Superfluid Hydrodynamics, Thermal Partition Function and Lifshitz Scaling

Shira Chapman, Carlos Hoyos and Yaron Oz

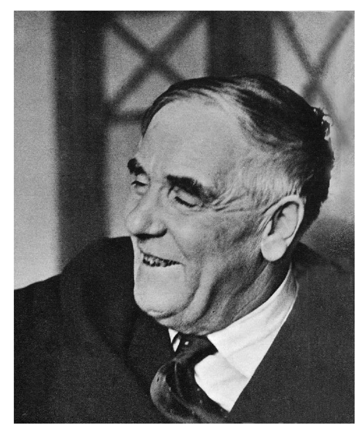
Tel Aviv University, Israel

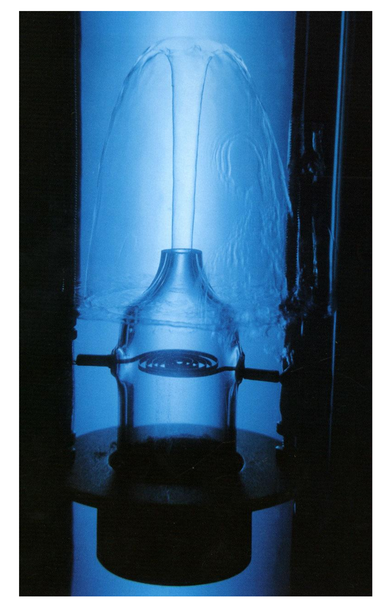
Abstract: We present the results of two separate studies, both in the field of superfluid hydrodynamics (with small modifications the results can be applied also to superconductors). In the first work we present an algebraic framework for deriving Kubo relations for superfluids. These allow to evaluate superfluid transport coefficients in terms of correlation functions of Feyman diagrams of the microscopic theory. In the second work we present the results of our studies on the hydrodynamics of superfluids that obey a Lifshitz scaling symmetry. This symmetry is believed to underlie the exotic properties of heavy fermion compounds and other materials including high T_C superconductors.

Superfluids

Superfluidity

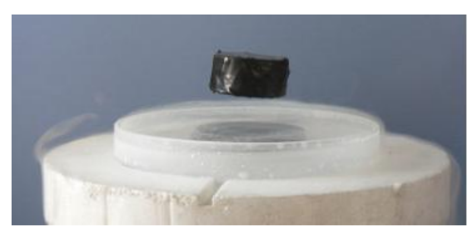
- Liquid state at very low temperatures.
- Flows with zero viscosity, zero entropy.
- Ability to self-propel and travel against the forces of gravity and surface tension.
- First observed in ^4He at 2.17^0C, by Pyotr Leonidovich Kapitsa in 1937.
- Origin of the effect – Bose Einstein condensation. Collective mode.





Superconductivity

- Zero resistance to DC current.
- Meissner effect – expulsion of magnetic fields.
- Condensation of Cooper pairs.

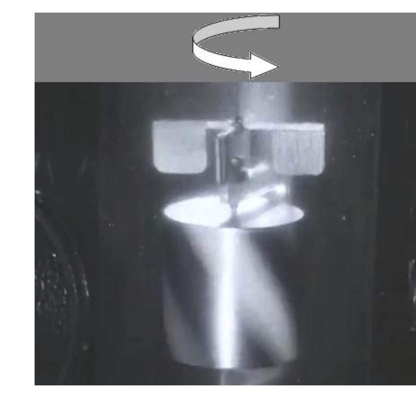


Two Fluid Picture

- 1st Experiment: leaking glass with ultra-fine capillarities → no viscosity.
- 2nd experiment: rotating cylinder induces rotation in rotating wheel → non zero viscosity.

Resolution: effective fluid picture due to Landau: two fluid flows $\vec{v}_n$, $\vec{v}_s$ (normal excitation, superfluid excitation).

- Fountain effect – zero entropy carried by superflow part.
- Two fluid picture valid also in superconductors.



Relativistic Superfluids

- Condensation → VEV of charged scalar operator.
- Phase of scalar field → massless Goldstone mode.
  - Does not disappear in averaging → participates in the hydrodynamics.
- Fluid variables:
  - T - Temperature
  - μ - Chemical Potential
  - u^μ - Normal flow component 4 velocity
  - $\xi^\mu = -\partial_\mu \phi$ - Gradient of Goldstone phase
  - $u_s{}^\mu$ - superflow component 4 velocity
- New thermal parameter ξ^μ, modified thermal relations: $dP = sdT + q_n d\mu + \frac{f}{2} d\xi^2$
- Stress energy tensor and current:

$$T^{\mu\nu} = \varepsilon_n u^\mu u^\nu + P(\eta^{\mu\nu} + u^\mu u^\nu) + \varepsilon_s u_s{}^\mu u_s{}^\nu + \pi^{\mu\nu}$$

$$J^\mu = q_n u^\mu + q_s u_s{}^\mu + j^\mu_{diss}$$

- Hydrodynamics – long wavelength (derivative) expansion. $\pi^{\mu\nu}$, j^μ_{diss} - first derivative corrections.

Two ways to obtain hydrodynamic corrections:

1. Require positive entropy production rate:

$$\exists\, s^\mu = su^\mu + \ldots \text{ with } \partial_\mu s^\mu \geq 0$$

entropy current is related to the stress-energy tensor and current → constraints on the dissipative corrections.

2. Write the **most general equilibrium** (time independent) **partition function** (effective action) for superfluids on **curved background**:

$$ds^2 = -e^{2\sigma(\vec{x})}(dt + a_i(\vec{x}))^2 + g_{ij}(\vec{x}) dx^i dx^j$$

Use the equilibrium partition function to derive corrections to stress tensor and currents:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \ln Z}{\delta g^{\mu\nu}}; \qquad J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta \ln Z}{\delta A_\mu}$$

- **Result**: corrections to the current:

$$j^\mu_{diss} = \sigma_\perp E^\mu_\perp + \sigma_\parallel E_\parallel \cdot \xi^\mu / \xi + \tilde{\kappa}_B B^\mu + \tilde{\kappa}_\omega \omega^\mu + \ldots$$

where E^μ, B^μ are the electric and magnetic fields, ω^μ is the vorticity.

Superfluid Kubo Formulas from Partition Function [1310.2247]

Goal

- Get expressions for transport coefficients in terms of correlation function of stress tensors and currents. These are the **Kubo formulas**.
- This allows to evaluate them in a microscopic theory using Feynman diagrams (at weak coupling) or holography (at strong coupling).

Method

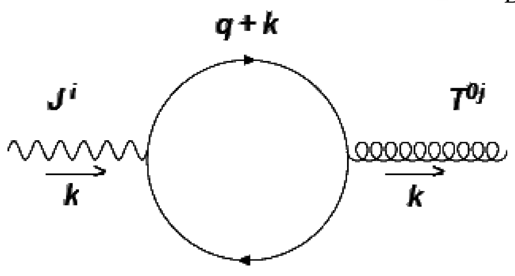
- For transport coefficients that are manifested in time independent flow one can do this by multiple differentiation of the equilibrium partition function.

Results

- We derived Kubo formulas for e.g. $\tilde{\kappa}_B$ and $\tilde{\kappa}_\omega$:

$$\tilde{\kappa}_B(T,\mu,\xi^2) = \lim_{k_z \to 0} \frac{i}{k_z} \left\langle J^x(k_z) J^y(-k_z) \right\rangle_{\substack{\omega=0 \\ \vec{\xi}=\xi_z}}$$

$$\tilde{\kappa}_\omega(T,\mu,\xi^2) = \lim_{k_z \to 0} \frac{2i}{k_z} \left\langle J^x(k_z) T^{0y}(-k_z) \right\rangle_{\substack{\omega=0 \\ \vec{\xi}=\xi_z}}$$

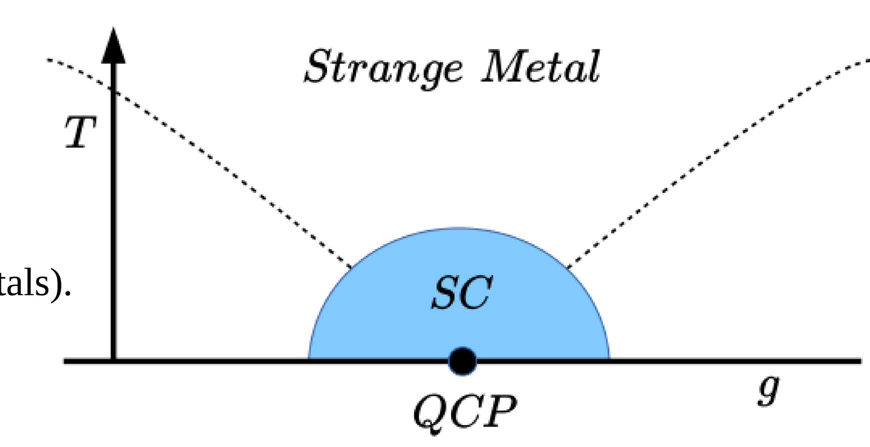


- The superflow velocity should be taken parallel to the external momentum.
- The role of the spatial superfluid velocity is similar to chemical potential.
- Substitution rules for propagators: $q^\mu \to (i\omega_n + \mu, \vec{q} + \vec{\xi})$

Outlook: evaluate in weakly coupled theory or holographic model.

Quantum Critical Points and Lifshitz Scaling

- Anisotropic Weyl – Lifshitz scaling symmetry: $t \to \Omega^z t$, $x^i \to \Omega x^i$
  - z – dynamical critical exponent. Measured from correlators.
- Must be accompanied by **broken boost invariance**.
- Phase transitions at zero temperature:
  - Driven by quantum fluctuations.
  - Quantum tuning parameter [B, doping, pressure].
- Infinite correlation length – scale invariance.
- Influence of QCP felt way above T=0.
- Strange metal behavior $\rho \sim T$ ($\rho \sim T^2$ in normal metals).
- Characteristic of high T_c superconductors.



Hydrodynamics of Lifshitz Superfluids [1402.2981]

- The currents of Lorentz symmetry obey: $\partial_\mu J^{\mu\alpha\beta} = T^{\alpha\beta} - T^{\beta\alpha}$
- **Broken boost invariance → antisymmetric part in energy-momentum tensor** in the direction of broken boosts.
- Assumption: fluid can be described by former variables.
  - No need for external time vector – the direction of time is determined by the normal fluid four velocity.
- $T^{[\mu\nu]} = u^{[\mu} V_A^{\nu]}$ for some vector V_A^μ.
- Stress tensor:

$$T^{\mu\nu} = (\varepsilon_n + p) u^\mu u^\nu + p\eta^{\mu\nu} + \varepsilon_s u_s^\mu u_s^\nu + \pi^{(\mu\nu)} + u^{[\mu} V_A^{\nu]}$$

- In the **Clark Putterman frame** one can decompose:

$$\pi^{(\mu\nu)} = \underbrace{\left(Q^\mu u^\nu + Q^\nu u^\mu\right)}_{\text{Heat flow}} + \underbrace{\Pi P^{\mu\nu}}_{\text{Trace part}} + \underbrace{\Pi_t^{\mu\nu}}_{\text{Traceless}}$$

where $P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ is the transverse projector and Q^μ represents the **heat flow**.

Goal

- Determine dissipative corrections: $Q^\mu, \Pi, \Pi_t^{\mu\nu}, V_A^\mu$

Method

- Require entropy increase (positive squares, $a^\mu \equiv u^\nu \partial_\nu u^\mu$ is the acceleration):

$$\partial_\mu s^\mu = -\frac{\left[\Pi\left(\partial_\mu u^\mu\right) + \Pi_t^{\mu\nu} \sigma_{\mu\nu}\right]}{T} - \frac{Q_\mu}{T}\left[a^\mu + P^{\mu\nu} \frac{\partial_\nu T}{T}\right] - \frac{V_{A\mu}}{2T}\left[a^\mu - P^{\mu\nu} \frac{\partial_\nu T}{T}\right] + \ldots \geq 0$$

Results

- Parity even analysis only.
- **8 new transport coefficients in the T-preserving sector**, 6 in the T-breaking sector (compared to non-Lifshitz superfluids).
- In the non relativistic limit - heat flow: $Q^i = \begin{pmatrix} \text{temperature} \\ \text{gradients} \end{pmatrix} + q_1 (\vec{\omega} \cdot \vec{a}) \omega^i + q_2 \left(\delta^{ij} - \frac{\omega^i \omega^j}{\omega^2}\right) a_j + \ldots$

where the counterflow: $\vec{\omega} = \vec{v}_s - \vec{v}_n$

- Heat flow proportional to **acceleration of normal flow.**
- **Anisotropy** between parallel/perpendicular directions w.r.t counterflow.
- Hard to disentangle from effect of shear viscosity.
- Perhaps some unusual frequency dependence would reveal the effect in superconductors.

Outlook

- Suggest a measurement in superconductors.
- Include parity violating effects.
- Construct holographic model for Lifshitz superfluid.