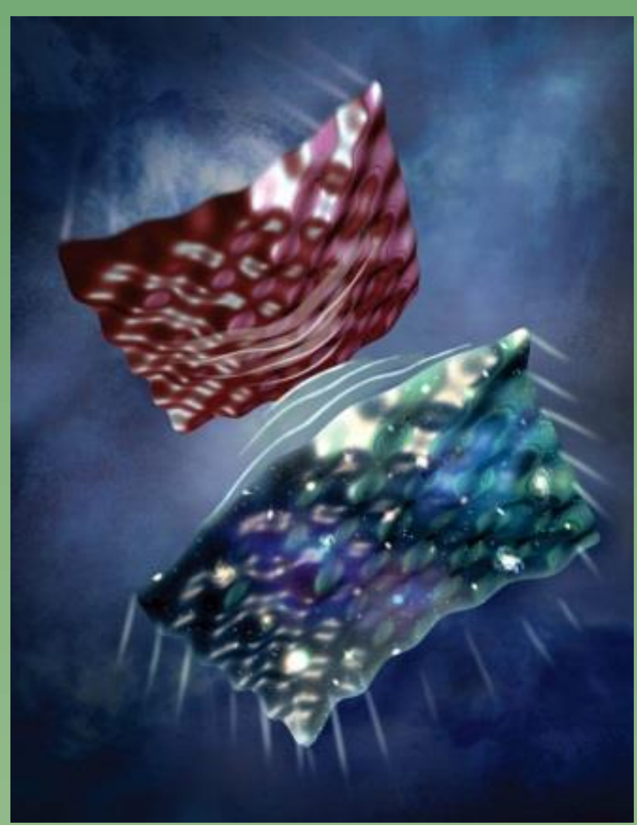


Result: **Quark-Gluon Plasma**, state of matter that lasts for $\tau \sim 10$ fm/c.



⋆ Long timescales: gluons are short-lived.
⋆ Strong interaction timescales: Gluons dominate the dynamics.

Holographic dual:
Collision of planar gravitational shockwaves in a higher dimensional AdS$_5$ spacetime, and the formation of a black hole.

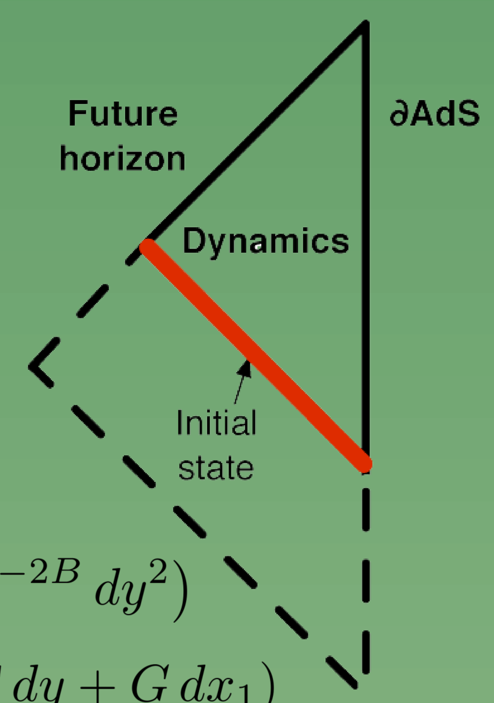


Characteristic Formulation of GR:

1) Eddington-Finkelstein coordinates with null r.
2) Determinant of spatial metric is a function (Σ).
3) Derivatives along outgoing null rays.

Bondi, Sachs

The structure of AdS makes these computations feasible



- Metric ansatz:

$$ds^2 = 2dtdr - A\,dt^2 + \Sigma^2\left[\cosh D\left(e^{B-C}\,dx_1^2 + e^{C-2B}\,dy^2\right) + \sinh D\left(2e^{B/2}\,dx_1dy\right) + e^B dx_2^2\right] + 2dt(F\,dy + G\,dx_1)$$

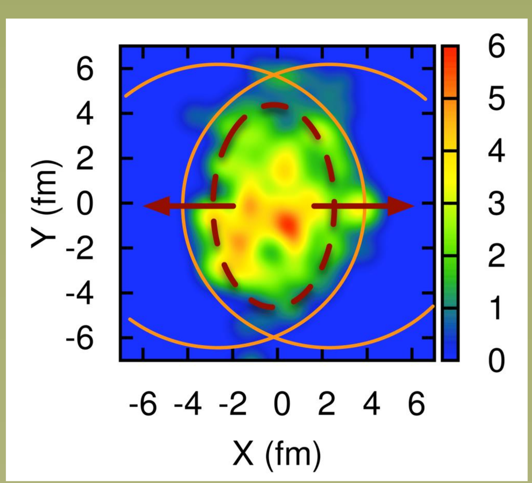
Chesler, Yaffe

- Boundary Conditions: A, F, G subject to evolution equations:

$$\partial_t a_4 = -\tfrac{4}{3}\,\partial_y f_4\,,$$
$$\partial_t f_4 = -\tfrac{1}{4}\partial_y a_4 - 2\partial_y b_4\,,$$
$$\partial_t \delta a_4 = -\tfrac{4}{3}\,(\partial_y \delta f_4 + ik\,\delta g_4)\,,$$
$$\partial_t \delta f_4 = -\tfrac{1}{4}\partial_y \delta a_4 - 2\partial_y \delta b_4 + \partial_y \delta c_4 + ik\,\delta d_4\,,$$
$$\partial_t \delta g_4 = -\tfrac{1}{4}ik\,\delta a_4 + ik\,\delta b_4 - ik\,\delta c_4 + \partial_y \delta d_4\,.$$

- Initial Conditions: spatial part of the metric except Σ: B, C, D at initial time slice.

Introduction of Inhomogeneities



$$h(r,t,y,x_1) = h_0(r,t,y) + e^{ikx_1}\delta h(r,t,y)$$

where $C_0=0$, $D_0=0$, $G_0=0$. $[k\in\mathbb{R}]$

(δh terms are treated as perturbations)

Motivation:

- The experiments are not homogeneous at all.
- Make contact with elliptic flows, etc...
- Generalize the spectrum of QNM to non-zero momentum.
- See if transverse expansion rate is faster or slower than the longitudinal.
- Since symmetry is not forced, we may see turbulence.

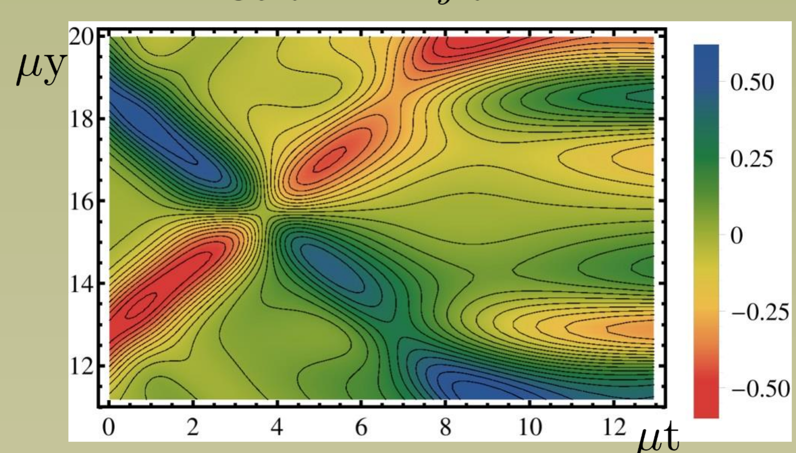
Summary of Results

Output: Examine the evolution of the post-collision stress-energy tensor:

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \mathcal{E} & \mathcal{S}_y & \mathcal{S}_{x_1} & 0 \\ \mathcal{S}_y & \mathcal{P}_y & \mathcal{T} & 0 \\ \mathcal{S}_{x_1} & \mathcal{T} & \mathcal{P}_{x_1} & 0 \\ 0 & 0 & 0 & \mathcal{P}_{x_2}\end{pmatrix}$$

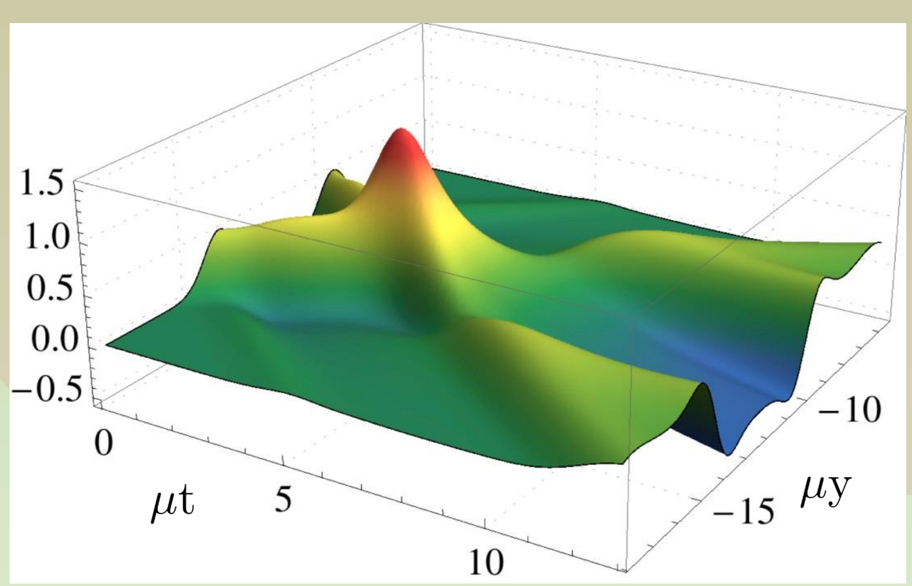
which can be read off from the asymptotics.

Inhomogeneity on longitudinal energy flux $\delta\mathcal{S}_y$ for $k=0.5$:

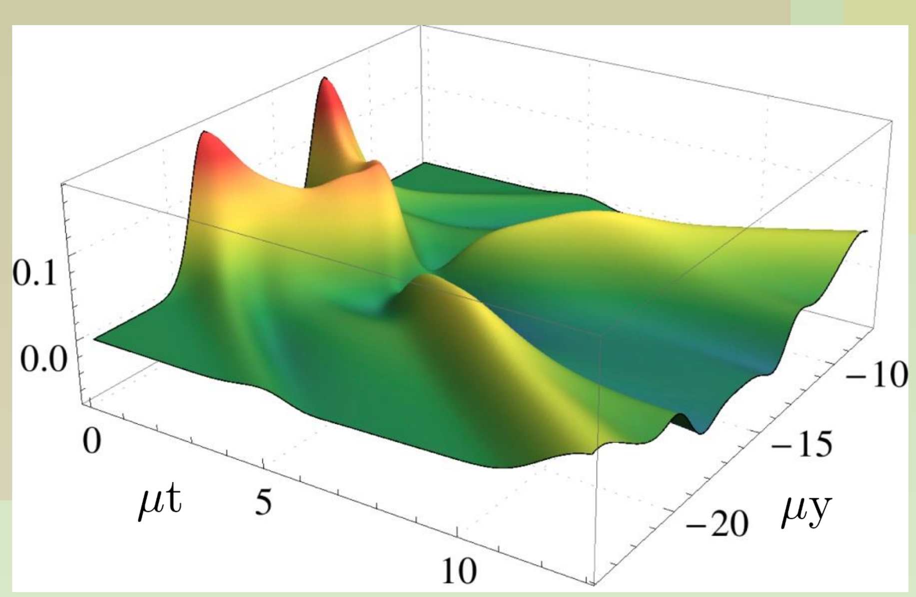


Common feature: An uneven profile in these inhomogeneities still remains.
Transverse dynamics $\Rightarrow$ Longer thermalization time?

Inhomogeneity on the energy density $\delta\mathcal{E}$, for $k=0.2$:

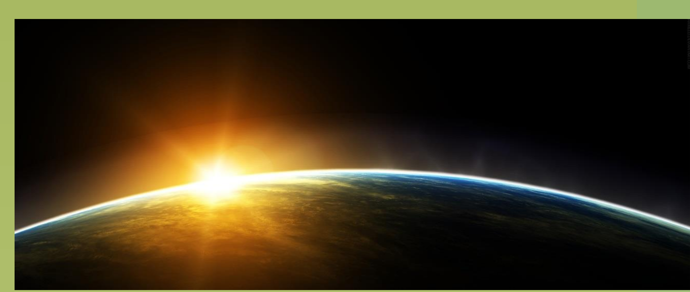


Inhomogeneity on the pressure anisotropy $\delta\mathcal{P}$, for $k=0.2$:



Apparent Horizon

Def:
Surface where outgoing light rays are trapped.



$\longrightarrow$ Its position must be fixed, via:
Residual gauge freedom: $r \to r + \xi(x^\mu)$

$\Rightarrow$ Choose $\xi(t,y)$ so that $r_{\rm AH}=1$.

Assuming AH to lie at a fixed $r_{\rm AH}$, $\left(\text{If } r_{\rm AH}(t,y)\text{, modify: } F \to F + \frac{\partial r_{\rm AH}}{\partial y}\right)$

$$3\Sigma^2\,\dot{\Sigma} - \partial_y(e^{2B}\,F\,\Sigma) + \tfrac{3}{2}e^{2B}\,F^2\,\partial_r\Sigma\Big|_{r=r_{\rm AH}(y)} = 0$$

Initial Data

- Profile of a planar shock: $\mathcal{H}(t,y) \equiv \frac{\mu^3}{\sqrt{2\pi w^2}}e^{-(t\mp y)^2/2w^2}$ $\Rightarrow$ Extract $\begin{cases} B(t=0,r,y) \\ a_4(t=0,y) \\ f_4(t=0,y)\end{cases}$
- Choice for initial perturbations:
$a_4 \to a_4(1+\epsilon e^{ikx_1})$, $f_4 \to f_4(1+\epsilon e^{ikx_1})$ and $\delta g_4 = 0$

Einstein eqs. fix the radial dependence for:

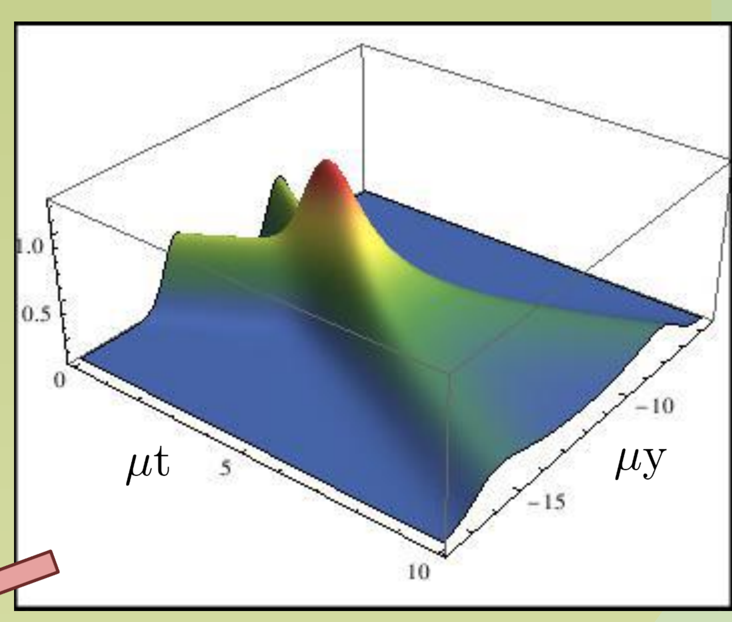
$$\delta B(0,r,y) = \frac{a_4(0,y)}{4r^4}, \qquad \delta C(0,r,y) = \delta D(0,r,y) = 0$$

- Background energy density: $\delta = 0.075\mu^4$

Numerical Evolution:

GR's coupled partial differential eqs. $\Longrightarrow$ Nested set of linear ordinary differential eqs.

For this data, energy is spread out by $t \sim 10/\mu$ for the background



Dynamic background of B

Main References

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- P. Chesler and L. Yaffe, *Phys.Rev.Lett.* **106**, 021601 (2011), [arXiv:1011.3562].
- M. Heller, R. Janik, and P. Witaszczyk, *PRL* **108**, 201602, [arXiv:1103.3452].
- J. C.-Solana, M. Heller, D. Mateos, W.v.d. Schee, *PRL* **111**, [arXiv:1305.4919].
- Wilke van der Schee, *Phys. Rev. D* **87**, 061901 (2013), [arXiv:1211.2218].