REVIEW

Extra Dimensions and Warped Geometries

Lisa Randall

The field of extra dimensions, as well as the hypothesized sizes of extra dimensions, have grown by leaps and bounds over the past few years. I summarize the new results and the reasons for the recent activity in this field. These include the observations that extra dimensions can be macroscopic or even infinite in size. Another new development is the application of extra dimensions to the determination of particle physics parameters and properties.

We generally take it for granted that we live in a world where there are three infinite spatial dimensions. In fact, we rarely give this fact much thought; we readily refer to left-right, forward-backward, and up-down.

Yet the most exciting developments in particle physics in the past few years have involved the recognition that additional dimensions might exist and furthermore might play a role in determining our observable world. New theoretical discoveries are evolving at a very rapid rate. As we will see, the potential implications range from experimental signatures of extra dimensions, to understanding fundamental questions about the nature of gravity, to new insights into the evolution of our universe.

One of the chief motivations for considering additional dimensions came from string theory, which in turn is motivated by the failure of classical gravity to work at very short distance scales or, equivalently, at very high energies, where quantum mechanical effects cannot be neglected. The only known way to consistently reconcile quantum mechanics with Einstein's theory of gravity is string theory, in which the fundamental objects that constitute our universe are not particles but (very tiny) extended objects: strings. In what follows, I use very little of the full formalism that has been developed to describe string theory. However, I am motivated by one very important fact. It appears that one can only have a consistent string theory that can describe the known particles if there are many additional spatial dimensions: six or seven, depending on how one looks at it. The question is, then, why don't we see these additional directions? What has become of them? Can they play any role in the physics we see? And is there any chance we will observe them soon?

Here I describe the two known ways to incorporate additional dimensions of space that are consistent with what we see, or rather what we don't see; namely Kaluza's (*1*, *2*) original idea of curling them up into little balls (compactification) or the more recent proposal by Sundrum and myself of focusing of the gravitational potential in a lower dimensional subspace (localization) (*3*, *4*). These ideas are important in and of themselves; one or the other would be the reason we so far haven't observed evidence for extra dimensions, should they exist. I then go on to explore another exciting development in the field of extra dimensions. This is the fact that even though the dimensions have not yet been seen directly, their existence might explain important features of the observed standard model and will be observed in the near future should these conjectures prove correct.

Hidden Dimensions

Before proceeding, it is useful to describe several of the ways in which we determine that there are three dimensions of space. Certainly, that is all that we see at a casual glance, but science often consists of probing beyond what is manifestly "evident." The key to understanding the experimental and astrophysical determination of the dimensions of space is to consider the gravitational force law that says that the force between two masses is $G_N m_1 m_2/r^2$, where r is the separation between the two masses. I use units where $(h/2\pi) = c = 1$ (h, Planck's constant; c, the speed of light). Newton's constant, G_N, is $(10^{-33}\ \text{cm})^2$, which is very small. It is inversely proportional to M_P^2, where M_P, the Planck mass, is about 10^{19} GeV. This mass scale appears because it is associated with the graviton coupling, where the graviton is the spin-2 particle that mediates the gravitational force (*5*). Such energies can only be probed at the very tiny Planck length, 10^{-33} cm. The position dependence of the force law is readily understood as a consequence of the isotropy of space—the fact that the laws of physics do not distinguish any particular direction. Imagine that one can draw the gravitational force as a set of lines emanating in all directions from a massive source, so that the density of lines determines the strength of gravity.

It is clear that as we measure the force at increasing distance r, the strength is proportional to $1/r^2$, or the inverse of the surface area of a sphere drawn at the distance r (because all force lines penetrate the sphere's surface). Because the gravitational force between two masses is proportional to the product of their masses, the $1/r^2$ form of the force law has important consequences for heavy macroscopic objects, such as planets. The force law is also measured on very small scales with much smaller objects. Here, the weakness of gravity is in evidence, and other, stronger forces can interfere with the measurement. The best measurement to date comes from an impressively accurate experiment by Adelberger's group at the University of Washington (*6*), where it has been determined that the $1/r^2$ force law persists down to distances on the order of a 10th of a millimeter. Deviations from this form on shorter distance scales are not excluded.

This means that according to detailed experimental observations, physics appears to reflect three spatial dimensions on distance scales ranging from a 10th of a millimeter to astrophysical and probably cosmological distances. Were there more than three spatial dimensions, the gravitational force should spread out in all these dimensions, and the force law would fall off faster, $1/r^3$ say, for one additional spatial dimension. Somehow, in order to agree with what we observe, this better not be the case, and was one of the first issues that needed to be addressed when additional dimensions were suggested.

Kaluza first proposed an additional dimension in his attempt to reconcile electromagnetism and gravity (*1*). His additional dimension had finite size; there are three infinite spatial dimensions, but a fourth one is trapped in a circle of size r_c in the extra dimension. Einstein, the referee of the paper, objected that the size of this additional dimension was not determined. Publication was delayed until 1926, when Klein observed that a circle of size determined by the Planck scale, 10^{-33} cm, is completely unobservable (*2*). This is readily understood intuitively; dimensions whose size we cannot resolve are observationally indistinguishable from no extra dimension at all. One common example to illustrate this is a garden hose; as viewed at a distance, it appears to be one dimensional. However, up close, one readily perceives that there are three dimensions. From the vantage point of the force law argument given above, the force law can be consistent with what we observe because isotropy is violated; the dimensions of finite size are readily distinguished from the "compactified" circular extra dimension. The force lines can only

Jefferson Physical Laboratory, Harvard University, Cambridge, MA 02138, USA. E-mail: randall@physics.harvard.edu

spread a distance r_c in the extra dimension. At larger distances, the force appears to be just that of a four-dimensional (three space plus one time) universe. This argument can also be used to see how the Planck scale of a four-dimensional world is related to that of a higher dimensional world. In the n-dimensional world, the force law dies off with r as $1/r^{n-2}$ and is inversely proportional to the higher dimensional "Planck" mass that determines the strength of the higher dimensional gravity, raised to the power $n - 2$. Because we average over the extra dimensions to see an effective lower dimensional theory, this scale, M, is related to the measured four-dimensional Planck mass scale by $M_P^2 = M^{n-2}V$, where V is the volume of the additional $n - 4$ dimensions (in the simple Kaluza example, the circumference of the circle. This means that although physics at short distances appears to be higher dimensional, as reflected in a gravitational force that goes as $1/(Mr)^{n-2}$, at distances longer than r_c, one would observe $1/(M_P r)^2$ (Fig. 1).

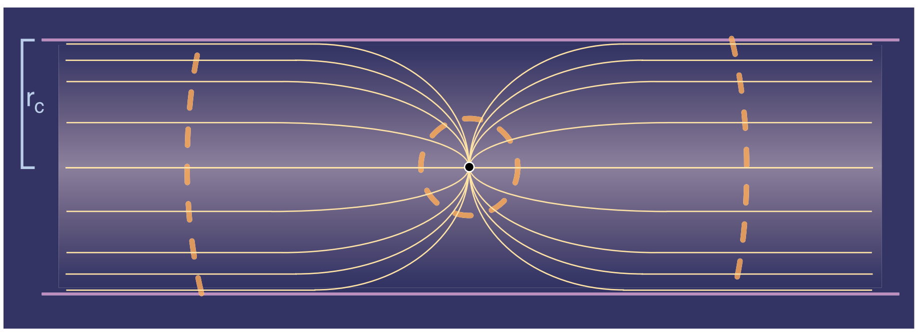

Fig. 1. For $r > r_c$, force lines only spread in the infinite-sized extra dimensions. For $r < r_c$, force is that of higher dimensional space (solid line is equipotential).

We conclude that additional dimensions are acceptable so long as they are of sufficiently small size. It was taken as conventional wisdom by string theorists that the length scale associated with the additional dimensions is the Planck length, 10^{-33} cm. That is, the additional six (or seven) dimensions are curled up in a manifold of extremely small (and therefore unmeasurable) size. The reason why the Planck length was the proposed scale is because there appears to be a single mass scale in string theory, and one would expect other mass scales to be of similar magnitude. Because string theory's major accomplishment is to permit a consistent formulation of quantum gravity, the Planck scale associated with the gravitational force is the natural scale.

However, we don't yet understand the physics leading to our four-dimensional world. From low-energy particle physics, we know there is much physics other than gravitational physics that is relevant to our world, and furthermore, mass scales much lower than the Planck mass are relevant. So it is too limiting to restrict ourselves to Planck-scale compactification. If other possibilities exist, they should be explored. In fact, experimental constraints tell us only that the extra dimensions must be smaller than about 10^{-16} mm, corresponding to the mass scale 10^3 GeV probed by current particle experiments. This means that from an experimental point of view, we really have no idea whether the dimensions are as small as the Planck length. They can be 16 orders of magnitude larger [see (*7*) for example].

In fact, that is not the last word on the possible size of extra dimensions. One of the new observations of the past few years was that of Arkani-Hamed, Dimopoulos, and Dvali (*8*) (ADD), who pointed out that radii 10^{16} times larger still (about a millimeter) are also consistent with all known observations if gravity only, and not other particles and forces, existed in the additional dimensions. Dimensions of this size could, however, be relevant to particle physics, as I explain in the next section. This observation was a consequence of another ingredient from string theory that has gained prominence over the past 10 years: the existence of branes.

In the string theoretical context, branes are precisely formulated extended objects that are necessary for the consistency of string theory, the importance of which was demonstrated by Polchinski (*9*). The term "branes" derived from "membranes"; it is perhaps simplest to envision these objects as membranes floating in a higher dimensional space. The properties of branes that are relevant to the study of extra dimensions are that (i) matter and forces can be confined to branes and (ii) branes carry energy, or tension. The first property means that we can have, for example, a brane with only three spatial dimensions, even though the full bulk space might have many extra dimensions. If a photon is stuck on this brane, it would not explore the extra dimensions. This implies that as far as electromagnetism is concerned, the extra dimensions do not exist. Clearly, this relaxes the constraints on the size of additional dimensions, because electromagnetism, which is extremely well studied, behaves just as it would in four dimensions, regardless of the size of the additional dimensions. However, the mechanism that confines electromagnetism to the brane cannot be applied to gravity; gravity must exist in the "bulk"; that is, the full spacetime. One way of understanding this is that the graviton is connected to the full spacetime geometry and is coupled to energy anywhere.

So the picture of ADD is that space is bounded by branes, where the standard model particles (but not gravity) are confined. This form of space had also been considered previously by Horava and Witten (*10*, *11*), with the extra dimension being larger than the Planck scale but not nearly so large as the size proposed by ADD. The reason why the ADD bound is precisely that of Adelberger's experiment (*12*) is that having thrown out the constraints from ordinary particle experiments, it is the direct test of the form of gravity at short distances that limits the size of the additional dimensions.

An Infinite (but Hidden) Extra Dimension

In fact, extra dimensions can be larger still; they can be infinite in size. This even more revolutionary idea also follows from the existence of branes, as Sundrum and I proposed. Gravity can be "trapped" and extra dimensions can have infinite spatial extent (*4*). This idea follows from a second property of branes: the fact that they carry energy. Because branes are not isotropic, the strength of the gravitational force varies according to distance from the brane. It turns out that this dependence on position is very strong if there is one additional dimension; the strength decreases exponentially with distance from the brane. A more precise formulation is the following: If there is an energetic four-dimensional flat brane in a five-dimensional spacetime, the five-dimensional space does not consist of flat, uniform, extra dimensions.

To accomodate a flat brane requires that in addition to the tension of the brane itself, there is a bulk vacuum energy, closely aligned to the brane tension. The solution to Einstein's equations is then described locally as anti–de Sitter (AdS) space, a space with a negative vacuum energy. And in this space, although it is fundamentally five dimensional and there is therefore a five-dimensional graviton, there exists a bound-state mode of the graviton that is highly concentrated on the brane and acts as if it were a four-dimensional graviton. In this geometry, the length of a yardstick depends on position. The spacetime is "warped"; it appears that the strength of the apparent four-dimensional gravity decreases exponentially with distance away from the "Planck brane" that traps the graviton. Another way of stating this is that although the strength of the gravitational coupling is the same everywhere, physical mass scales decrease exponentially with distance from the brane, so that gravity far from the brane is weak. The fifth dimension does not have to be finite in size, because unlike the case with flat extra dimensions, the gravitational force is spreading very little in the direction perpendicular to the brane.

To derive this form for the gravitational force, one solves Einstein's equations of general relativity in the presence of the brane. General relativity tells us that not only do gravitational forces affect matter, but matter determines the surrounding gravitational potential. In this case, the presence of the massive brane leads to a gravitational force that is highly concentrated near the brane. So although the extra dimension can be very large (or even infinite), the gravitational force is highly concentrated near the brane (Fig. 2).

One is only sensitive to a short distance scale (the scale over which the gravitational force decreases), rather than the size of the extra dimension. This is why the hidden dimension can even be infinite in size (*4*). Gravity is localized near the brane.

The model of (*4*) (known as RS2) is a concrete example in which space has five dimensions, but the world looks four dimensional. If one sits on the Planck brane, the world looks four dimensional up to very high energies, on the order of the Planck scale. As one ventures out into the fifth dimension, one would still measure a four-dimensional force law, but it would only apply for successively lower energies.

More recently, with Karch, I studied a theory that is in some ways even more surprising (*13*). In that theory, space looks four dimensional on the brane and at some distance away from the brane. This is because there is again a mode that looks like a four-dimensional graviton. However, the majority of the space is not sensitive to the force mediated by this four-dimensional graviton, because it only couples in a small portion of the space. The part of the space where the trapped mode does not couple sees itself as five dimensional. This leads one to consider the possibility of gravitational "islands"; the dimensionality of space you think you see depends on where you are in the bulk. The brane can be considered to be a four-dimensional sinkhole. This is truly a possibility of nature; we only ever see a finite region of space, even with cosmological observations. Our observation that the world we see looks four dimensional can be merely an accident of our location. The rest of the universe can be five-, or even 10 dimensional, and we wouldn't necessarily know it. Another interesting possibility with an infinite flat extra dimension was proposed by Dvali, Gabadadze, and Porrati (*14*, *15*). Clearly, there remain many possibilities to explore.

Extra Dimensions and the Hierarchy Problem

So far, I have focused on gravity itself and how one can accommodate a richer spacetime manifold than we think about intuitively, leading to new possibilities for what can be our true geometry. In this section, I discuss one way in which additional dimensions might be relevant to determining four-dimensional physics. The possibilities presented here generally involve a further layer of speculation in order to incorporate the physics that ties extra dimensions to observable low-energy scales. Such theories, in which the extra dimensions are tied to relatively low-energy scales, have the enticing possibility that they can be observed in the next generation of colliders. So whether or not we believe these scenarios, experiments in the near future should determine their validity.

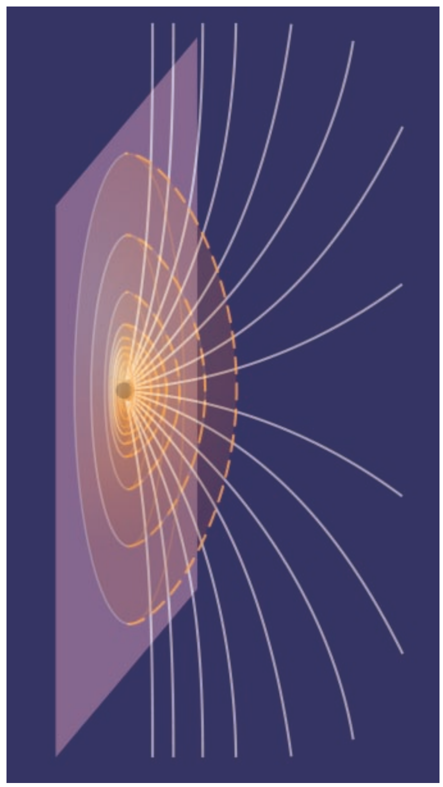

Fig. 2. Force lines and equipotential surfaces for the warped geometry. Force lines are denser near the brane.

One of the major goals of particle physicists in the past 20 years has been to solve a problem known as the hierarchy problem. The hierarchy problem is the fact that the electroweak mass scale of about TeV = 10^3 GeV is 16 orders of magnitude smaller than the Planck mass scale of 10^{19} GeV. The electroweak scale is the scale where standard-model particle masses (quarks and leptons) are generated. The Planck scale that we have referred to several times is the mass scale that determines Newton's constant. It is also the scale where quantum gravity effects should become important and new physics such as string theory should be relevant. In fact, we have already seen that Newton's constant is proportional to the inverse of the Planck mass squared. The hierarchy problem can be restated as the problem of why gravity is so exceptionally weak. The problem is that without any additional structure, there is no reason for two mass scales in the same theory to be so different. The problem is not only that we don't understand this mass scale, but it is difficult to consistently maintain this separation of scales in the context of the field theory through which we study the electroweak forces. Any interaction would tend to align these two highly disparate mass scales. Before the study of extra dimensions, it was thought that there was only one consistent resolution to this problem that has not already been excluded experimentally: namely, supersymmetry. Supersymmetry postulates the existence of additional particles that have the same mass as and related interactions with the known particles. These new particles cancel effects that tend to align masses, so that the electroweak scale can be as low, as we observe. However, we have no firm evidence yet that weak-scale supersymmetry resolves the hierarchy problem; this will only be known with the exploration of higher mass scales at the Tevatron, currently operating in Batavia, Illinois, and in the future with the Large Hadron Collider, located at CERN, near Geneva. One potential piece of evidence for supersymmetry is that the three couplings associated with the strengths of the strong, electromagnetic, and weak couplings unify at a common value at a high energy scale close to the Planck scale if the further assumption of no additional charged particles between the TeV and Planck scales is made (*16*–*20*). However, we will see that extra-dimensional theories can also be consistent with unification. Furthermore, there are potential problems with supersymmetric models. From experiment, we know that supersymmetry is not exact, so the masses of supersymmetric partners are not precisely degenerate with their known standard-model counterparts. There is no simple credible theory of supersymmetry breaking that satisfactorily explains all known observations. Until experiment narrows the field, it is certainly worthwhile to consider alternative explanations for the hierarchy.

The first proposal to address the hierarchy problem in the context of extra dimensions was also by ADD (*8*). Their fundamental insight was that the gravity and electroweak scales can be so different because of additional structure in the theory of gravity, as opposed to all previous ideas that tried to introduce new structure in the particle physics sector associated with the electroweak scale. ADD observed that with sufficiently large dimensions, one can equate the fundamental gravitational and weak interaction mass scales. This follows from the relation between Planck scales given above; a large volume permits M_P to be large, whereas M, the gravitational scale in the higher dimensional theory, is far lower, on the order of 10^3 GeV. This does not resolve the hierarchy but transforms it into a different problem, that of explaining the very large size of the extra dimensions. This proposal has many interesting experimental consequences. It turns out that with two extra dimensions, their size would be on the order of a millimeter, which is precisely the size that is explored in current precision tests of gravity. This was one of the chief reasons for the excitement associated with these theories and motivated the work of Adelberger (*6*), which ruled out deviations from Newton's law on scales of a millimeter. Furthermore, large extra dimensions that address the hierarchy problem would lead to observable consequences at the same mass scale we mentioned above in association with supersymmetry. The same experiments that search for supersymmetry can also search for large extra dimensions. For the ADD scenario, the signature would be missing energy; particles can collide to produce gravitational particles that escape into the extra dimension and are therefore not observed. Phenomenological and astrophysical constraints and implications of this scenario were considered in (*21*, *22*).

Certainly one unsatisfying feature of the large-dimension proposal is the difficulty in stabilizing large extra dimensions. But if one has uniform isotropic extra dimensions, the large volume is essential to explain the hierarchy. The weakness of gravity that we see as four-dimensional observers is due precisely to the fact that the gravitational force is spread out over a large volume. Sundrum and I, in a theory referred to as RS1 (*3*), realized that the very different geometry we had found, given a brane in a single extra dimension, can also address the hierarchy but with a rather modestly sized extra dimension if there is a second brane some distance away from the first. The geometry is very similar to RS2 but with space ending on the second brane.

This is due to the form of gravity; the strength of gravity decreases exponentially with distance from the brane because of the exponential rescaling of masses. The strength of gravity is not uniform; the gravitational force is weak away from the brane even without diluting the force over a large volume. The proposal is the following. Suppose that in addition to the Planck brane, which traps gravity, there is an additional brane separated from the first. Quarks, leptons, photons, and other ingredients of the standard model are stuck on this brane. Then the electroweak force sees only the second (TeV) brane, while gravity probes the entire space. Because the electroweak mass scale decreases exponentially with distance from the brane that traps gravity, a hierarchy in masses on the order of 10^{16} only requires a distance scale of order $\log 10^{16} \approx 35$. If one can naturally stabilize the length at this value, there is a natural solution to the hierarchy problem. The large number that separates the TeV and Planck scales arises from the fact that the gravitational coupling changes so rapidly (exponentially) over this relatively modest distance. Unlike the previous scenario, this is not a very large extra dimension but one of a relatively natural size. In this picture, there are separate physical theories confined to the two different branes. The TeV brane on which we live would house all the ingredients of the standard model. The Planck brane could be host to all sorts of other interactions we don't see. The only reason why the Planck brane is important to us is that it traps gravity, thereby explaining the hierarchy (Fig. 3).

However, because this scenario relied crucially on the separation of branes, it was essential to have a mechanism that could stabilize this distance. Goldberger and Wise (*23*) showed that this stabilization could be achieved in the presence of an additional scalar field, which is a particle whose energy is minimized for a particular value of the size of the fifth dimension. Subsequently, much work was done on this scenario. Recently, Giddings *et al*. (*24*) showed an example of a stabilized hierarchy derived explicitly from string theory based on an idea of Verlinde (*25*).

As with the large extra dimension scenarios, the experimental consequences of this warped geometry scenario (RS1) are quite dramatic. Although in the simplest scenario no new physics effects will occur in gravity experiments at a millimeter, there will be significant effects in high-energy particle physics experiments, should this scenario be correct. In the version of our theory presented in (*3*), there would be particles associated with the graviton (those that carrymomentum in the fifth dimension) that would be observed to decay in the detector into known particles such as an electron and positron that we can observe. This is a very distinctive signature; these particles would have spin 2, like the graviton, and would come with definite mass relations. There are other possibilities as well. In a variant of the original proposal (*26*), in which the second brane does not end space but resides in an infinite extra dimension (essentially combining RS1 and RS2), one would have missing energy signatures identical to those one would obtain with six large ADD-type extra dimensions. Other ranges of parameters for which low-energy tests, such as tests of gravity over short distances, might be relevant were considered (*27*).

Another remarkable feature of the warped metric solution to the hierarchy problem (RS1) is that the unification of couplings at a high energy scale can be readily incorporated (*28*, *29*). This is possible because, unlike the large extra dimension scenario, the TeV scale is not the highest energy scale accessible to the full higher-dimensional theory. Incorporating this feature means that RS1 can be considered as a theory with all forces unified, thereby achieving a major goal of particle physics.

Another interesting feature of this scenario is that because of the inclusion of high-energy scales, conventional inflation (*30*) can readily be incorporated. Moreover, it has also been shown to reproduce the known low-energy cosmology (*24*). This makes this theory a realistic candidate for the solution to the hierarchy.

Other Implications for Particle Physics

Extra dimensions can have other important ramifications for particle physics in our observable world. We have already discussed two ways in which they might address questions about the relative size of mass scales. There is another big difference between phys-

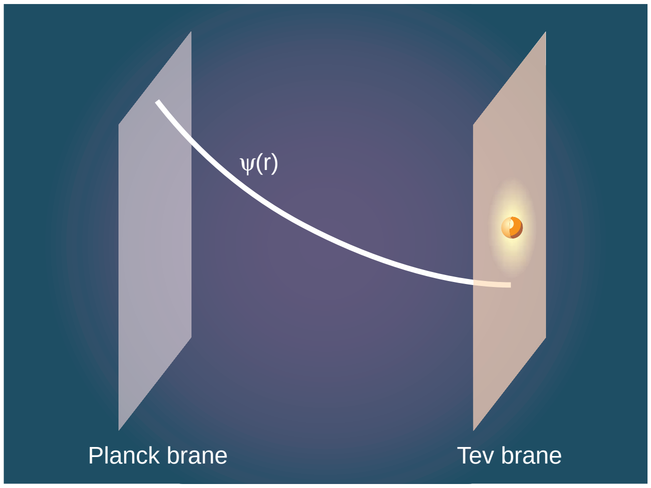

Fig. 3. $\Psi(r)$ is the graviton wavefunction. Gravity is weak because of the exponential suppression of $\Psi(r)$ on the TeV brane.

ics with additional dimensions and that with four: Particles and forces can reside somewhere else in an additional dimension. We briefly explain why this "sequestering" can help address many questions about why certain symmetries are more accurate than we would expect in a universe with only four dimensions.

In four-dimensional field theory, once a symmetry is violated in one interaction, it will filter into all possible symmetry-violating terms. And basically, anything that is not prevented by symmetry will be present. Even though supersymmetry is very constraining, once supersymmetry is broken, one has a plethora of interactions that are permitted. Particularly dangerous is the fact that in a broken supersymmetric theory, it is difficult to prevent "flavor violation": interactions terms that change one type of quark into another. Because there are strong experimental constraints on such processes, they should be forbidden or suppressed by the theory.

Sequestering (*31*) supersymmetry breaking can suppress the unwanted flavor-violating interactions. The idea is that the supersymmetry-breaking sector resides on a different brane than the observed standard-model particles. Isolating the physics in this way prevents the interactions that couple the supersymmetry-breaking and standard-model sectors that can violate flavor. There is another interesting feature of these theories. Because direct interactions between the supersymmetry-breaking and standard-model sectors are forbidden, supersymmetry breaking might be communicated only by particles associated with gravity. When this is true, there is a predictive spectrum, because the supersymmetry-breaking masses are determined by the known gravitational coupling and the known standard-model couplings. The simplest models have phenomenological difficulties, but these have been addressed [(*32–37*), for example]. Nonetheless, the models give a sufficiently different spectrum from the usual supersymmetric models to be distinguished experimentally (*31*, *38*).

Sequestering can also prevent dangerous symmetry violations in other contexts. For example, separating leptons and quarks (*39*) can sufficiently suppress dangerous violations of baryon number that would otherwise lead to proton decay. And one has the possibility of safely generating quark and lepton masses without introducing the flavor violation that we know to be suppressed (*40*).

Higher Dimensions: Are You in or out?

It should be clear at this point that we are learning fundamentally new things about new solutions to Einstein's equations; that is to say, viable gravitational backgrounds. We are also learning new ways to address problems in particle physics. Some of the more interesting recent work has sought to apply lessons we learned from extra dimensions to four-dimensional particle physics directly. There are essentially two ways in which this has been done.

The first, known as deconstruction, (*41*, *42*), originates with the observation that if a nongravitational higher dimensional theory is put on a lattice, with only the extra dimensions being discretized, one has a theory that is fundamentally four dimensional but has an energy regime cutoff in the ultraviolet by the lattice spacing and in the infrared by the dimension of the lattice volume (these parameters are determined by the number of lattice sites and the gauge coupling), in which the theory appears to be higher dimensional. This is revealed by the spectrum, which agrees with that for an extra dimensional theory over this finite energy regime. It is clear that the theory is four dimensional at high energies, where all you see are local four-dimensional forces corresponding to lattice sites in addition to particles transforming under these forces. At low energies, the theory is four dimensional, because one cannot resolve distance scales necessary to see an additional dimension (equivalently, there are no Kaluza-Klein modes). However, at intermediate energy scales, the theory appears to be five (or higher) dimensional. The idea of sequestering readily carries over to this scenario. Other lessons from extra dimensions can in principle be reproduced, and theories with no higher dimensional realization can be implemented.

The second means for constructing new and interesting four-dimensional theories relies on a fact that is now well studied by string theorists: the fact that a five-dimensional AdS space is equivalent to a four-dimensional scale-invariant field theory, in the sense that all properties of the four-dimensional theory can be computed from the five-dimensional gravitational theory, and in principle one can learn about the gravitational theory from the conformal field theory (this is known as a holographic correspondence). The localized gravity theories we discussed earlier were constructed in an AdS space. The exact statement of the correspondence is modified by the presence of branes (*43–46*). The five-dimensional theory with branes corresponds to a scale-invariant theory with an ultraviolet cutoff (corresponding to the Planck brane) and possibly an infrared cutoff (when there is a second brane). This gives a tool for translating extra-dimensional nongravitational properties into purely four-dimensional features. This has been applied, for example, to deduce a four-dimensional model of sequestering supersymmetry breaking (*47*, *48*).

Finally, it is of interest to understand how the four-dimensional theory reproduces some of the strange features that might happen in "locally localized" gravity. These include the existence of a four-dimensional domain in a higher dimensional space and, amazingly, a massive graviton. These features have been understood from a four-dimensional vantage point (*49*). It is very likely that AdS/CFT correspondence between a four-dimensional conformal field theory (CFT) and a five-dimensional AdS space will enlighten us further about gravity and string theory.

Conclusions

The study of extra dimensions is a rapidly evolving field. New theoretical properties are being discovered at a very fast pace. Some of these teach us fundamental features of gravitational theories. Some have shed light on possible ways to address problems in particle physics. And some have revealed new ideas in the context of purely four-dimensional physics. Finally, it is possible that one or several of these ideas will be relevant to the question of how string theory evolves from a higher dimensional theory to one that reproduces observed four-dimensional physics. Possible ideas along these lines are discussed (*50*, *51*). Although it is clear that not all the ideas I have presented will be realized in nature, it is not at all out of the question that some of them are. The ideas that gravity can be localized and that large parameters can be generated in a large or highly curved bulk might well have important implications for our world.

And it could be that the universe has a very rich structure, with many different branes, on which there exist very different physics, living in an as yet unknown geometry.

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REVIEW

Measuring Spacetime: From the Big Bang to Black Holes

Max Tegmark

Space is not a boring static stage on which events unfold over time, but a dynamic entity with curvature, fluctuations, and a rich life of its own. Spectacular measurements of the cosmic microwave background, gravitational lensing, type Ia supernovae, large-scale structure, spectra of the Lyman α forest, stellar dynamics, and x-ray binaries are probing the properties of spacetime over 22 orders of magnitude in scale. Current measurements are consistent with an infinite flat everlasting universe containing about 30% cold dark matter, 65% dark energy, and at least two distinct populations of black holes.

Traditionally, space was merely a three-dimensional (3D) static stage where the cosmic drama played out over time. Einstein's theory of general relativity (*1*) replaced this concept with 4D spacetime, a dynamic geometric entity with a life of its own, capable of expanding, fluctuating, and even curving into black holes. Now, the focus of research is increasingly shifting from the cosmic actors to the stage itself. Triggered by progress in detector, space, and computer technology, an avalanche of astronomical data is revolutionizing our ability to measure the spacetime we inhabit on scales ranging from the cosmic horizon down to the event horizons of suspected black holes, using photons and astronomical objects as test particles. The goal of this article is to review these measurements and future prospects, focusing on four key issues: (i) the global topology and curvature of space, (ii) the expansion history of spacetime and evidence for dark energy, (iii) the fluctuation history of spacetime and evidence for dark matter, and (iv) strongly curved spacetime and evidence for black holes. In the process, I will combine constraints from the cosmic microwave background (CMB) (*2*), gravitational lensing, supernovae Ia, large-scale structure (LSS), the hydrogen Lyman α forest (LyαF) (*3*), stellar dynamics, and x-ray binaries. Although it is fashionable to use cosmological data to measure a small number of free "cosmological parameters," I will argue that improved data allow raising the ambition level beyond this, testing rather than assuming the underlying physics. I will discuss how, with a minimum of assumptions, one can measure key properties of spacetime itself in terms of a few cosmological functions—the expansion history of the universe, the spacetime fluctuation spectrum, and its growth.

Before embarking on a survey of spacetime, a brief review is in order of what it is we want to measure, the basic tools at our disposal (*4*, *5*), and the general picture of how spacetime relates to the structure of the universe. According to general relativity theory (GR), spacetime is what mathematicians call a manifold, characterized by a topology and a metric. The topology gives the global structure (Fig. 1, top), and we can ask: Is space infinite in all directions or multiply connected, like say a hypersphere or doughnut, so that traveling in a straight line could in principle bring you back home—from the other direction? The metric determines the local shape of spacetime (i.e., the distances and time intervals we measure) and is mathematically specified by a 4 × 4 matrix at each point in spacetime.

GR consists of two parts, each providing a tool for measuring the metric. The first part of GR states that, in the absence of nongravitational forces, test particles (objects not heavy enough to have a noticeable effect on the metric) move along geodesics in spacetime, in generalized straight lines, so the observed motions of photons and astronomical objects allow the metric to be reconstructed. I will refer to this as *geometric* measurements of the metric. The second part of GR states that the curvature of spacetime (expressions involving the metric and its first two derivatives) is related to its matter content—in most cosmological situations simply the density and pressure, but sometimes also bulk motions and stress energy. I will refer to such measurements of the metric as *indirect*, because they assume the validity of the Einstein field equations (EFE) of GR.

The current consensus in the cosmological community is that spacetime is extremely smooth, homogeneous, and isotropic (translationally and rotationally invariant) on large ($\sim 10^{23}$ to 10^{26} m) scales, with small fluctuations that have grown over time to form objects like galaxies and stars on smaller scales. CMB observations (*2*) have shown that space is almost isotropic on the scale of our cosmic horizon ($\sim 10^{26}$ m), with the metric fluctuating by only about one part in 10^5

Department of Physics, University of Pennsylvania, Philadelphia, PA 19104, USA. E-mail: max@physics.upenn.edu