Compressible quark matter in $\mathcal{N} = 4$ SYM

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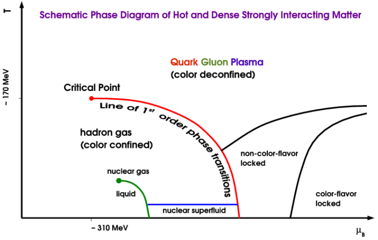
To appear, with Anton Faedo, David Mateos, Javier Tarrio

Introduction: AdS/QCD

The strong nuclear interactions between quarks and gluons are described by **Quantum Chromodynamics (QCD)**.



- Non-perturbative aspects remain challeging: confinement, chiral symmetry breaking, phase transitions etc.
- Some progress achieved using lattice simulations: restrictions due to the *sign problem*.

QCD string dual

Study using the **gauge/string duality**?

$\rightarrow$ String duals of many gauge theories are known, but QCD itself is difficult to construct.
$\rightarrow$ Try to extract 'universal' behaviour, predictions that are robust enough to apply to QCD as well.

In this talk

- Study **d=4 SYM** coupled to massless quarks at finite charge density and finite temperature.
- String dual described by the **D3-D7 system** with an **electric flux** on the D7's.

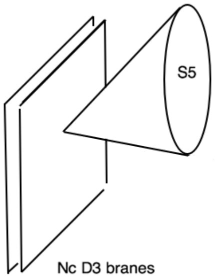
Outline

1. Introduce the various ingredients step-by-step:
   - D3 branes
   - Add D7 flavour branes
   - Add charge

2. Discuss thermodynamics of the system.

4-dimensional SYM

N_c D3-branes in flat space $\Leftrightarrow$ d=4 $\mathcal{N}$=4 $SU(N_c)$ SYM theory



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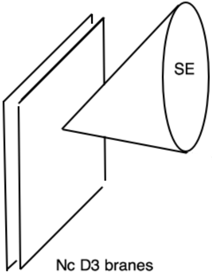


- Susy can be reduced to $\mathcal{N}$=1 by generalising the internal manifold to a SE: quiver gauge theories.
- Dimensionless coupling constant: $g^2_{YM} \sim (length)^{d-4}$, the gauge theory is conformal ($\beta = 0$).

- Degrees of freedom in the adjoint... quarks? Need to add D7 "flavour" branes. [Karch,Katz]

Adding flavour

Add N_f **D7 branes** corresponding to massless quarks.

	x^1	x^2	x^3	r	SE				
D3	×	×	×	·	·	·	·	·	·
D7	×	×	×	×	×	×	×	·	·

→ To simplify, smear the D7's in the internal directions: introducing quarks with different quantum numbers. [Bigazzi et al.]
→ Solving the BPS equations for $S = S_{sugra} + S_{sources}$, including backreaction, one finds that: $\beta \sim \frac{N_f}{N_c} > 0$ [Benini et al.]

- Flavour is irrelevant in the IR: (log) AdS_5 *at low energies.*
- The UV is altered significantly: *the theory develops a Landau pole, described by a hyperscaling violating metric with* $\theta = 7/2$.

Landau Pole physics: UV cut-off

Questions:

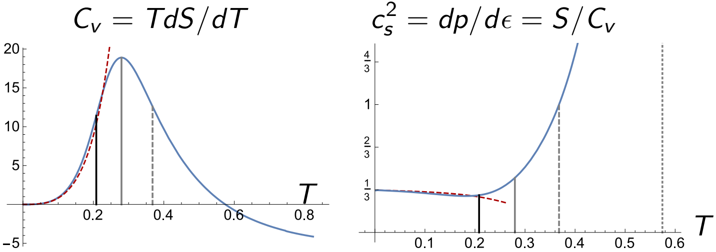
1. How is the UV **cut-off** manifested in the RG flow? the $D = 5$ metric, g, is non-monotonic. [CP et al.]

- maximum number of degrees of freedom $n \sim g_{xx}^{3/2}$.
- the radial proper distance is finite: $\int^{UV} \sqrt{g_{\rho\rho}} d\rho$ converges.

2. Are the solutions valid? Some of the effects of the Landau Pole are within the region where supergravity **can be trusted**.

Landau pole physics: Thermodynamics

$\rightarrow$ Add temperature (numerics).
$\rightarrow$ Study thermodynamics: regularise action.



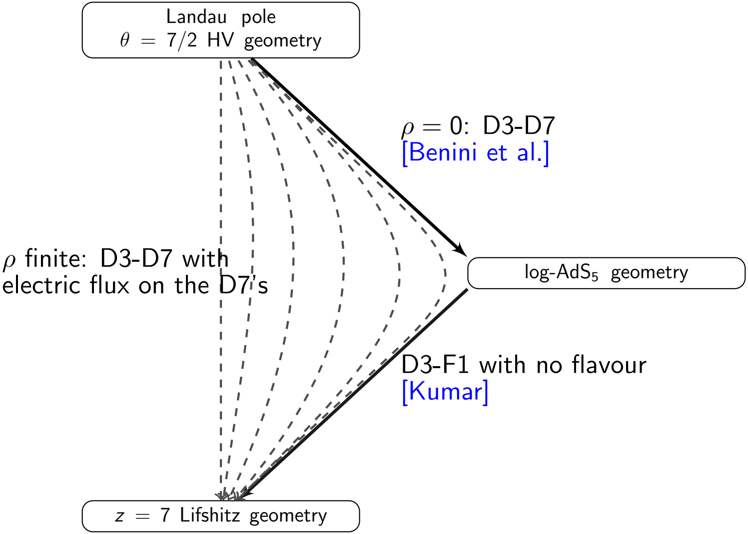
- Specific heat: becomes negative, signaling an instability.
- Speed of sound: grows above 1/3 (conformal value) and, in fact, diverges at some finite temperature.

Adding charge

Turn on a chemical potential by adding N_{st} units of **electric flux** on the flavour branes: $F \sim dt \wedge dr$ [Witten]

- Include full backreaction with $S = S_{sugra} + S_{sources}$.
- Only parameter appearing in equations is $\rho \sim \frac{N_c^{1/4} N_{st}}{4 N_f^{1/2}}$.
- The charge is relevant in the UV: doesn't change the asymptotics.
- Conformality in the IR is broken by the new scale: the theory now *flows to a Lif solution with* $z = 7$: $t \to \lambda^7 t, \quad x \to \lambda x$.

Pictorial representation RG flows



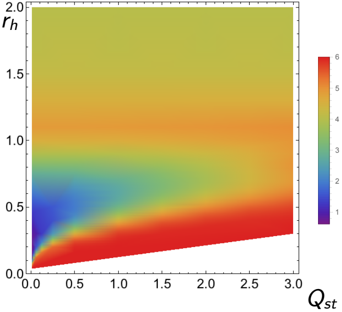
Constructing the flows

→ Solve **numerically** a set of coupled ODEs, for various values of the quark density.
→ We recover the expected behaviour:

Scaling power of the dilaton at the horizon:

$$c = \frac{r\,(e^{\phi})'}{e^{\phi}}\Big|_{r=r_h}$$

- AdS: $e^{\phi} \sim const \Rightarrow c = 0$
- LP: $e^{\phi} \sim r^4 \Rightarrow c = 4$
- Lif: $e^{\phi} \sim r^6 \Rightarrow c = 6$



Towards the Phase diagram

$\rightarrow$ Study thermodynamics: renormalise the action, like before.

$\rightarrow$ Stability properties?

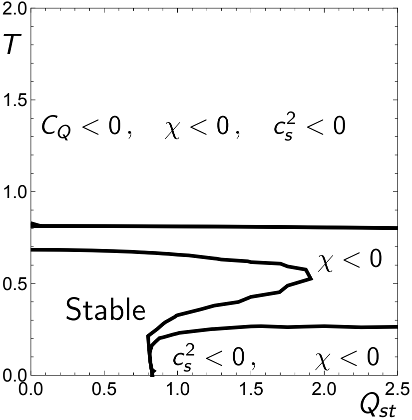
- Thermodynamic stability requirement: Hessian is positive definite.

$$C_Q = T \frac{\partial s}{\partial T}\Big|_{Q_{st}} > 0\,, \qquad \chi = \frac{\partial Q_{st}}{\partial \mu}\Big|_{T} > 0\,.$$

- Dynamical stability: How does the speed of sound, c_s^2, behave?

$$c_s^2 > 0\,.$$

The Phase diagram



- Unstable at high T due to LP, as in chargeless case.
- Unstable at low T and high Q_{st} : inhomogeneous phase.
- $c_s^2 < 0$: the sound mode goes unstable.

Conclusions

We managed to model a d=4 $\mathcal{N}$=1 SYM theory with dynamical quarks at finite density and finite temperature.

- Hints toward spatially modulated phase transitions. Construct them?
- Study Colour Flavour Locking superconductors? consider instantons ($F \wedge F \neq 0$) on this background: desolved D3's in the D7's.

Thank you!

References:

- A. F. Faedo, A. Kundu, D. Mateos and J. Tarrio, "(Super)Yang-Mills at Finite Heavy- Quark Density," JHEP 1502 (2015) 010 [arXiv:1410.4466 [hep-th]].
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- A. F. Faedo, D. Mateos, C. Pantelidou and J. Tarrio, Holography with a Landau pole," JHEP 1702 (2017) 047 [arXiv:1611.05808 [hep-th]].