Matrix Quantum Mechanics

FZZT branes and Spin Calogero models

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Motivation: Non-singlet sectors in MQM

- Singlet sector of gauged Matrix Quantum Mechanics (MQM) $\Leftrightarrow c = 1$ Liouville theory/ 2D non critical string theory in a linear dilaton background [Olga's talk...]
- Other states? Black Holes? $(SL(2,\mathbb{R})_k/U(1)$ coset [Mandal, Sengupta, Wadia, Witten, Rabinovici...]) $\Leftrightarrow$ Non-singlet sectors of MQM
- Proposal in terms of winding modes [Kazakov-Kostov-Kutasov] $\Rightarrow$ Non-local action/ no clear Lorentzian/ microscopic description
- Important to understand BH microstates/ dynamical questions (formation-scrambling)/ BH interior
- Adjoint representation $\Leftrightarrow$ Long-strings [Bars ...] that extend along Liouville [Maldacena, Kostov, ...] Presence of FZZT branes

Is it possible to connect these threads?

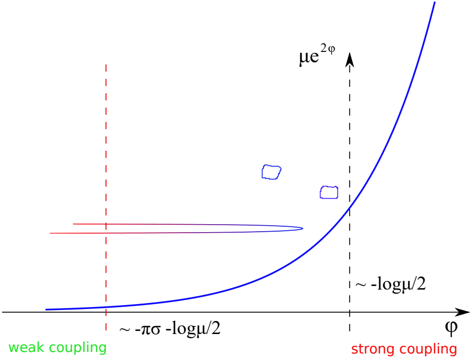
Liouville theory and Long strings

- The Liouville action on a worldsheet with boundaries is

$$S = \int_R d^2 z \sqrt{g} \left(\frac{1}{4\pi} g^{ab} \partial_a \phi \partial_b \phi + \frac{1}{4\pi} Q R \phi + \mu e^{2b\phi} \right) + \int_{\partial R} ds g^{1/4} \left(\frac{QK\phi}{2\pi} + \mu_B e^{b\phi} \right)$$

K is extrinsic curvature and μ, μ_B the bulk-boundary cosmological constants. For $c_{matter} = 1 \Rightarrow b = 1, Q = 2$ and $\mu_B = \sqrt{\mu} \cosh(\pi\sigma)$.

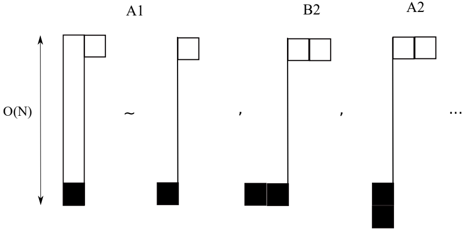
- Consider scattering of closed and open strings.
- Closed strings see the bulk Liouville wall. Open strings have their endpoints pinned near the weak coupling region.
- A very energetic open string can stretch a lot (large σ), before it scatters back



Long strings and FZZT branes

[Maldacena, Kostov, ...]

- To compute the scattering amplitude one introduces FZZT branes that extend along Liouville. The long strings end on them. The scattering amplitude can be computed from the worldsheet disk two point function [Fateev, Zamolodchikov^2]
- This amplitude is reproduced by scattering in the adjoint rep sector of MQM [Maldacena, Fidkowski...]
- One long folded string $\Rightarrow$ one "impurity" in the fermi sea interacting via a Calogero term with the rest
- States containing n folded strings
  $\Rightarrow$ Irreps with a Young-Tableaux of n-boxes and n-anti-boxes.



ZZ vs FZZT branes

[Zamolodchikov, Zamolodchikov, Fateev, Teschner]

- Liouville theory has two types of boundary states
- The ZZ boundary state (D_0 brane anchored at large ϕ)

$$
\begin{aligned}
|m,n\rangle &= \int_{-\infty}^{\infty} d\nu \Psi_\nu(m,n)|\nu\rangle\,, \\
\Psi_\nu(m,n) &= \sinh(2\pi m\nu/b)\sinh(2\pi n\nu b)(\mu)^{-i\nu/b}\frac{\Gamma(1+2i\nu b)\Gamma(1+2i\nu/b)}{2^{-3/4}(-2i\pi\nu)}
\end{aligned}
$$

- MQM describes the physics of N D_0-branes [McGreevy, Verlinde] $\Rightarrow$ closed strings in the double scaling limit
- The FZZT boundary state (brane extending along Liouville)

$$
\begin{aligned}
|B_\sigma\rangle &= \int_{-\infty}^{\infty} d\nu e^{2\pi i\nu\sigma}\Psi_\nu(\sigma)|\nu\rangle\,, \\
\Psi_\nu(\sigma) &= (\mu)^{-i\nu/b}\frac{\Gamma(1+2i\nu b)\Gamma(1+2i\nu/b)\cos(2\pi\sigma\nu)}{2^{1/4}(-2i\pi\nu)}
\end{aligned}
$$

- To describe these one can add extra (anti)-fundamental fields (quarks) $\Rightarrow$ this leads to the open string sector

The model

Similar models by [Minahan, Polychronakos, Gaiotto, Dorey, Tong...]

- Consider the (gauged) MQM action with the $N \times N$ Hermitian matrices $M(t)$ and $A(t)$ (a non dynamical gauge field).

$$S = \int dt \, \mathrm{tr} \left(\frac{1}{2} (D_t M)^2 - V(M) \right) , \quad V(M) \sim -M^2 \text{ double scaling limit}$$

- Describe open strings between N-ZZ and N_f-FZZT branes $\Rightarrow$ Extend by adding $N_f \times N$ (anti)- fundamental fields $\chi_{\alpha i}, \psi_{\alpha i}$. In 1-d they can be either fermions or bosons

$$S_f = \int dt \sum_{\alpha}^{N_f} \mathrm{tr} \left(i\psi_\alpha^\dagger D_t \psi_\alpha - m_\alpha \psi_\alpha^\dagger \psi_\alpha + i\chi_\alpha^\dagger D_t^* \chi_\alpha - m_\alpha \chi_\alpha^\dagger \chi_\alpha \right) ,$$

- One can also add the Chern-Simons term $S_{CS} = k \int dt \, \mathrm{tr} \, A$
- This model has a $U(N)$ gauge symmetry and a $SU(2N_f)$ global symmetry broken by the mass terms to a subgroup.

Reduction to Spin-Calogero

- The gauss-law constraint gives

$$:i[M\,,\dot{M}]_{ij} :=: J_{ij} := -k\delta_{ij} + \sum_{\alpha}^{N_f}\left[\psi^{\dagger}_{\alpha j}\psi_{\alpha i} - \chi^{\dagger}_{\alpha i}\chi_{\alpha j}\right] .$$

- The Hamiltonian is then

$$\hat{H} = \sum_{i}^{N} -\frac{1}{2}\frac{\partial^2}{\partial\lambda_i^2} + V(\lambda_i) + \frac{1}{2}\sum_{i\neq j}\frac{J_{ij}J_{ji}}{(\lambda_i-\lambda_j)^2} + \sum_{i,\alpha}^{N,N_f} m_\alpha\psi^{\dagger}_{\alpha i}\psi_{\alpha i} + m_\alpha\chi^{\dagger}_{\alpha i}\chi_{\alpha i}$$

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- The fundamentals thus “feed” non-trivial representations
- This model can be written as a spin Calogero model [Polychronakos] using $\Psi^\dagger_{\tilde{\alpha} i} = (\psi^\dagger_{\alpha i}, \chi_{\alpha i})$, $S_i^{\tilde{A}} = \Psi^\dagger_{\tilde{\alpha} i} T^{\tilde{A}}_{\tilde{\alpha}\tilde{\beta}} \Psi_{\tilde{\beta} i}$ and $\tilde{k} = k \mp N_f$

$$\hat{H}_C = \frac{1}{2}\sum_{i\neq j}\frac{\tilde{k}(2N_f \pm \tilde{k})/2N_f \pm 2S_i^{\tilde{A}}S_j^{\tilde{A}}}{(\lambda_i-\lambda_j)^2} + \sum_{i\tilde{A}} B^{\tilde{A}} S_i^{\tilde{A}}$$

with $T^{\tilde{A}}$, the $SU(2N_f)$ generators and the “magnetic” field $B = \sum_{\tilde{A}} B^{\tilde{A}} T^{\tilde{A}}$ (specifically $B = \text{diag}\{m_\alpha, -m_\alpha\}$)

Chiral variables and simple gauge invariant excitations

- One can introduce the "chiral variables" [Alexandrov, Kazakov, Kostov]

$$\hat{X}_{\pm} = \frac{\hat{M} \pm \hat{P}}{\sqrt{2}}$$

- The simplest gauge invariant excitations are

$$J_-^n = \mathrm{tr}\, X_-^n, \quad J_+^n = \mathrm{tr}\, X_+^n$$

These are known to correspond to left/right moving tachyons (closed strings) in the dual string theory.

- One can then define the "spin-currents"

$$(S^n)_-^{\tilde{A}} = \mathrm{tr}\, \Psi_{\tilde{\alpha}}^{\dagger} X_-^n T_{\tilde{\alpha}\tilde{\beta}}^{\tilde{A}} \Psi_{\tilde{\beta}}\,, \quad (S^n)_+^{\tilde{A}} = \mathrm{tr}\, \Psi_{\tilde{\alpha}}^{\dagger} X_+^n T_{\tilde{\alpha}\tilde{\beta}}^{\tilde{A}} \Psi_{\tilde{\beta}}$$

which should correspond to open-strings with possible tachyonic dressing.

- These spin currents obey an $SU(2N_f)_{\tilde{k}}$ Kac-Moody algebra, where the central extension term is known to arise at large-N from the constraint/commutator of $X_{\pm}$ [Dorey, Tong, Turner]

Partition function

In terms of gauge field zero modes

The canonical thermal partition function can be written as ($q = e^{-\omega\beta}$)

$$Z_N^{(N_f)} = \frac{e^{\beta MN}}{N!} \int_0^{2\pi} \prod_i^N \frac{d\theta_i}{2\pi} e^{-ik\theta_i} \prod_{i<j} \left| e^{i\theta_i} - e^{i\theta_j} \right|^2 \frac{q^{\frac{N^2}{2}} \prod_{i,\alpha} \left(1+e^{-\beta m_\alpha} e^{i\theta_i}\right)\left(1+e^{-\beta m_\alpha} e^{-i\theta_i}\right)}{\prod_{i,j} \left(1-q e^{i\theta_i - \theta_j}\right)}$$

- It can also be written in terms of fundamental and adjoint $U(N)$ characters ($z_w(\beta) = \sum_\alpha e^{-\beta m_\alpha}$)

$$\begin{aligned} Z_N^{(N_f)} &= \int_{U(N)} \mathcal{D}U \det U^{-k} e^S \\ S &= \sum_{l=1} \left[\frac{(\pm 1)^{l+1}}{l} z_w(l\beta) \left(\operatorname{tr} U^l + \operatorname{tr} U^{-l}\right) + \frac{q^l}{l} \operatorname{tr}(U^l) \operatorname{tr} U^{-l} \right] \end{aligned}$$

- Similar to "mixed actions" in QCD [Creutz, Ogilvie, Samuel...] and reductions of gauge theories on compact spaces [Aharony, Minwalla, Papadodimas, Wadia...]
- It can be also understood as the partition function of $c = 1$ MQM with winding mode perturbations of arbitrary power.

Canonical Partition function

- A similar partition function also appeared in a matrix model related to non-Abelian quantum Hall states and the chiral WZW model [Dorey, Tong, Turner]
- One can rewrite the partition function in terms of Kostka polynomials and Schur functions as a sum over reps R (with $x_{\tilde{\alpha}} = e^{-\beta m_{\tilde{\alpha}}}$)

$$Z = q^{N^2/2} \prod_{j=1}^{N} \frac{1}{1-q^j} \sum_R K_{R,(\tilde{k}^N)}(q) s_R(x)$$

- The partition function in the large N limit is

$$\mathcal{Z}_{N\to\infty} \to q^{E_0} \prod_{j=1}^{N} \frac{1}{1-q^j} \chi_{R_{\tilde{k},C}}(q,x)$$

  with $N = 2N_f L + C$, $L \Rightarrow \infty$ with C fixed.
- The representation appearing is rectangular (the $\tilde{k}$-fold symmetrization of the C'th antisymmetric rep or vice versa.)
- For $C = 0$ it is the vacuum character of chiral WZW.

Canonical Partition function

"Veneziano limit"

- Only one rep at large N, to see further non-trivial structure let the FZZT's backreact on the ZZ's $\Rightarrow$ study $N, N_f \to \infty$ with $N_f/N = c$ fixed $\Rightarrow$ saddle point approximation
- A large number of FZZT branes condense causing a large backreaction on the closed string background of the ZZ branes.
- Adjoint characters $\to$ shift of the parameters of fundamental characters [Samuel, Ogilvie...]. A very similar case was studied by [Schnitzer...]
- Introduce density of states $\rho(\theta)$. The saddle point is given by self-consistently solving

$$\frac{\partial S(\theta)}{\partial \theta} = \sum_{n=1}^{\infty} b_n \sin(n\theta) = -\int_{\mathcal{C}} d\theta' \rho(\theta') \cot \frac{1}{2}(\theta - \theta')$$

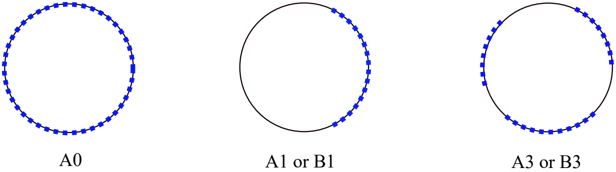
with

$$b_n = q^n \rho_n + \frac{N_f}{N} x^n$$

where $\rho_n = \int_{\mathcal{C}} d\theta \rho(\theta) \cos n\theta$

Phase transition

- the Vandermonde makes the eigenvalues to spread on the circle, while the potential terms clump them
- The result depends on the density of eigenvalues along the circle $\rho(\theta)$. This can have support on the full circle (A_0 case) or in an arc of the circle (A_1 case). It can also saturate in some arcs. The most general case is described by [Jurkiewicz, Zalewski, Mandal, Wadia...]



- If one keeps only the first winding mode $\Rightarrow$ [Gross, Wadia, Witten] phase transition between A_0 and A_1 phase. It happens for

$$q + \frac{2N_f}{N} x = 1$$

- We are trying to analyse all the winding modes simultaneously

Limit of 2d Black hole Matrix Model

Model of [Kazakov, Kostov, Kutasov]

- Take a double scaling limit (assuming $m_\alpha = m$)

$$N_f \to \infty, \quad m \to \infty, \quad \text{with} \quad N_f e^{-\beta m} = \tilde{t}, \quad \text{finite}$$

- Limit of "heavy quarks" / strong magnetic field (Calogero picture)
- The only winding modes surviving in this case: $\exp\left(\tilde{t}\,\mathrm{tr}\,U + \tilde{t}\,\mathrm{tr}\,U^\dagger\right)$, are identical to those studied in the matrix model conjectured to describe the physics of the 2-d black hole ($SL(2,R)/U(1)$ coset)

  This goes via the 2-d black hole - sine Liouville correspondence of [Fateev, Zamolodchikov^2] (valid for $R = 3/2$)
- In this limit bosons and fermions behave in the same way
- The same limit can be taken on the Liouville side [Maldacena] (loop-loop correlator) $\Rightarrow$ identification $\sigma = 2m$.
- Our approach expresses the couplings $\tilde{t}$ in terms of Liouville theory/Matrix model parameters N_f, $\sigma = 2m$.

Grand Canonical ensemble

- In the Grand–Canonical ensemble the partition function is a τ function obeying discrete soliton equations [Date, Jimbo, Miwa] (that do not affect k - conjugate "zero time")

$$\mathcal{Z}_G = e^{\sum_{n=1}^{\infty} n t_n t_{-n}} \tau(t; \beta, \mu, k) \,, \quad t_n = \frac{(\pm)^{n+1} N_f e^{-\beta \sigma n}}{q^{-n/2} - q^{n/2}}$$

- Need to be supplemented with Virasoro constraints that relate different k's. These are due to reparametrisations of the partition function
- We have also found the form of these constraints (for fundamental [Morozov...] + adjoint characters), but proven hard to solve
- The vacuum τ function has an interesting combinatorial interpretation $\Rightarrow$ grand Free energy of vortices/ anti-vortices in accord with expectations [Boulatov Kazakov] and matches Liouville loop-loop correlator result

$$\exp\left(\sum_{n\geq 1} n t_n t_{-n}\right) = \prod_{\alpha=1}^{N_f} \sum_{\lambda} s_\lambda(y^{(\alpha)}) s_\lambda(y^{(\alpha)}) \,, \quad y_i^{(\alpha)} = e^{-\beta(m_\alpha + \omega \epsilon_i)}$$

$$\sum_{n=1}^{\infty} n t_n t_{-n} = N_f^2 \sum_{n=0}^{\infty} (n+1) \log(1 - e^{-\beta\sigma} q^{n+1})$$

Future directions I - Second quantization

collective field theory [Avan, Das, Jevicki] vs bi-local field theory [Dhar, Mandal, Wadia]

- Understand better the string backgrounds these models describe $\Rightarrow$ Collective field theory picture ("hydrodynamic description")

$$\begin{aligned} \phi(x,t) &= \operatorname{tr}\delta(x-M(t)) = \alpha_+(x,t)-\alpha_-(x,t)\,, \\ \mathcal{J}_{\tilde\alpha\tilde\beta}(x,t) &= \operatorname{tr}\Psi^\dagger_{\tilde\alpha}\delta(x-M(t))\Psi_{\tilde\beta} = \mathcal{J}^+_{\tilde\alpha\tilde\beta}(x,t)-\mathcal{J}^-_{\tilde\alpha\tilde\beta}(x,t) \end{aligned}$$

$$\begin{aligned} H_{coll} &= \int dx\frac{1}{6}(\alpha_+^3-\alpha_-^3)+(\mu-\frac{x^2}{2})(\alpha_+-\alpha_-)+(\alpha_+T_+^{\mathcal{J}}-\alpha_-T_-^{\mathcal{J}}) \\ &+ \int dxdy\frac{\mathcal{J}_{\tilde\alpha\tilde\beta}\mathcal{J}_{\tilde\beta\tilde\alpha}}{(x-y)^2} + \text{cubic - higher spin terms} \end{aligned}$$

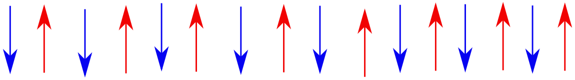
- The stress energy tensor $T^{\mathcal{J}} = \frac{d^{\tilde A\tilde B}\mathcal{J}^{\tilde A}\mathcal{J}^{\tilde B}}{2(1+2N_f)}$ can deform the fermi-sea of the linear dilaton $\phi_0(x)\sim\sqrt{\mu-\frac{x^2}{2}}$
- An interesting problem is to find time (in)-dependent solutions of collective field theory and/or develop the appropriate bi-local fermionic field theory [Dhar, Mandal, Wadia...]

Statics and some dynamics through spins

Preliminary picture (Mainly $SU(2)$ intuition)

$$\hat{H}_C = \frac{1}{2}\sum_{i\neq j}\frac{\tilde{k}(2N_f \pm \tilde{k})/2N_f \pm 2S_i^{\tilde{A}}S_j^{\tilde{A}}}{(\lambda_i - \lambda_j)^2} + \sum_{i\tilde{A}} B^{\tilde{A}} S_i^{\tilde{A}}$$

- Half of the spins try to align and half to anti-align with the constant magnetic field. This is the leading effect for $m \to \infty$, exept at coincidence points $\lambda_i = \lambda_j \Rightarrow$ two kinds of eigenvalues/fermi surfaces

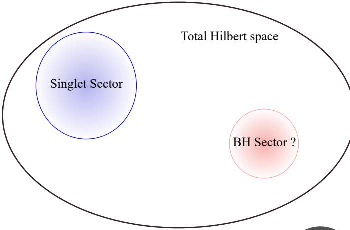


- All the spins between themselves try to (anti) - align $\Leftrightarrow$ (anti)-ferro magnet $\Rightarrow$ competition/frustration! $\Rightarrow$ degeneracies
- A black hole might be thought of as a big impurity made out of lots of smaller impurities/boxes. One might try dynamically to construct it in a scattering process.
- Fermi vs non-fermi liquid behaviour. Intuition from the multichannel Kondo model?

Future directions III

- IP, IOP models [Iizuka, Okuda, Polchinski] do not have the expected signs of chaos (4-point OTOC [Michel, Polchinski, Rosenhaus, Suh]), but contain a single fundamental, we have argued that one needs a large number of flavors for black hole-like physics in a similar adjoint/fundamental model
- Compute real time correlators. Signs of chaos? (might be possible at large N,N_f)
- More clear picture of possible Black hole formation, evaporation process (dynamics)
- State dependence [Papadodimas, Raju] in 2d string theory?

- The singlet sector enough to describe linear dilaton background + tachyons
- Other backgrounds $\Rightarrow$ non-singlet sectors
- The non-triviality comes from the mapping Matrix model $\rightarrow$ String field theory



Thank you!