Orthodox Academy of Crete

9^{th} Crete Regional Meeting in String Theory

Crete Center for Theoretical Physics

UNIVERSAL BOUNDS ON DIFFUSION

arXiv:1612.05500

arXiv:1705.01766

with B. Gouteraux, E. Kiritsis and W.Li + ……..

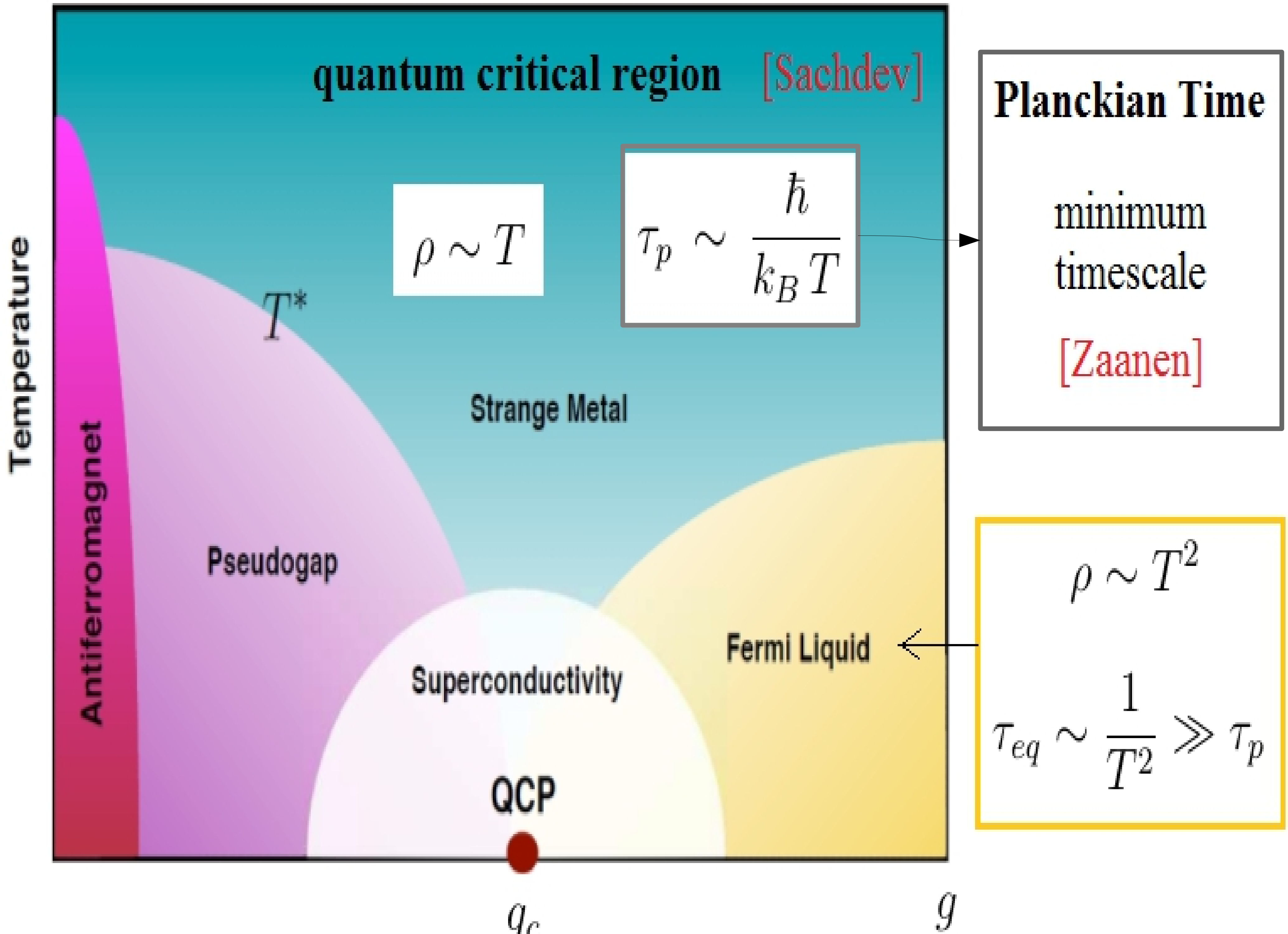
Matteo Baggioli

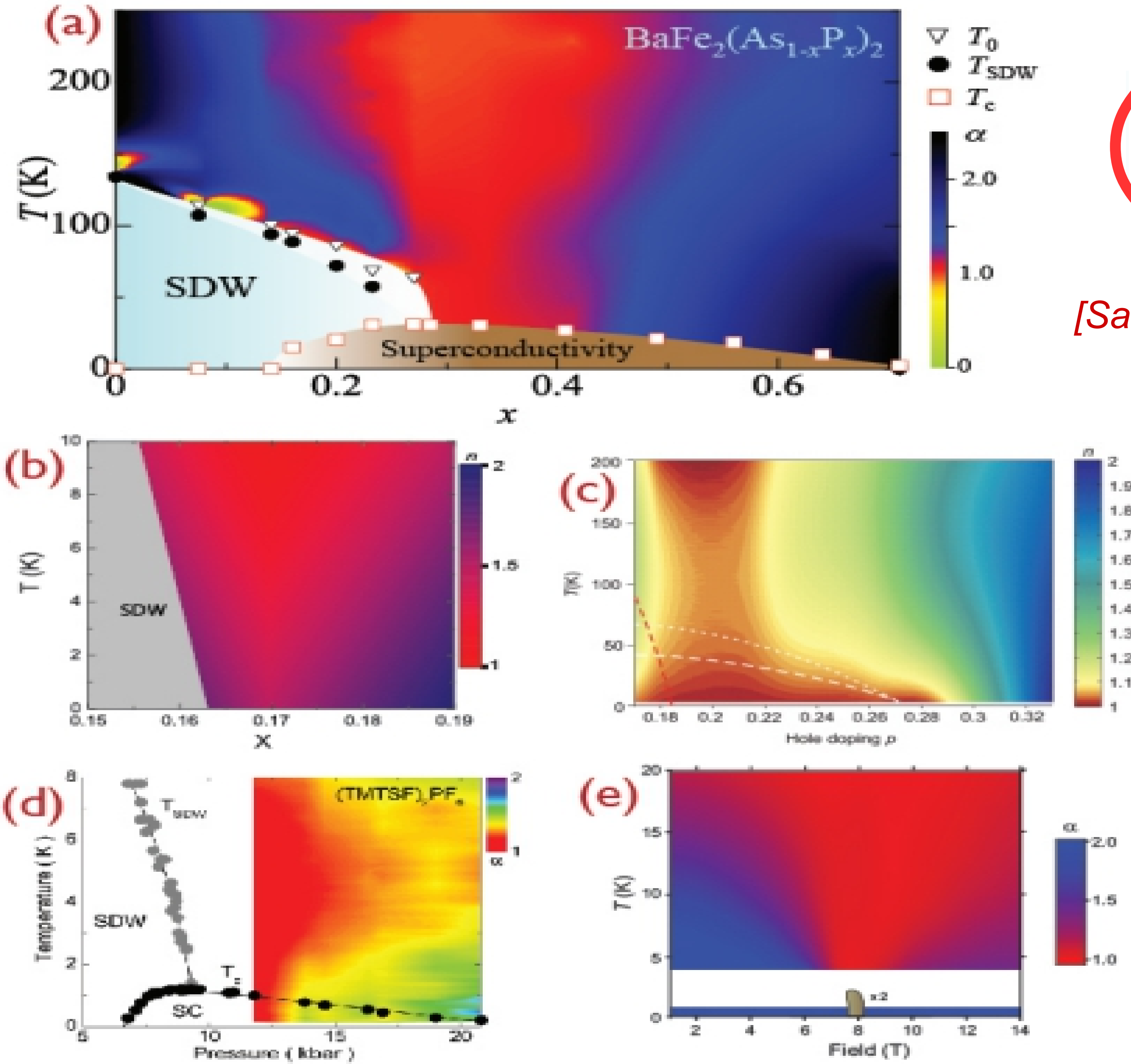
UOC & Crete Center for Theoretical Physics

Is there a miminum (Planckian) timescale in nature ?

Imprints on transport ??

Bounds on transport !??


quantum critical region
[Sachdev]
Planckian Time
minimum
timescale
[Zaanen]
$\rho \sim T$
$\tau_p \sim \frac{\hbar}{k_B T}$
T^*
Temperature
Antiferromagnet
Pseudogap
Strange Metal
Superconductivity
Fermi Liquid
QCP
$\rho \sim T^2$
$\tau_{eq} \sim \frac{1}{T^2} \gg \tau_p$
g_c
g

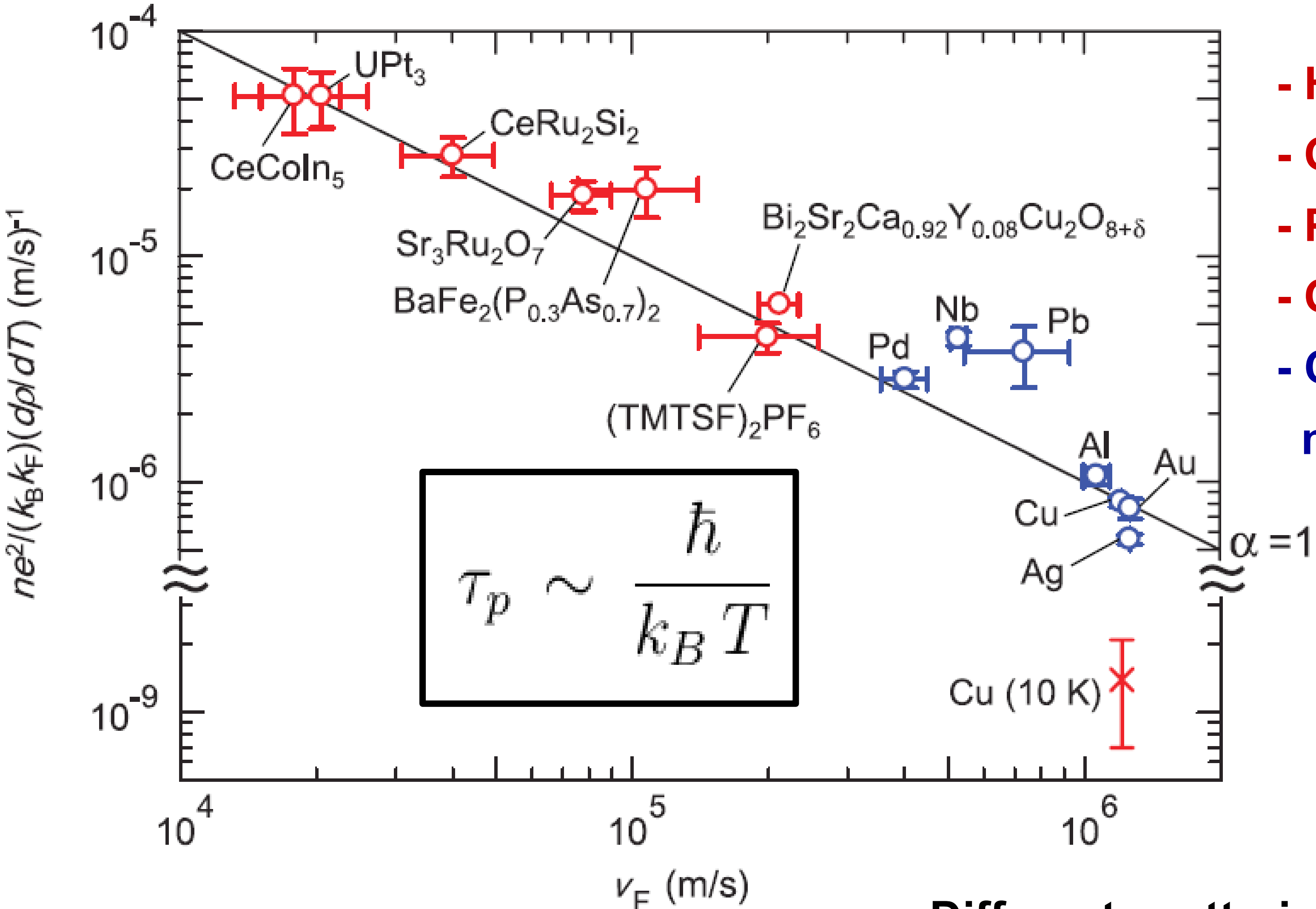


$$\rho \sim T$$

[Sachdev, Keimer]

UNIVERSAL PLANCKIAN SCATTERING TIME

[Bruin JA, Sakai H, Perry RS, Mackenzie AP., Science 2013]



- Heavy fermions

- Oxides

- Pnictides

- Organic metals

- Conventional metals

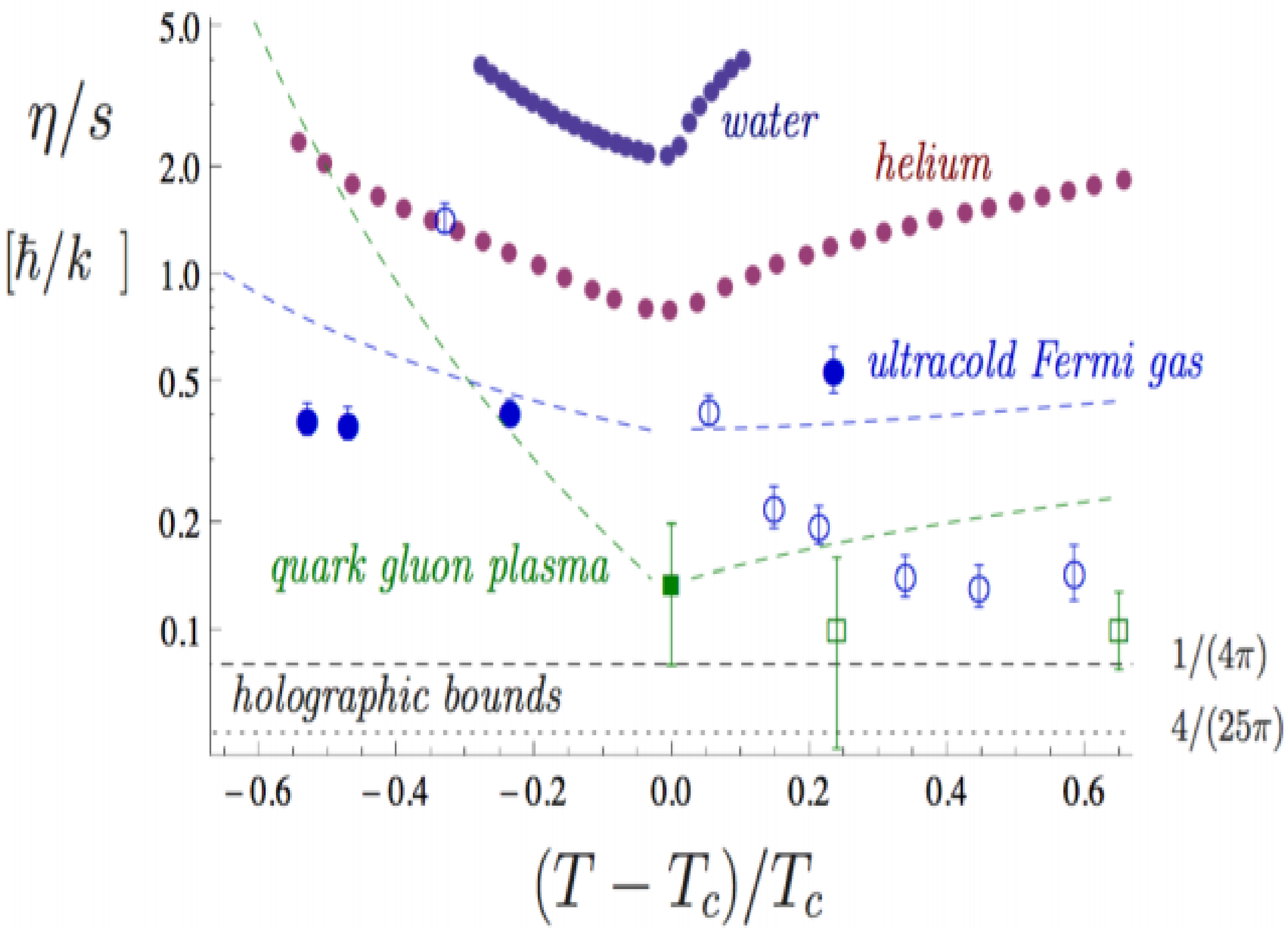
$$\frac{1}{\tau} = \frac{e^2\rho}{hd}\sum_i k_{Fi} v_{Fi} \qquad (T\tau)^{-1} \sim \alpha \frac{k_B}{\hbar}$$

Different scattering mechanisms

2 order of magnitude variations in the Fermi velocity

KSS BOUND

[Kovtun,Son,Starinets,Policastro, 2004]



Minimum Planckian Time

↓

UNIVERSAL BOUND(S)

$$\frac{\eta}{s} \geq \boxed{\frac{1}{4\pi}} \frac{\hbar}{k_B} \quad \longleftrightarrow \quad \tau_p \sim \frac{\hbar}{k_B T}$$

BOUNDS ON DIFFUSIVITIES

[Hartnoll, 2014]

$$[D] \equiv [v^2] * [t]$$

Diffusion can't be arbitrarily fast

$$D \geq v^2 \, \tau_p = v^2 \, \frac{\hbar}{k_B T}$$

Generically

$$\tau_{eq} \gg \tau_p$$

Fast(est) dissipation
Strong coupling

KSS story (v=c) →

$$D_{\mathcal{P}} \equiv \frac{\eta}{s\,T} \geq \left(\frac{1}{4\,\pi}\right) \frac{\hbar}{k_B T}$$

Charge and Energy Diffusivities (??) →

$$D_c = \frac{\sigma}{\chi} \qquad D_e = \frac{\kappa}{c_v}$$

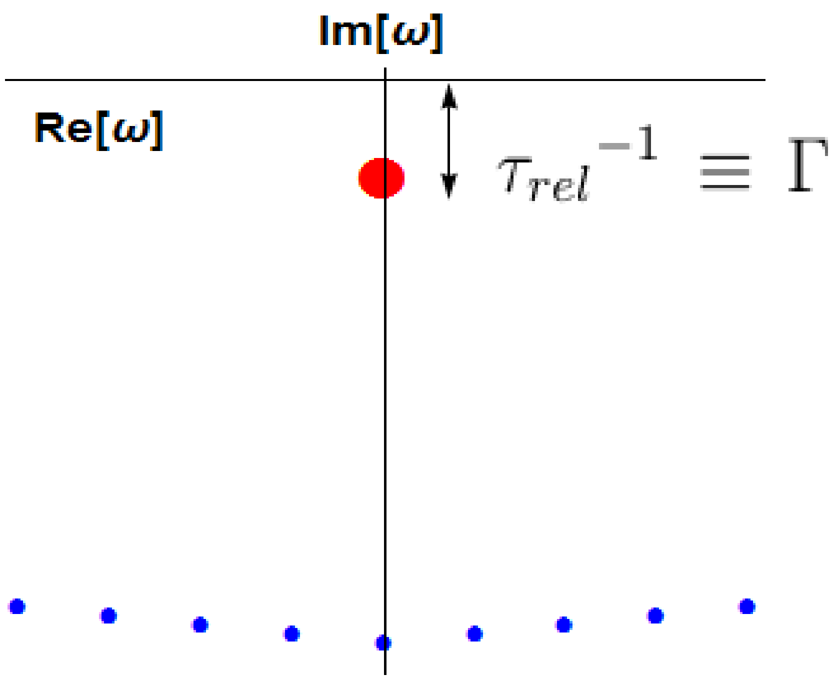
COHERENT METALLIC TRANSPORT

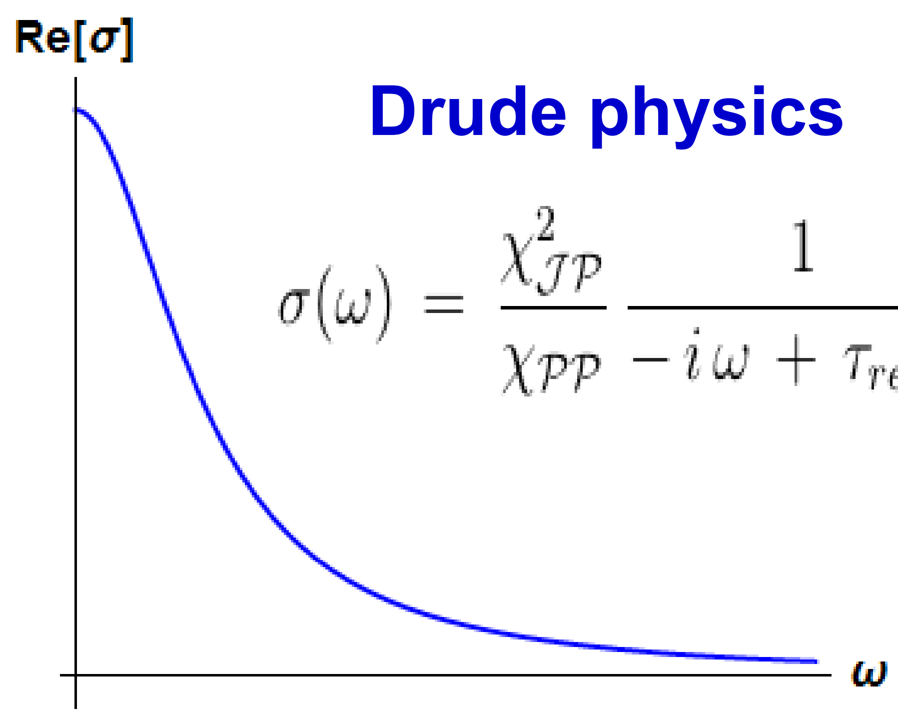
$$\text{electric current } \mathcal{J} \;+\; \text{momentum } \mathcal{P}$$

They overlap because a finite charge density !!

Momentum relaxation : $\langle \mathcal{P}(t) \rangle \sim e^{-t/\tau_{rel}}$

Weak momentum relaxation
(long lived momentum) $\tau_{rel} \gg E \sim k_B T$





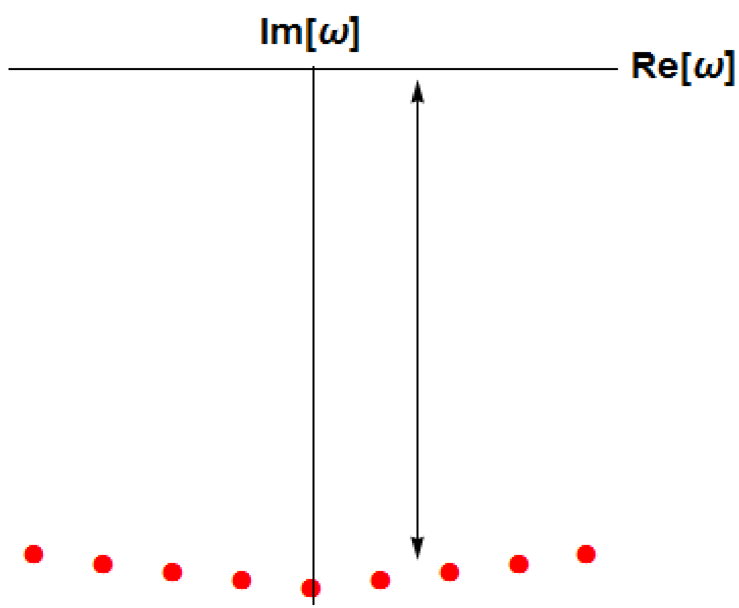
Drude physics

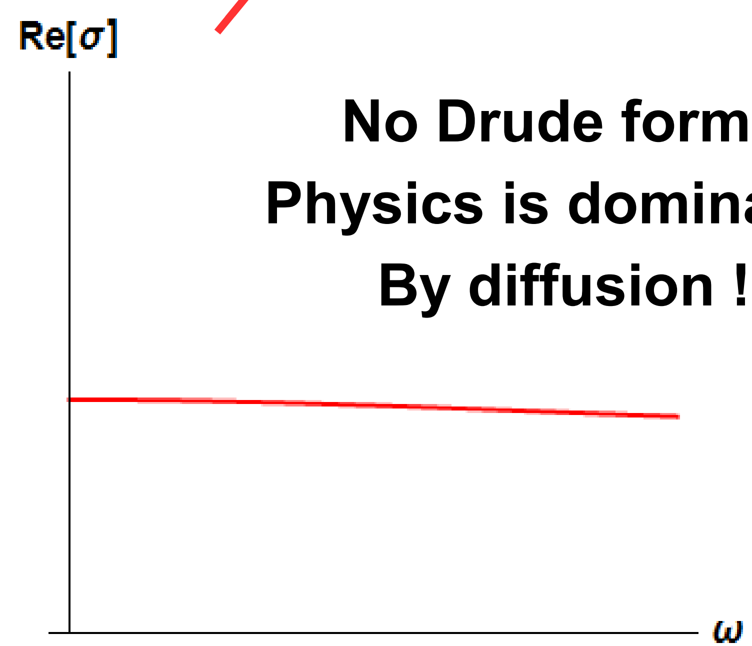
$$\sigma(\omega) = \frac{\chi^2_{\mathcal{J}\mathcal{P}}}{\chi_{\mathcal{P}\mathcal{P}}} \frac{1}{-i\,\omega + \tau_{rel}{}^{-1}}$$

INCOHERENT METALLIC TRANSPORT

no long-lived quantities overlapping with the current operators.

Fast momentum relaxation $\tau_{rel} \not\gg E \sim k_B T$





No Drude form !
Physics is dominated
By diffusion !

In this limit we should approach the minimum timescale $\frac{\hbar}{k_B T}$

In this limit $D \geq v^2 \tau_p$ **should be saturated !!**

The butterfly velocity and quantum chaos

Who is the velocity v ??
It can't be the Fermi velocity !!

~~Mott-Ioffe-Regel bound~~
~~Wiedemann–Franz law~~
~~Quasiparticles~~

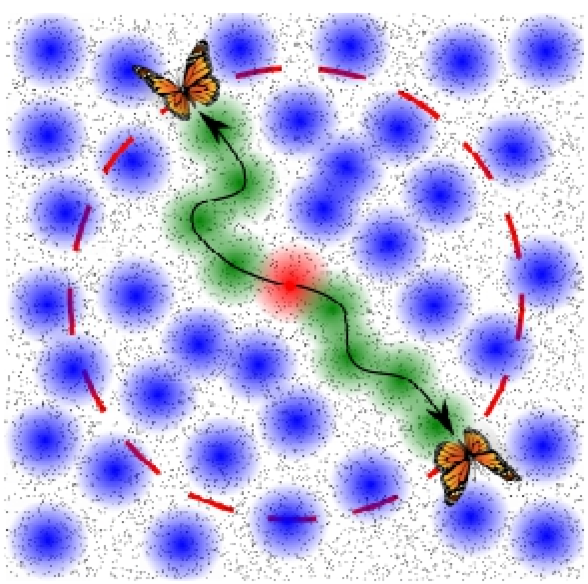
v IS THE BUTTERFLY VELOCITY = SPEED OF INFORMATION PROPAGATION

(HOL.) QUANTUM CHAOS

[Shenker, Stanford, Susskind, Swingle, Maldacena, Blake, Roberts, Douglas, ...]

OUT-OF-TIME CORRELATOR

$$\langle [\mathcal{V}(x,t)\,\mathcal{W}(0,0)]^2 \rangle_\beta \sim e^{\lambda_L (t-t^* - |x|/v_B)},$$



PROPOSAL

$$\frac{D}{v_B^2} \geq C\,\frac{\hbar}{k_B T}$$

[Blake, 2016]

Momentum relaxing Holography

BOUNDARY | **BULK**

Stress Tensor $T_{\mu\nu}(\vec{x})$ ⟷ **Metric** $g_{\mu\nu}(\vec{x}, z)$

$\partial_\mu T^{\mu\nu} = 0$ ⟷ **Diffeomorphism Invariance**

In order to break translational invariance in the CFT
We need to break (spatial) diffeomorphisms in the bulk

[Vegh,'13]

Generic effective holographic theory
With momentum dissipation ⟶ **MASSIVE GRAVITY**

[Vegh, Tong, Blake]

Translations broken
Energy conserved ⟶ **LORENTZ VIOLATING MASSIVE GRAVITY**

Hol. Massive gravity Phenomenology

The graviton mass is setting the momentum relaxation time !!

[Davison]

$$\partial_i T^{ij} = -\frac{1}{\tau_{rel}} T^{tj} \neq 0 \qquad \frac{1}{\tau_{rel}} \sim \mathcal{M}_h^2(T,\, k,\, q,\, g_i\,, \dots)$$

Access to DC transport analytically !!

[Donos, Gauntlett, Amoretti,Magnoli, Musso, Lucas, Blake, Gouteraux, ...]

$$\sigma = \sigma^{\mathcal{I}} + \frac{q^2}{\mathcal{M}_h^2}\,, \qquad \alpha = \bar{\alpha} = \frac{s\,q}{\mathcal{M}_h^2}\,, \qquad \bar{\kappa} = \frac{s^2\,T}{\mathcal{M}_h^2}\,.$$

INCOHERENT LIMIT → **LARGE GRAVITON MASS LIMIT**

[Gouteraux, Davison, Hartnoll, Kim^2, Jin-Sin, Seo, ...]

Simplest holographic theory

$$S = \int d^{d+1}x\,\sqrt{g}\left[R - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^I\right]$$

$$\phi^I = k\,x^I$$

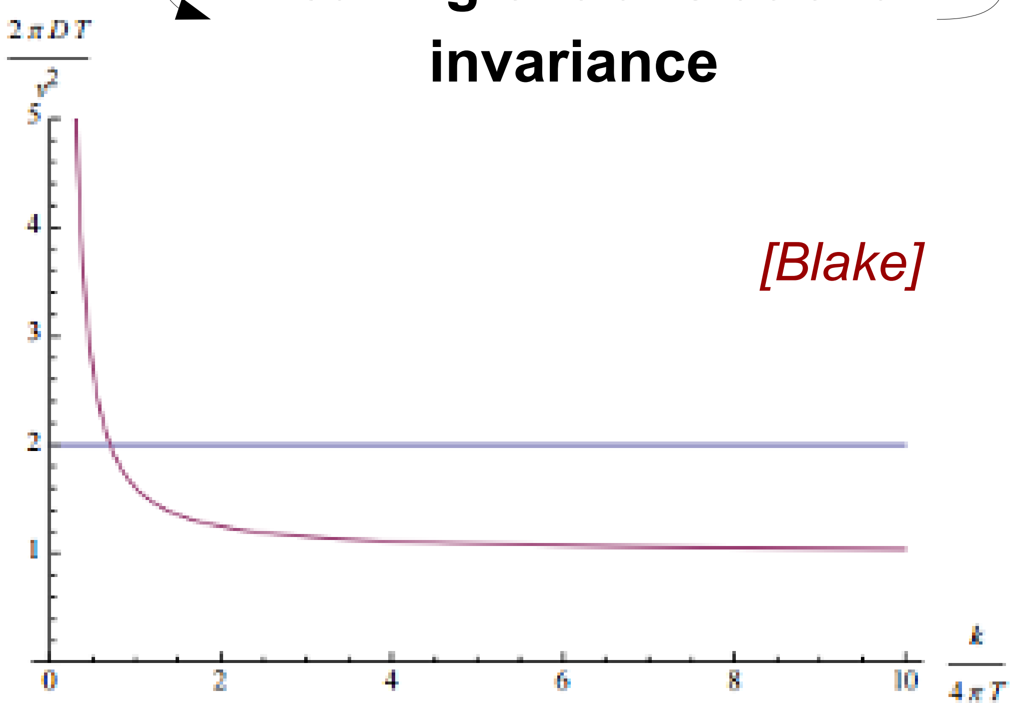
~~Axions~~

Stueckelberg fields

[Andrade, Withers,'13]

Breaking of translational invariance

$$\tau_{rel} \sim \frac{1}{k^2} \qquad \sigma = 1 + \frac{\mu^2}{k^2}$$



[Blake]

$$D_c = \frac{v_B^2}{\pi T} \qquad D_e \approx \frac{v_B^2}{2\pi T}$$

BOUNDS :

Higher derivatives check part I

What happens if we modify the charge sector With higher derivative couplings to the Momentum dissipating sector?

[MB, Gouteraux, Kiritsis, Li, 2016]

[MB, Pujolas, 2014]

$$\mathcal{X}^{\mu}{}_{\nu} \equiv \frac{1}{2} \sum_{I=x,y} \partial^{\mu}\phi^{I}\partial_{\nu}\phi^{I}$$

$$\mathcal{L} = \ldots - \frac{\mathcal{J}}{4} Tr\left[\mathcal{X}\, F^2\right]$$

$$\mathcal{L} = \ldots - \frac{F^2}{4}\left(1 + \mathcal{K}\, Tr\left[\mathcal{X}\right]\right)$$

RESULTS

CHARGE SECTOR

$$\frac{D_c}{v_B^2} = \mathcal{A}(g_i)\,\frac{\hbar}{k_B T}\,, \qquad g_i = \mathcal{J}, \mathcal{K}, \ldots$$

$$\mathcal{A}\left(g_i^*\right) = 0$$

ENERGY SECTOR

$$\frac{D_e}{v_B^2} \geq \mathcal{C}\,\frac{\hbar}{k_B\, T} \qquad \mathcal{C} > 0$$

Also for: $V\left(\mathcal{X}\right)$

More and more checks

Lifshitz - Hyperscaling ✓ → $D_T = \frac{z}{2z-2} v_B^2 \tau_L,$

[Blake, Sachdev, Davison]

AdS2 horizons ✓ → $D_T = E v_B^2 \tau_L, \qquad 1/2 < E \leq 1$

[Blake, Donos]

Gauss- Bonnet ✓ → $L^2 \rightarrow L^2_{eff}(\lambda_{GB})$

[Wu,Wang,Ge,Tian]

SYK ✓ → $D_2 = \frac{v_B^2}{2\pi T}.$

[Davison,Fu, Gu, Georges, Sachdev, Jensen]

Weakly coupled Fermi Liquids ✓

[Aleine, Faoro, Ioffe]

Bose-Hubbard models ✓

[Bohrdt, Endrel, Mendes, Knap]

Diffusive metals ✓

[Swingle, Chowdhury]

Electron-Phonon bad metals ✓

[Werman, Kivelson, Berg]

Critical Fermi Surfaces ✓ → $D_T \sim v_B^2 \lambda_L^{-1}$

[Patel, Sachdev]

Higher derivatives check part II

Higher derivative couplings Charge sector - Momentum relaxing sector → ~~**CHARGE BOUND**~~

[MB, Gouteraux, Kiritsis, Li, 2016]

What about Gravity – Momentum Relaxing sector ??

$$S = \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F^2 - \frac{1}{2} \left(g^{\mu\nu} - \underline{\gamma G^{\mu\nu}} \right) \sum_{i=1}^{2} \partial_\mu \phi^i \partial_\nu \phi^i \right)$$

Holographic Horndeski theories

[MB, Li, 2017]

Results: →

$$\frac{D_e}{v_B^2} \geq \mathcal{C} \, \frac{\hbar}{k_B T}$$

Conclusions

IS THIS BOUND REALLY UNIVERSAL ???

Higher derivative terms, large N, alpha' , etc ....
Beyond holography checks ....

[Gouteraux, Blake, Davison, Sachdev, Donos, Kiritsis,Patel,Li Kim, Ling,Wu,Jensen Tian, Wang, Swingle]

CAN WE PROVE IT ??
UNDERSTAND IT BETTER !!
vB?Strong coupling-Chaos?

[Hartnoll, Grozdanov, Lucas, Shenker, Liu, Stanford, Phillips, Ge, Niu, Amoretti, Jin-Sin, Musso, Magnoli, ...]

IF UNIVERSAL, CAN WE MEASURE IT ?? CAN WE TEST IT ?

Holography, Condensed Matter, Quantum Chaos, Hydrodynamics, Quantum Information, Random matrix theories, Black Holes ….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thank you