Chiral Transport and the Hydrodynamic Frames

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Based on works:

[1508.06879 1509.08878 1612.0864 170?.????]

In collaboration with

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0. Outline

- Relativistic hydro
- Motivation for hydrodynamic chiral transport
- Non–dissipative feature of chiral transport
- Spectrum of chiral hydro modes
- Chiral hydro modes from Kinetic theory
- Frame choice and hydro modes

1. Hydrodynamics

Response of the system to perturbations:

at low energy and Long wave-length limit

Equations of hydro are local conservation equations

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$
$$\partial_\mu J^\mu = 0$$

variables → $T(x),\ \mu(x),\ u^\mu(x)$

$(u^\mu u_\mu = -1)$

Main idea of hydro: **14** variables in terms of **5** variables

2. Dual gravity picture?

Early studies:

Membrane Paradigm: Fluid on the horizon

[Damur; Thorne Price Macdonald 1970’s]

New viewpoint:

Fluid–Gravity Duality: Fluid on the boundary

[Bhattacharyya Hubeny Minwalla Rangamani 2007]

3. Extensions of Fluid–Gravity:

1. Forced Fluid [Bhattacharyya Loganayagam Minwalla Nampuri Trividi 2007]
2. Non-Relativistic Fluid [Bhattacharyya Minwalla Wadia 2008]
3. **(Chirally) Charged Fluid** [Erdmenger Haack Kaminski Yarom 2008]

[Banerjee Bhattacharyya Bhattacharyya Dutta Loganayagam Surowka 2008]

$$S = \frac{1}{16\pi G_5}\int \sqrt{-g_5}\left[R + 12 - F_{AB}F^{AB} - \underbrace{\frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}}_{\text{anomaly}}\right]$$

anomaly

$$T_{\mu\nu} = p(\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\eta\sigma_{\mu\nu} + \ldots$$

$$J_\mu = n\, u_\mu - \mathfrak{D}\, P_\mu^\nu \mathcal{D}_\nu n + \xi\, l_\mu + \ldots$$

vorticity: $l^\mu \equiv \epsilon^{\nu\lambda\sigma\mu} u_\nu \partial_\lambda u_\sigma$

5. Hydro with triangle anomalies:

Motivated by Fluid / Gravity:

In the presence of anomalies one may add parity odd terms:

$$J^\mu = nu^\mu - \sigma T P^{\mu\nu}\partial_\nu \left(\frac{\mu}{T}\right) + \sigma E^\mu \boxed{+ \xi \omega^\mu + \xi_B B^\mu}$$

$$\partial_\mu j^\mu = C E^\mu B_\mu,$$
$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

[Son Surowka 2009]

[Kharzeev Yee ; **Neiman Oz** 2011]

[Jensen Loganayagam Yarom 2012]

In Landau–Lifhitz frame

$$\xi = \mathcal{C}\mu^2 \left(1 - \frac{2}{3}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) + \mathcal{D}T^2\left(1 - \frac{2\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right)$$
$$\xi_B = \mathcal{C}\mu\left(1 - \frac{1}{2}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) - \frac{\mathcal{D}}{2}\frac{\bar{n}T^2}{\bar{\epsilon}+\bar{p}}$$

6. Chiral transport is non–dissipative:

Ohm Law: $\boldsymbol{J} = \sigma \boldsymbol{E}$ (-) (-)(+) dissipative

London 2nd eq.: $\frac{\partial \boldsymbol{J}_s}{\partial t} = \sigma_s \boldsymbol{E}$ (+) (+)(+) non-dissipative

Transport in system of
Single right-handed fermions: $\boldsymbol{J} = \xi_B \boldsymbol{B}$ $(-)$ $(+)(-)$ non-dissipative

7. A more realistic model :

Chiral hydrodynamics with both vector and axial currents:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$
$$\partial_\mu J^\mu = 0$$
$$\partial_\mu J_5^\mu = \mathcal{C} E_\mu B^\mu$$

with

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$
$$J^\mu = n u^\mu + \xi\,\omega^\mu + \xi_B B^\mu$$
$$J_5^\mu = n_5 u^\mu + \xi_5\,\omega^\mu + \xi_{B5} B^\mu$$

[Gao Liang Pu Wang Wang 2012]

[Landsteiner Megias Pena-Benitez 2013]

$$\xi = 2\mathcal{C}\left(\mu\mu_5 - \frac{n\mu_5}{3w}\left(3\mu^2 + \mu_5^2\right)\right) - 2\mathcal{D}\,\frac{n\mu_5}{w} T^2$$
$$\xi_5 = \mathcal{C}\left(\mu^2 + \mu_5^2 - \frac{2n_5\mu_5}{3w}\left(3\mu^2 + \mu_5^2\right)\right) + \mathcal{D}\left(1 - \frac{2n_5\mu_5}{w}\right) T^2$$
$$\xi_B = \mathcal{C}\,\mu_5\left(1 - \frac{n\mu}{w}\right)$$
$$\xi_{5B} = \mathcal{C}\mu\left(1 - \frac{n_5\mu_5}{w}\right)$$

8. Hydro excitations in a chiral system:

Long-wavelength excitations around equilibrium:

$$u^\mu = \left(1,\ \boldsymbol{\Omega}\times\boldsymbol{x}\right) \qquad \Omega r \ll 1,$$
$$T = Const.,\ \ \mu = Const.,\ \ \mu_5 = Const.$$
$$\boldsymbol{B} = Const.$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda}J_\lambda$$
$$\partial_\mu J^\mu = 0$$
$$\partial_\mu J_5^\mu = \mathcal{C}E_\mu B^\mu$$

perturbing the hydro fields: →

$$\phi_a = (T,\ \boldsymbol{\pi_i},\ \mu,\ \mu_5), \qquad a = 1,2,...,6$$

$$M_{ab}^{B\Omega}(\boldsymbol{k},\omega)\delta\phi_a(\boldsymbol{k},\omega) = 0,$$

9. Fluid coupled to weak magnetic field

$$\begin{bmatrix} -i\alpha_1\omega & ik_j & -i\alpha_2\omega & -i\alpha_3\omega \\ i\alpha_1 v_s^2 k^i & -i\omega\delta^i_j - i\frac{\xi}{2\bar{w}}\left(\boldsymbol{B}\cdot\boldsymbol{k}\delta^i_j - B_j k^i\right) - \frac{\bar{n}}{\bar{w}}\epsilon^i{}_{jl}B^l & i\alpha_2 v_s^2 k^i & i\alpha_3 v_s^2 k^i \\ -i\beta_1\omega + \left(\frac{\partial\xi_B}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & \frac{\bar{n}}{\bar{w}} ik_j - \frac{\xi_B}{\bar{w}} i\omega B_j & -i\beta_2\omega + \left(\frac{\partial\xi_B}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & -i\beta_3\omega + \left(\frac{\partial\xi_B}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \\ -i\gamma_1\omega + \left(\frac{\partial\xi_{5B}}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & \frac{\bar{n}_5}{\bar{w}} ik_j - \frac{\xi_{5B}}{\bar{w}} i\omega B_j & -i\gamma_2\omega + \left(\frac{\partial\xi_{5B}}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & -i\gamma_3\omega + \left(\frac{\partial\xi_{5B}}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{bmatrix}$$

6 linear coupled equations give **6** hydro modes:

$\boldsymbol{k} \parallel \boldsymbol{B}$

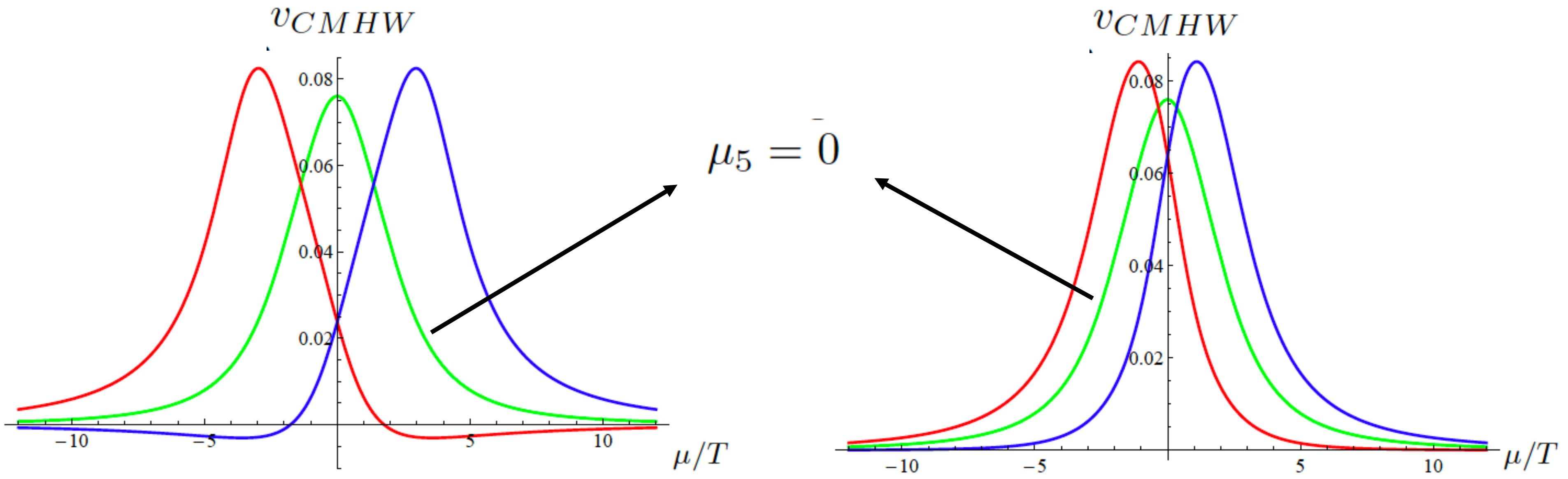
1,2: Chiral-Magnetic-Heat wave: $\omega_{1,2}(k) = -\frac{\mathcal{A}_1 \pm \sqrt{\mathcal{A}_1^2 - \mathcal{A}_2\mathcal{E}}}{\mathcal{E}}\boldsymbol{B}.\boldsymbol{k}$

3,4: Ordinary sound wave: $\omega_{3,4}(k) = \pm c_s k$

5,6: Chiral Alfven wave: $\omega_{5,6}(k) = \pm\frac{\xi}{2w}Bk$

10. Chiral –Magnetic–Heat Wave

Equation of state $$\epsilon = 3p = \frac{7\pi^2}{60}T^4 + \frac{1}{2}(\mu^2 + \mu_5^2)T^2 + \frac{1}{4\pi^2}(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4)$$



(a) $\frac{\mu_5}{T} = 3$ and $\frac{B}{T^2} = 0.5$

(b) $\frac{\mu_5}{T} = 1$ and $\frac{B}{T^2} = 0.5$

[N.A Allahbakhshi Davody Taghavi 2016]

11. CMHW in QGP:

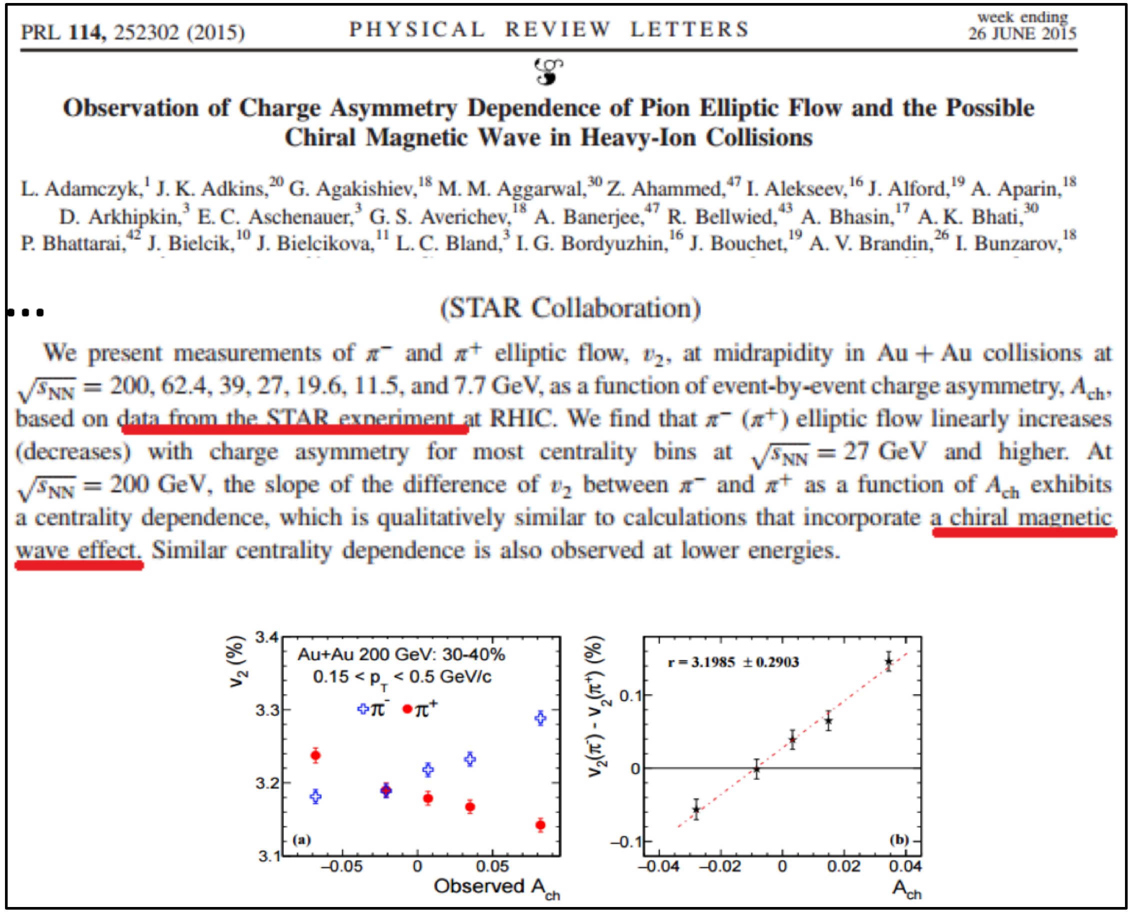
It is well–understood that the QGP produced in HIC in initially non–chiral: $\mu_5 = 0$

$$\omega_{1,2}(k) = -\frac{\mathcal{A}_1 \pm \sqrt{\mathcal{A}_1^2 - \mathcal{A}_2 \mathcal{E}}}{\mathcal{E}} \boldsymbol{B}.\boldsymbol{k}$$

$\mu_5 = 0$

CMW: [Kharzeev, Yee. 2012]

$$\omega_{1,2} = \pm \frac{Bk}{2\pi^2\chi} \frac{1 - \frac{\mu n}{w}}{\sqrt{1 - \frac{\mu n}{w} - \frac{n}{\chi w}\left(\frac{n}{c_s^2} - \chi\mu\right)}}$$



PRL **114,** 252302 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

Observation of Charge Asymmetry Dependence of Pion Elliptic Flow and the Possible Chiral Magnetic Wave in Heavy-Ion Collisions

L. Adamczyk,[1] J. K. Adkins,[20] G. Agakishiev,[18] M. M. Aggarwal,[30] Z. Ahammed,[47] I. Alekseev,[16] J. Alford,[19] A. Aparin,[18] D. Arkhipkin,[3] E. C. Aschenauer,[3] G. S. Averichev,[18] A. Banerjee,[47] R. Bellwied,[43] A. Bhasin,[17] A. K. Bhati,[30] P. Bhattarai,[42] J. Bielcik,[10] J. Bielcikova,[11] L. C. Bland,[3] I. G. Bordyuzhin,[16] J. Bouchet,[19] A. V. Brandin,[26] I. Bunzarov,[18]

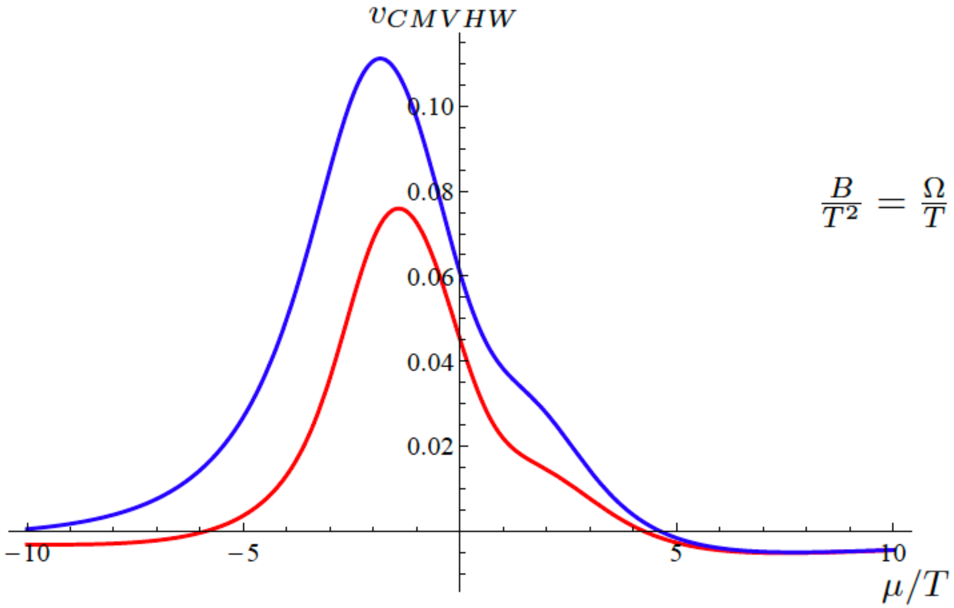
...

(STAR Collaboration)

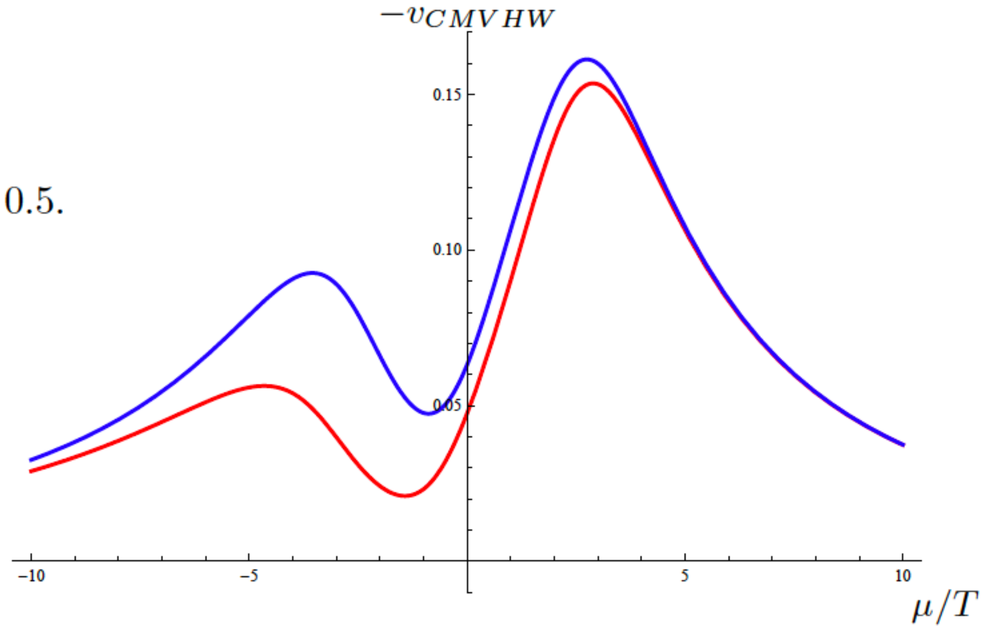
We present measurements of π^- and π^+ elliptic flow, v_2, at midrapidity in Au + Au collisions at $\sqrt{s_{NN}}$ = 200, 62.4, 39, 27, 19.6, 11.5, and 7.7 GeV, as a function of event-by-event charge asymmetry, A_{ch}, based on data from the STAR experiment at RHIC. We find that π^- (π^+) elliptic flow linearly increases (decreases) with charge asymmetry for most centrality bins at $\sqrt{s_{NN}} = 27$ GeV and higher. At $\sqrt{s_{NN}} = 200$ GeV, the slope of the difference of v_2 between π^- and π^+ as a function of A_{ch} exhibits a centrality dependence, which is qualitatively similar to calculations that incorporate a chiral magnetic wave effect. Similar centrality dependence is also observed at lower energies.

12. **Rotating** Fluid coupled to weak magnetic field

Scalar sector: mixed chiral-magnetic-vortical-heat wave



$\frac{B}{T^2} = \frac{\Omega}{T} = 0.5.$



blue curve:

$$v_{sum} := v_{CMHW} + v_{CVHW} = v_{CMVHW}|_{\mathbf{\Omega=0}} + v_{CMVHW}|_{\mathbf{B=0}}$$

$$v_{CMVHW} \neq v_{sum}.$$

13. Out of equilibrium dynamics from **Kinetic theory**

Chiral particles in kinetic theory: Chiral Kinetic Theory

[Stephanov Yee 2012]

[Son Yamamoto 2012]

$$I = \int_{t_i}^{t_f} (\boldsymbol{p}\cdot\dot{\boldsymbol{x}} + \boldsymbol{A}\cdot\dot{\boldsymbol{x}} - \Phi - |\boldsymbol{p}| - \boldsymbol{a_p}\cdot\dot{\boldsymbol{p}})dt$$

$$|\hat{\boldsymbol{a}}_p| = \frac{1}{p} \quad \Longrightarrow \quad \boldsymbol{b} = \boldsymbol{\nabla_p} \times \boldsymbol{a_p}$$

Berry Curvature → Berry Flux

Kinetic equation:

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \boldsymbol{x}}\dot{\boldsymbol{x}} + \frac{\partial f}{\partial \boldsymbol{p}}\dot{\boldsymbol{p}} = C[f]$$

$$\sqrt{G}\,\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} + \boldsymbol{E}\times\boldsymbol{b} + \boldsymbol{B}(\hat{\boldsymbol{p}}\cdot\boldsymbol{b}).$$

$$\sqrt{G}\dot{\boldsymbol{p}} = \boldsymbol{E} + \hat{\boldsymbol{p}}\times\boldsymbol{B} + \boldsymbol{b}(\boldsymbol{E}\cdot\boldsymbol{B}).$$

14. Conservation equations in CKT:

distribution function $f_{\pm}^{R,L}\left(p;\ \beta(x), \boldsymbol{u}(x), \mu_{R,L}(x)\right) = \dfrac{1}{e^{\beta(x)\left(\epsilon - \boldsymbol{p}.\boldsymbol{u}(x) \mp \mu_{R,L}(x)\right)} + 1}$

Integrating the kinetic equations after multiplying with collision invariant quantities:

Collision invariant.	Conservation Eq.	Corresponding Hydro Eq.
q_R	$\int_{\boldsymbol{p}} \left(\frac{df_+^R}{dt} - \frac{df_-^R}{dt}\right) = 0$	$\partial_\mu J^\mu = 0$
q_L	$\int_{\boldsymbol{p}} \left(\frac{df_+^L}{dt} - \frac{df_-^L}{dt}\right) = 0$	$\partial_\mu J_5^\mu = \mathcal{C} E_\mu B^\mu$
p^i	$\int_{\boldsymbol{p}} p^i \left(\frac{df_+^R}{dt} + \frac{df_-^R}{dt}\right) + (R \leftrightarrow L) = 0$	$\partial_\mu T^{\mu i} = F^{i\nu} J_\nu$
ϵ	$\int_{\boldsymbol{p}} \epsilon \left(\frac{df_+^R}{dt} + \frac{df_-^R}{dt}\right) + (R \leftrightarrow L) = 0$	$\partial_\mu T^{\mu 0} = F^{0\nu} J_\nu$

15. Hydro modes from CKT

For slowly-varying macroscopic fields $\beta(x), \boldsymbol{u}(x), \mu_{R,L}(x)$

we expand the distribution function around the thermo distribution:

$$f_{\pm}^{R,L}(p) = \frac{1}{e^{\beta\left(\epsilon \mp \mu_{R,L}\right)} + 1}$$

to find the linearized equations:

$$M_{ab}\, \delta\phi_a(\omega, \boldsymbol{k}) = 0$$

with: $\phi_a(x) = \left(\beta(x), \boldsymbol{\pi}(x), \mu_R(x), \mu_L(x)\right)$

16. Comparison between **Hydro in LL** and **CKT**

For a fluid of single right-handed fermions at $\mu = 0$:

kinetic theory ⬇ (No-drag fram) relativistic hydro ⬇ (Landau-lifshitz frame)

Type of mode	No-drag fram	Landau-lifshitz frame
CMHW	$v_1 = \left(\frac{\xi}{2w} + \left(\frac{\partial \xi_B}{\partial n}\right)_\epsilon\right) B$	$v_1 = \left(\frac{\partial \xi_B}{\partial n}\right)_\epsilon B$
Sound	$v_{2,3} = \pm c_s - \frac{c_s^2 - 1}{2w} \xi B$	$v_{2,3} = \pm c_s$
CAW	$v_{4,5} = 0$	$v_{4,5} = -\frac{\xi}{2w}\mathrm{B}$

$$\xi = \mathcal{D} T^2$$

$$\mathcal{D} = \frac{1}{12}$$

17. Hydrodynamic frames:

Beyond the zero order in derivative expansion, there is an ambiguity in defining hydro variables. One can make transformations

$$u^\mu \to u^\mu + \delta u^\mu$$
$$T \to T + \delta T$$
$$\mu \to \mu + \delta\mu$$
$$\mu_5 \to \mu_5 + \delta\mu_5$$

[Bhattacharya Bhattacharyya Minwalla Yarom 2011]

while physical quantities $T^{\mu\nu}$ J^μ and J_5^μ remaining invariant.

Constructed by independent one-derivative data $\nabla_\mu u^\mu,\ \partial_\mu^\perp \frac{\mu}{T},\ B^\mu,\ \omega^\mu, \ldots$

Each choice of frame is equivalent to one way of fixing this ambiguity.

18. In the absence of dissipation

Thermodynamic frame

$$u^\mu \to u^\mu$$

$$T \to T$$
$$\mu \to \mu$$
$$\mu_5 \to \mu_5$$

Landau-Lifshitz

$$u_\mu T^{\mu\nu}_{(1)} = 0$$

$$u^\mu \to u^\mu + (\# B^\mu)$$

$$T^{\mu\nu} = w u^\mu u^\nu + p g^{\mu\nu} + \sigma^{\mathcal{B}}_\epsilon u^{(\mu} B^{\nu)} + \sigma^{\mathcal{V}}_\epsilon u^{(\mu}\omega^{\nu)}$$
$$j^\mu = n u^\mu + \sigma^{\mathcal{V}} \omega^\mu + \sigma^{\mathcal{B}} B^\mu$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$
$$J^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu$$

$$\delta u^\mu = -\frac{1}{\epsilon + p}\left(\sigma^{\mathcal{B}}_\epsilon B^\mu + \sigma^{\mathcal{V}}_\epsilon \omega^\mu\right)$$

[Landsteiner. 2017]

19. Hydrodynamic modes are **not** frame invariant

Thermodynamic (Lab)	Landau-Lifshitz.	Boost
$u^\mu = (1, \mathbf{0})$	$u^\mu = (1, \mathbf{0})$	$v_{LL} = \dfrac{v_{Lab} - \tilde{v}_{rel}}{1 - v_{Lab}\, \tilde{v}_{rel}}$ $\tilde{v}_{rel} = -\dfrac{\sigma_\epsilon^{\mathcal{B}} B + \sigma_\epsilon^{\mathcal{V}} \omega}{w}$
$u^\mu = (1, \mathbf{0})$	$u^\mu = (1, \tilde{\boldsymbol{v}}_{rel}), \quad u^\mu u_\mu = -1 + O(\tilde{\boldsymbol{v}}_{rel}^2)$	$v_{LL} = v_{Lab}$

Physical frame, consistent with **Vilenkin 89** results.

"No-drag frame: [Stephanov Yee 2015]"

20. QCD fluid:

In single chirality fluid: CAW does not exist. [Yamamoto 2105]

[N.A Davody Rezaei Hejazi 2016]

In QCD type fluid: CAW does exist:

$$v_{CAW} = \left(\mathcal{C}\mu\mu_5 - \frac{2n}{3w}\mathcal{C}\mu_5(3\mu^2 + \mu_5^2) - \frac{2n}{w}\mathcal{D}T^2 \right) B$$

$$\mathcal{C} = \frac{1}{2\pi^2}, \qquad \mathcal{D} = \frac{1}{6}$$

Gauge-gravitational anomaly in QGP.

21. Open questions

1. How would change the situation when the magnetic field is dynamical?

2. Will be the CAW propagate when considering the back-reaction effects?

3. The full spectrum of chiral magneto hydrodynamics?

Thank you