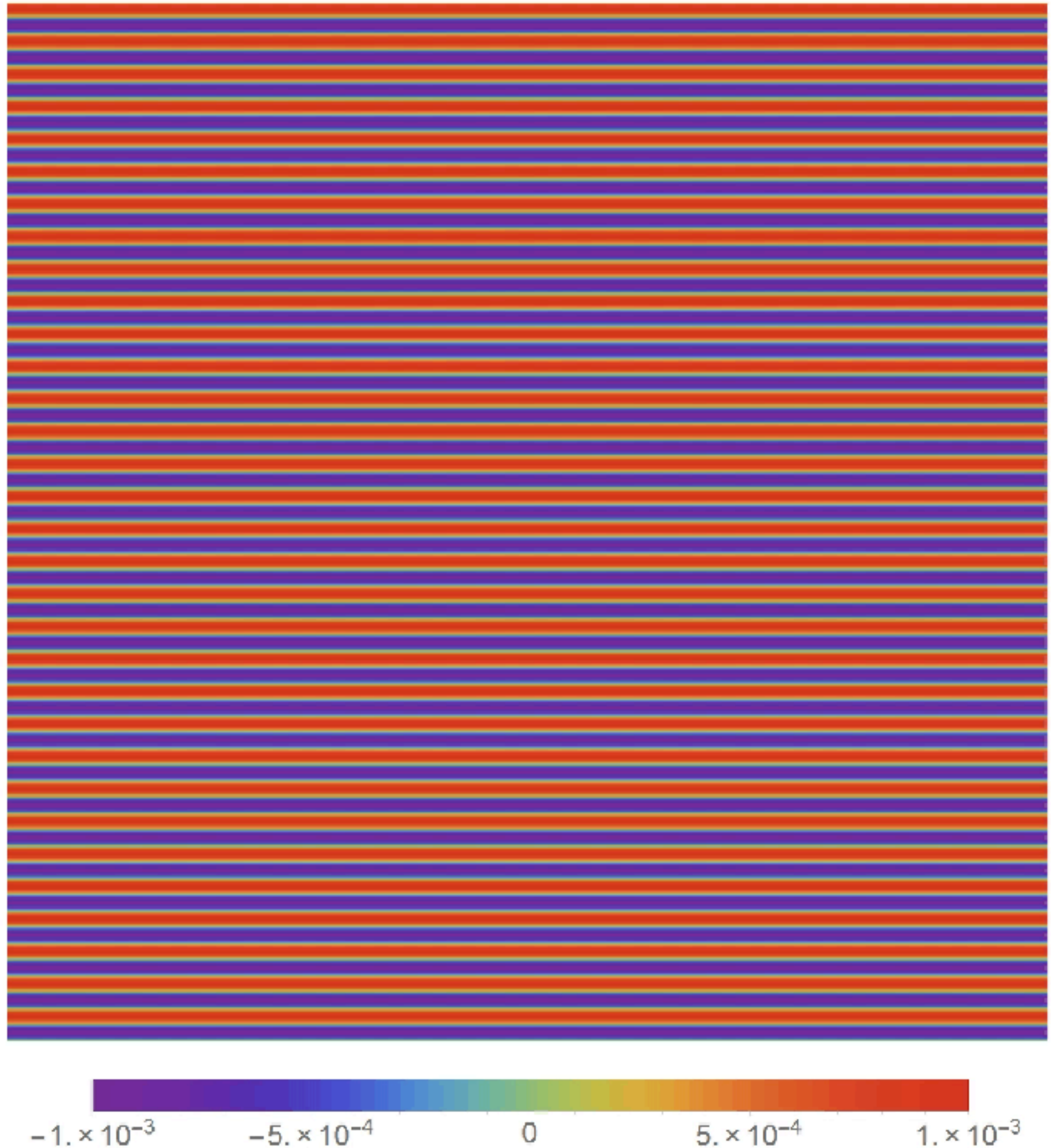
The large d limit of turbulence

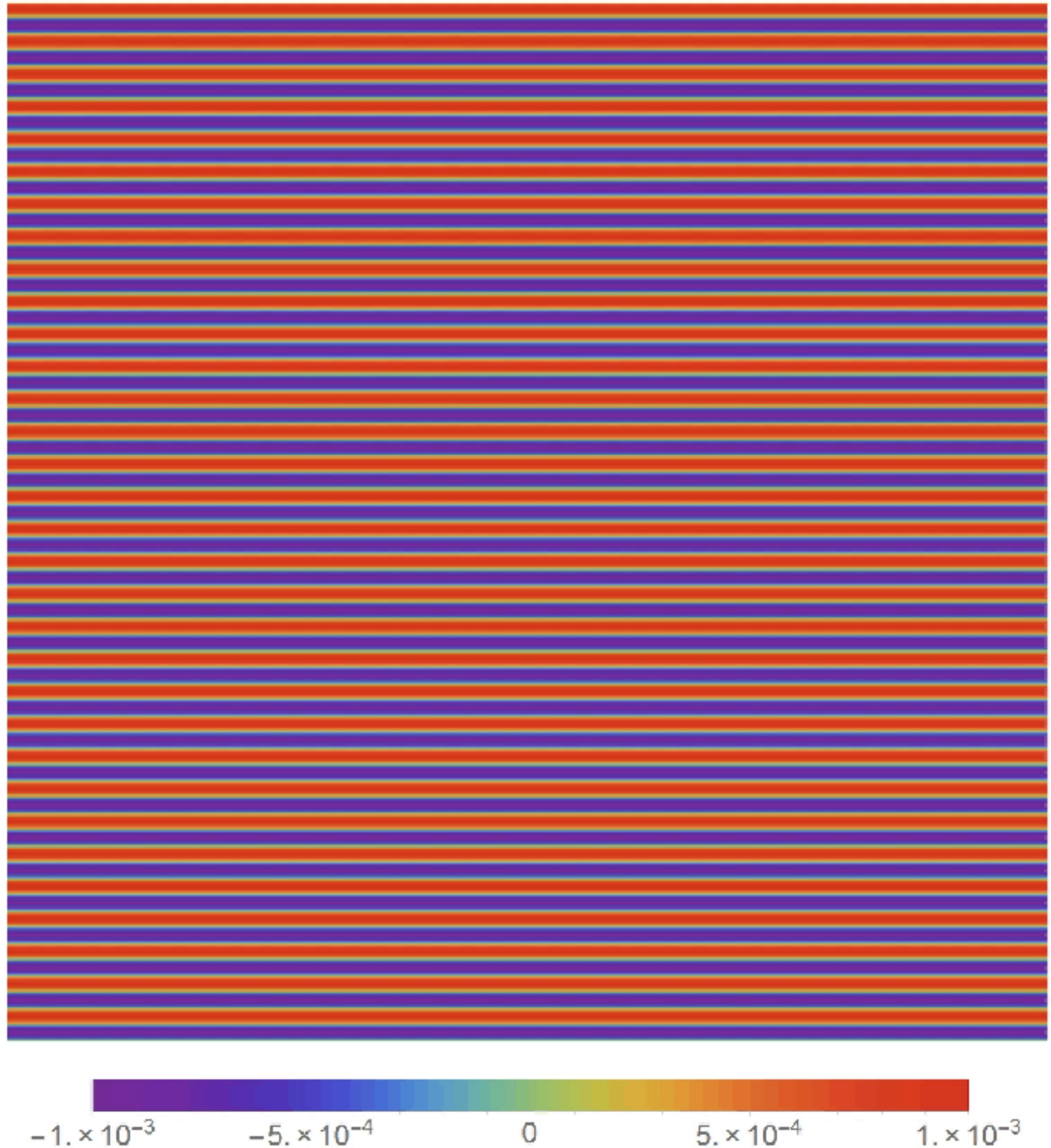
Amos Yarom

With: M. Rozali and E. Sabag

(to appear shortly)



$-1.\times10^{-3}$
$-5.\times10^{-4}$
0
$5.\times10^{-4}$
$1.\times10^{-3}$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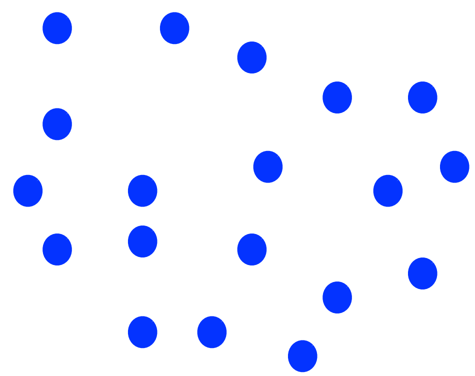
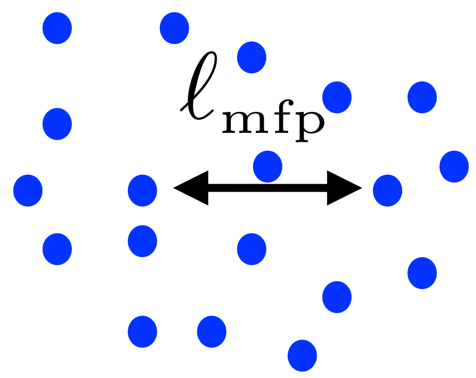
$1.\times10^{-3}$

- The large d limit of hydrodynamics
- The large d limit of holography
- Turbulent behavior of large d fluids

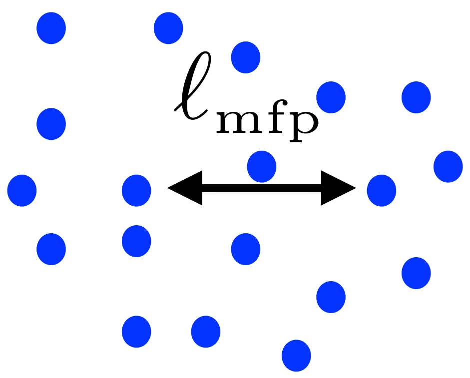
Hydrodynamics



Hydrodynamics

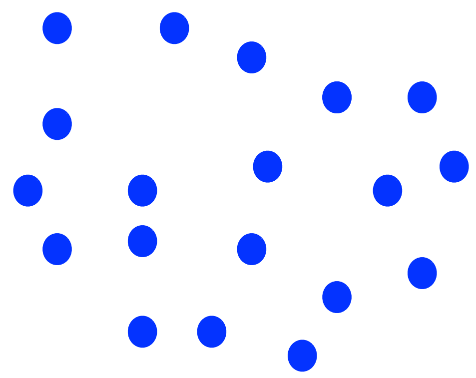


Hydrodynamics



$$L \gg \ell_{\rm mfp}$$

Hydrodynamics



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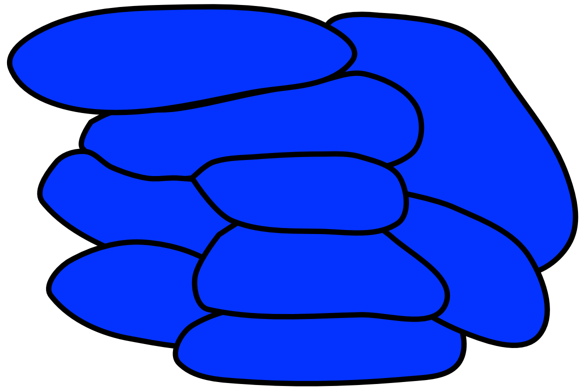
Hydrodynamics

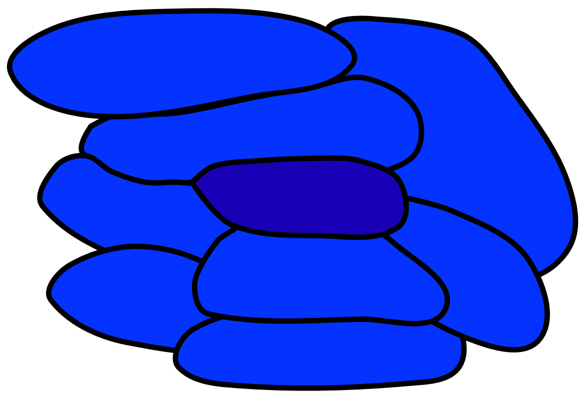


$$L \gg \ell_{\mathrm{mfp}}$$

Hydrodynamics

Variables:

$T(x^\mu)$

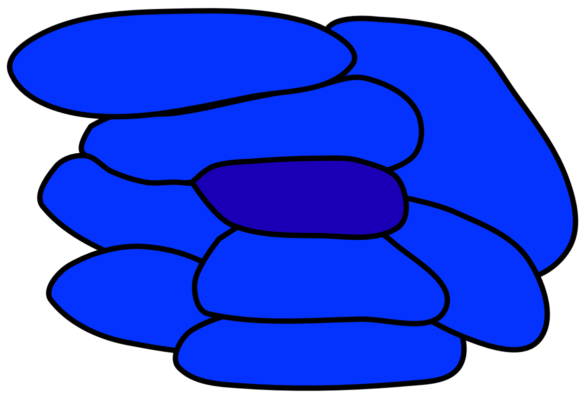


$L \gg \ell_{\rm mfp}$

Hydrodynamics

Variables:

$T(x^\mu)$ Temperature



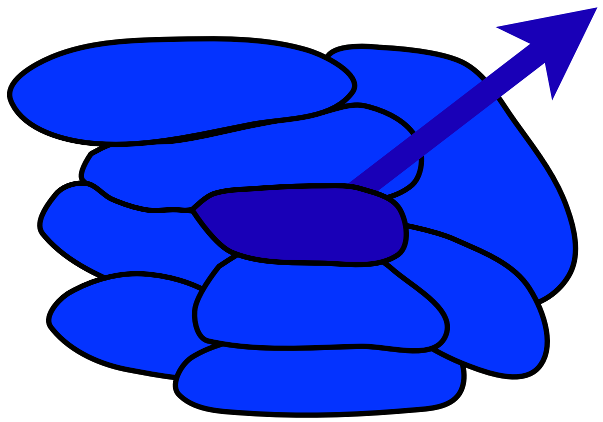
$L \gg \ell_{\rm mfp}$

Hydrodynamics

Variables:

$T(x^\mu)$ Temperature

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

Hydrodynamics

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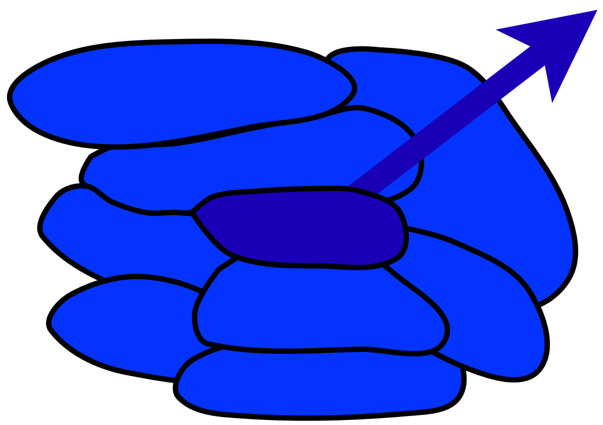

$T(x^\mu)$ Temperature

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

Constitutive relations:

$L \gg \ell_{\rm mfp}$

$T^{\mu\nu}[T, u^\alpha]$

Hydrodynamics

Variables:

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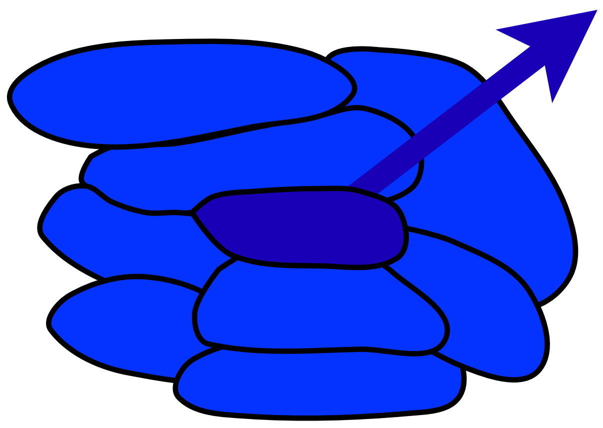

$L \gg \ell_{\mathrm{mfp}}$

Constitutive relations:

$T^{\mu\nu}[T, u^\alpha]$

Kinematic equations:

$\partial_\mu T^{\mu\nu} = 0$

Hydrodynamics

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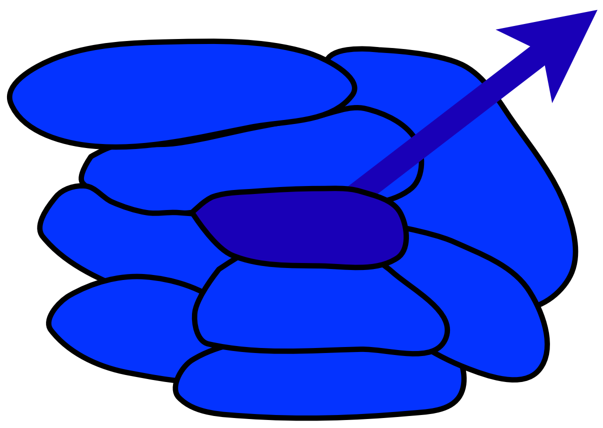

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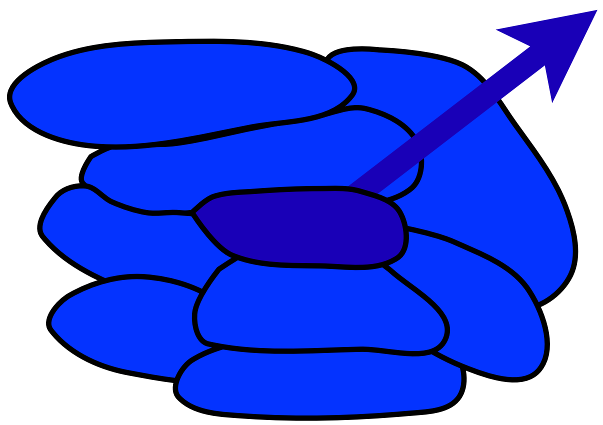


$L \gg \ell_{\rm mfp}$

Constitutive relations:

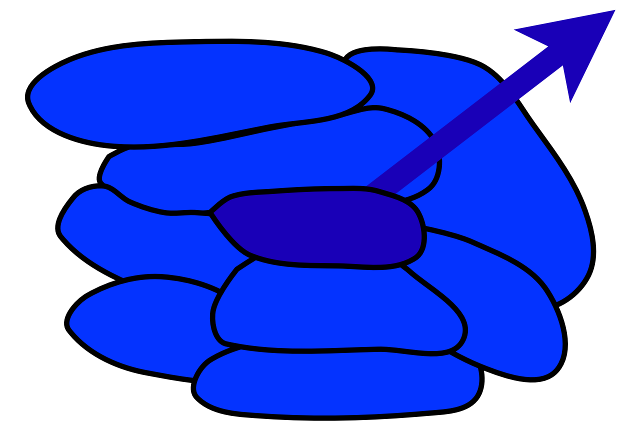
$$T^{\mu\nu} = \epsilon(T) u^\mu u^\nu + P(T)\left(\eta^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial)$$

Kinematic equations:

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

Variables:



$T(x^\mu)$ Temperature

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

Constitutive relations:

$L \gg \ell_{\mathrm{mfp}}$

$$T^{\mu\nu} = \epsilon(T)u^\mu u^\nu + P(T)\left(\eta^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial)$$

$\epsilon = (d-1)P$

Kinematic equations:

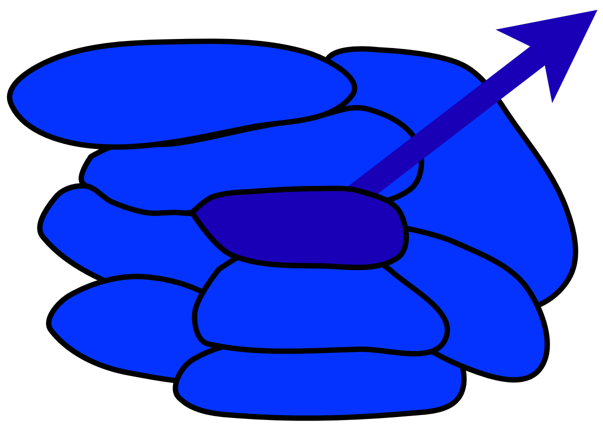
$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

Variables:

$T(x^\mu)$ Temperature

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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Constitutive relations:

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

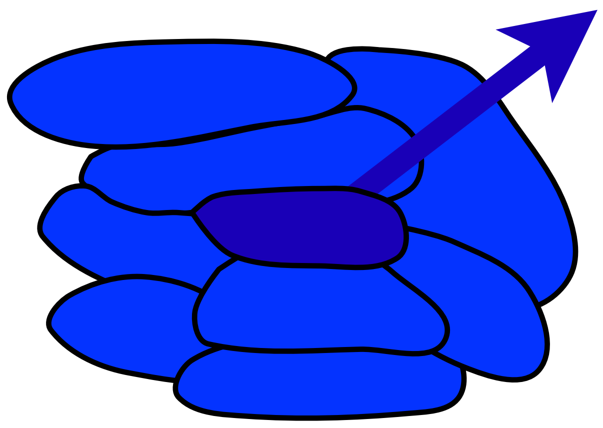
Kinematic equations:

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

Constitutive relations:

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right)$$



$$L \gg \ell_{\rm mfp}$$

Kinematic equations:

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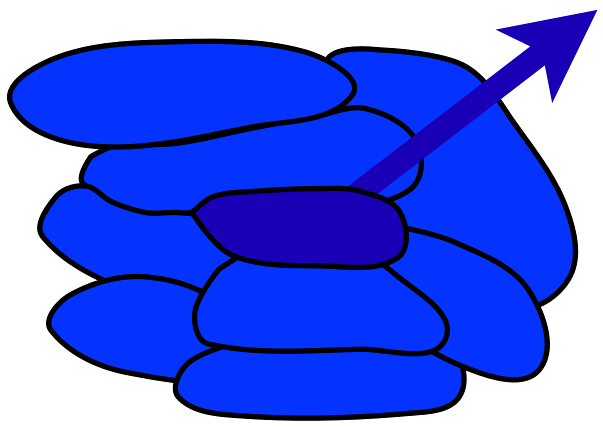
Hydrodynamics

Constitutive relations:



$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) - 2\eta(T)\sigma^{\mu\nu}$$

$$+\mathcal{O}(\partial^2)$$

$$L \gg \ell_{\mathrm{mfp}}$$

Kinematic equations:

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

Constitutive relations:

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^{\mu}u^{\nu} + \eta^{\mu\nu}\right) - 2\eta(T)\sigma^{\mu\nu}$$

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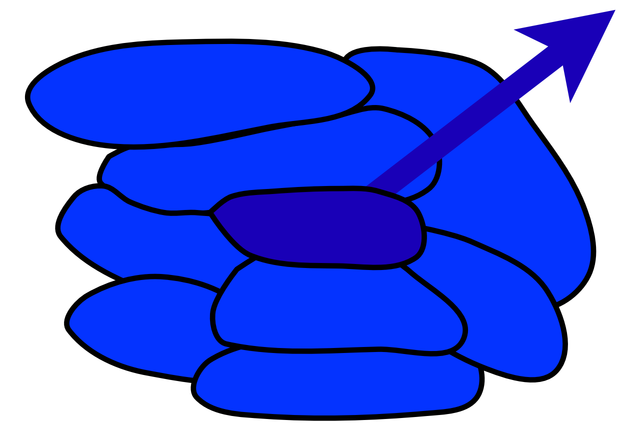


$$L \gg \ell_{\mathrm{mfp}}$$

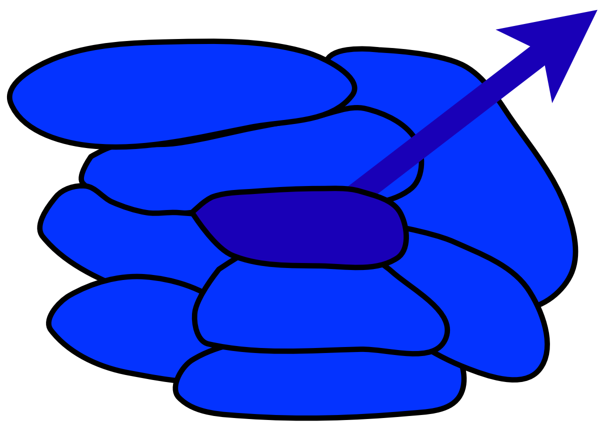
$$\sigma_{\mu\nu} = \langle \partial_{\mu} u_{\nu} \rangle$$

Kinematic equations:

$$\partial_{\mu} T^{\mu\nu} = 0$$

Hydrodynamics

Constitutive relations:

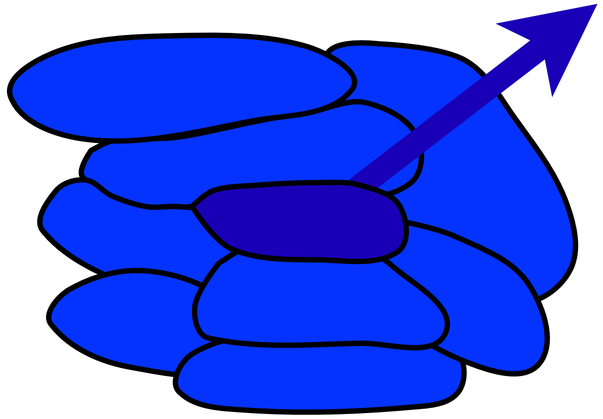


$$L \gg \ell_{\text{mfp}}$$

$$
\begin{aligned}
T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right) & - 2\eta(T)\sigma^{\mu\nu} \\
+ \Big[& \lambda_1 u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu} \\
& + \lambda_2 \left(\sigma^{\lambda\mu} \sigma_\lambda{}^\nu - \frac{\sigma^{\alpha\beta}\sigma_{\alpha\beta}}{d-1} P^{\mu\nu} \right) \\
& + \lambda_3 \left(\omega^{\mu\lambda} \sigma_\lambda{}^\nu + \omega^{\nu\lambda} \sigma_\lambda{}^\mu \right) \\
& + \lambda_4 \left(\frac{1}{2} \left(\omega^{\mu\lambda} \omega^\nu{}_\lambda + \omega^{\nu\lambda} \omega^\mu{}_\lambda \right) - \frac{\omega^{\alpha\beta}\omega_{\alpha\beta}}{d-1} P^{\mu\nu} \right) \Big] \\
& + \mathcal{O}(\partial^3)
\end{aligned}
$$

Hydrodynamics

Constitutive relations:



$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) - 2\eta(T)\sigma^{\mu\nu}$$

$$+ \Big[\lambda_1 u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu}$$

$$+\lambda_2 \left(\sigma^{\lambda\mu}\sigma_\lambda{}^\nu - \frac{\sigma^{\alpha\beta}\sigma_{\alpha\beta}}{d-1}P^{\mu\nu}\right)$$

$$+\lambda_3 \left(\omega^{\mu\lambda}\sigma_\lambda{}^\nu + \omega^{\nu\lambda}\sigma_\lambda{}^\mu\right)$$

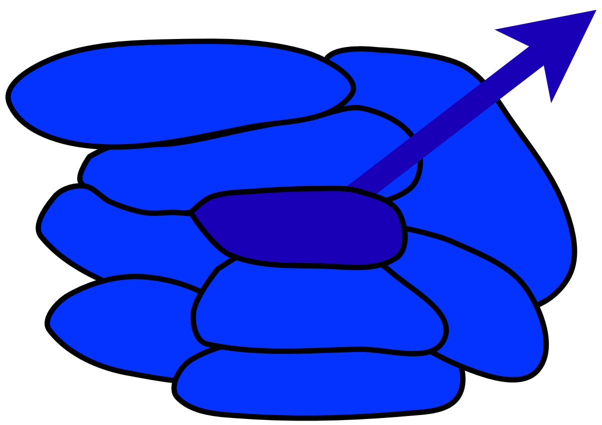
$$+\lambda_4 \left(\frac{1}{2}\left(\omega^{\mu\lambda}\omega^\nu{}_\lambda + \omega^{\nu\lambda}\omega^\mu{}_\lambda\right) - \frac{\omega^{\alpha\beta}\omega_{\alpha\beta}}{d-1}P^{\mu\nu}\right)\Big]$$

$$+\mathcal{O}(\partial^3)$$

$$L \gg \ell_{\mathrm{mfp}}$$

Large d Hydrodynamics

Constitutive relations:



$L \gg \ell_{\rm mfp}$

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right) - 2\eta(T)\sigma^{\mu\nu}$$

$$+ \Big[\lambda_1 u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu}$$

$$+\lambda_2 \left(\sigma^{\lambda\mu}{\sigma_\lambda}^\nu - \frac{\sigma^{\alpha\beta}\sigma_{\alpha\beta}}{d-1}P^{\mu\nu}\right)$$

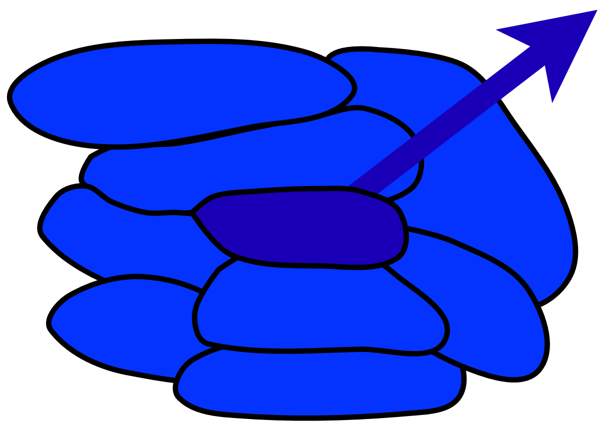
$$+\lambda_3 \left(\omega^{\mu\lambda}{\sigma_\lambda}^\nu + \omega^{\nu\lambda}{\sigma_\lambda}^\mu\right)$$

$$+\lambda_4 \left(\frac{1}{2}\left(\omega^{\mu\lambda}{\omega^\nu}_\lambda + \omega^{\nu\lambda}{\omega^\mu}_\lambda\right) - \frac{\omega^{\alpha\beta}\omega_{\alpha\beta}}{d-1}P^{\mu\nu}\right)\Big]$$

$$+\mathcal{O}(\partial^3)$$

Large d Thermodynamics

Constitutive relations:



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$$+\lambda_2\left(\sigma^{\lambda\mu}{\sigma_\lambda}^\nu - \frac{\sigma^{\alpha\beta}\sigma_{\alpha\beta}}{d-1}P^{\mu\nu}\right)$$

$$+\lambda_3\left(\omega^{\mu\lambda}{\sigma_\lambda}^\nu + \omega^{\nu\lambda}{\sigma_\lambda}^\mu\right)$$

$$+\lambda_4\left(\frac{1}{2}\left(\omega^{\mu\lambda}{\omega^\nu}_\lambda + \omega^{\nu\lambda}{\omega^\mu}_\lambda\right) - \frac{\omega^{\alpha\beta}\omega_{\alpha\beta}}{d-1}P^{\mu\nu}\right)\Big]$$

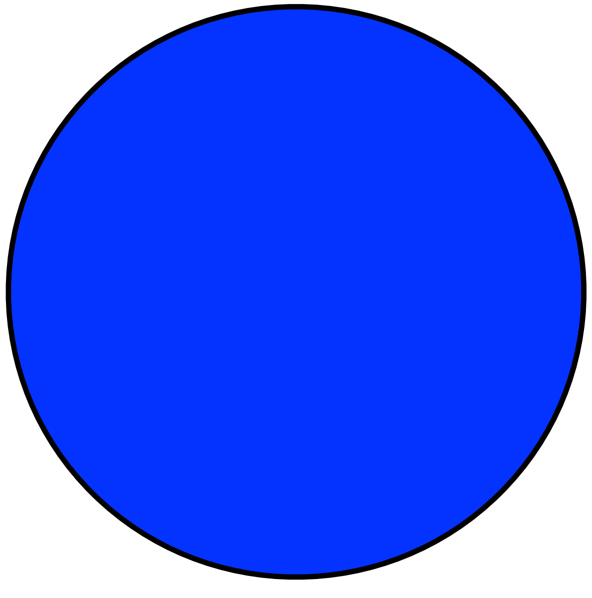
$$+\mathcal{O}(\partial^3)$$

$$L \gg \ell_{\mathrm{mfp}}$$

Large d Thermodynamics

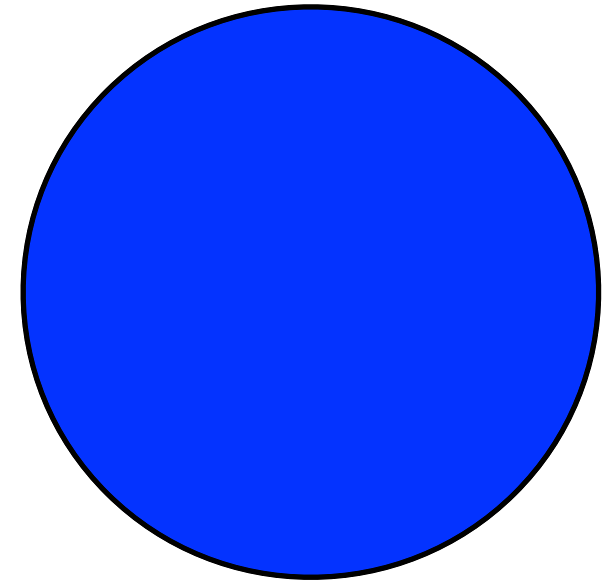
Equilibrium

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$



Large d Thermodynamics

Equilibrium

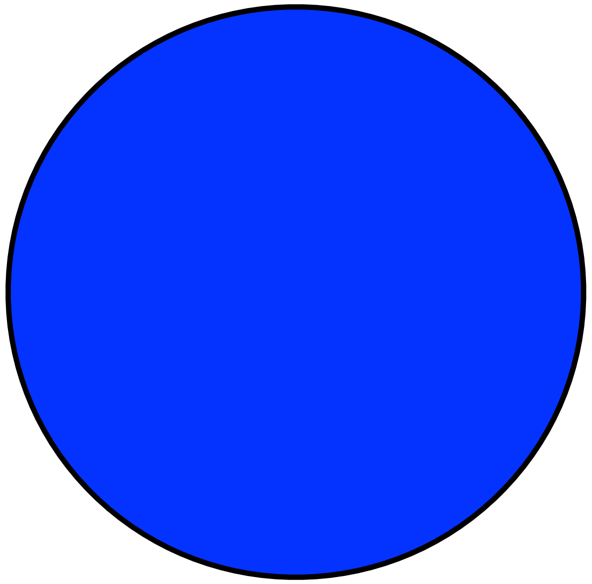


$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

$$ds^2 = -dt^2 + d\vec{x}^2$$

Large d Thermodynamics



Equilibrium

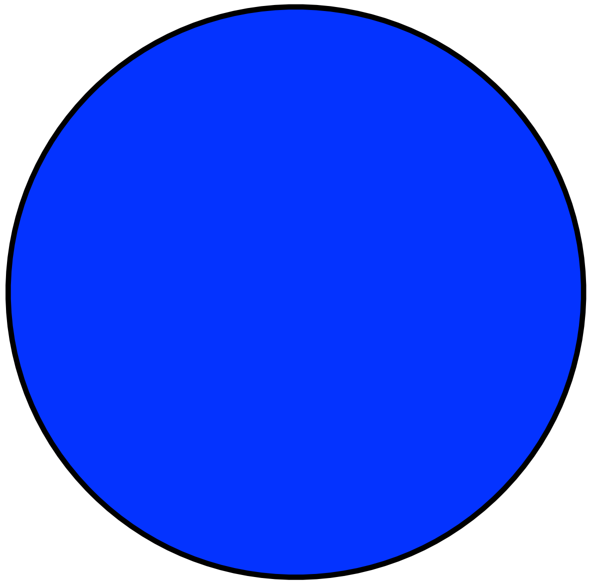
$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

$$ds^2 = -dt^2 + d\vec{x}^2 \qquad u^\mu = (1, 0, \ldots)$$

Large d Thermodynamics

Equilibrium

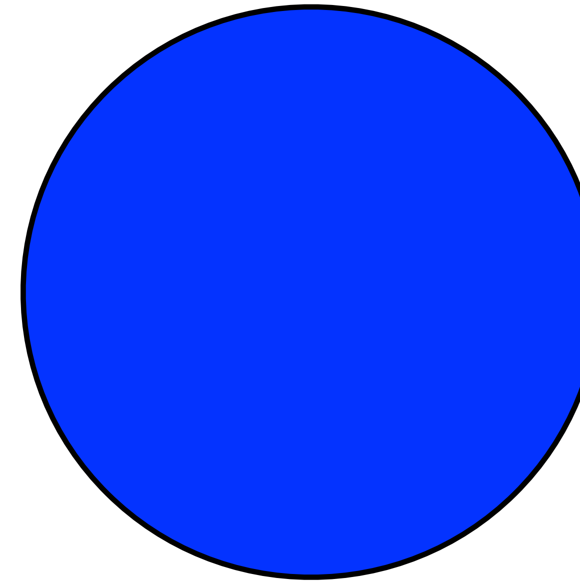


$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

$$ds^2 = -dt^2 + d\vec{x}^2 \qquad u^\mu = (1, 0, \ldots) \qquad \epsilon = \mathcal{O}(d^0)$$

Large d Thermodynamics



Equilibrium

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

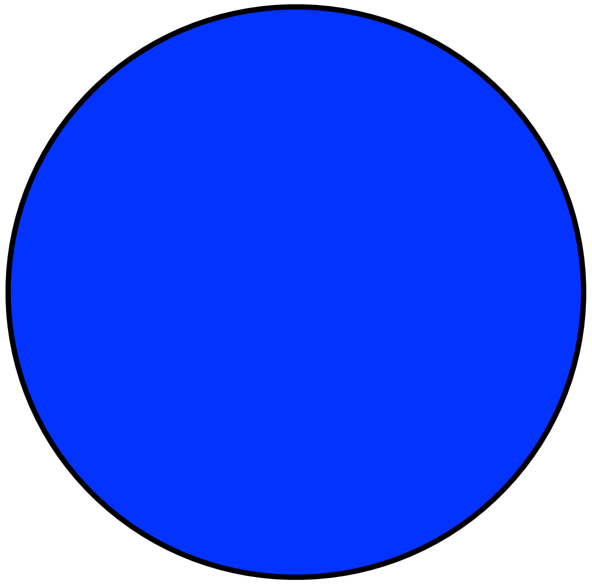
$$ds^2 = -dt^2 + d\vec{x}^2 \qquad u^\mu = (1, 0, \ldots) \qquad \epsilon = \mathcal{O}(d^0)$$

So that:

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & & & & \\ & \frac{\epsilon}{d-1} & & & \\ & & \frac{\epsilon}{d-1} & & \\ & & & \frac{\epsilon}{d-1} & \\ & & & & \ddots \end{pmatrix}$$

Large d Thermodynamics

Equilibrium



$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

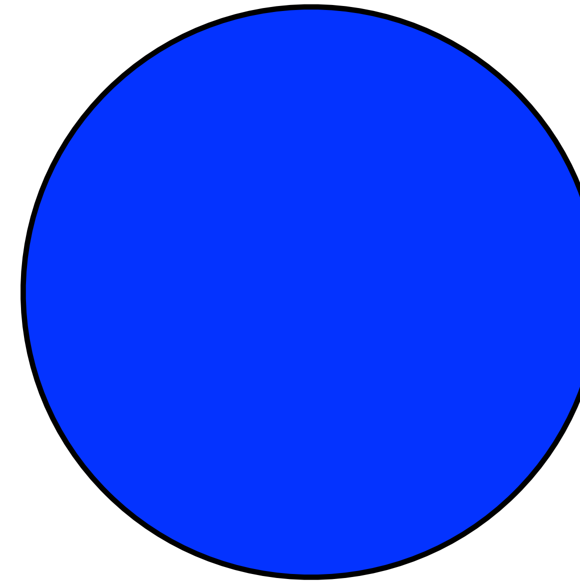
$$ds^2 = -dt^2 + d\vec{x}^2 \qquad u^\mu = (1, 0, \dots) \qquad \epsilon = \mathcal{O}(d^0)$$

$$\vec{x} = \frac{\vec{\chi}}{\sqrt{d}}$$

So that:

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & & & & \\ & \frac{\epsilon}{d-1} & & & \\ & & \frac{\epsilon}{d-1} & & \\ & & & \frac{\epsilon}{d-1} & \\ & & & & \ddots \end{pmatrix}$$

Large d Thermodynamics



Equilibrium

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

$$ds^2 = -dt^2 + d\left(\frac{\vec{\chi}}{\sqrt{d}}\right)^2 \quad u^\mu = (1, 0, \ldots) \qquad \epsilon = \mathcal{O}(d^0)$$

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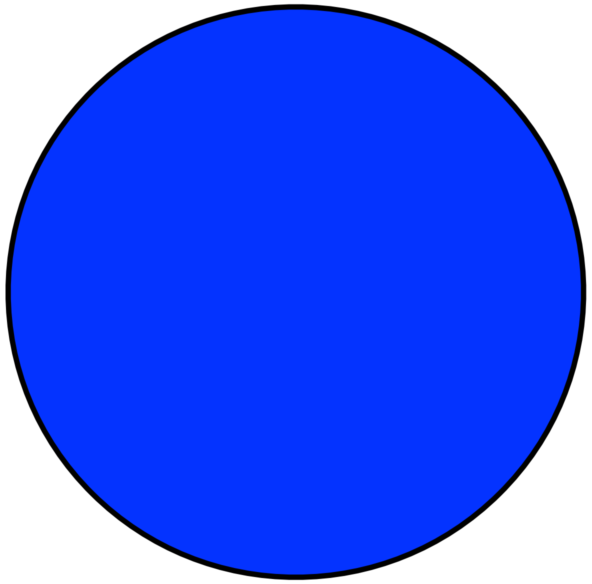
Large d Thermodynamics

Equilibrium



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Large d Thermodynamics

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$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

$$ds^2 = -dt^2 + d\left(\frac{\vec{\chi}}{\sqrt{d}}\right)^2$$

$$u^\mu = \frac{\left(1, \vec{\beta}\right)}{\sqrt{1 - \frac{\vec{\beta}\cdot\vec{\beta}}{d}}}$$

$$\epsilon = \mathcal{O}(d^0)$$

Large d Thermodynamics

Equilibrium

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

$$ds^2 = -dt^2 + d\left(\frac{\vec{\chi}}{\sqrt{d}}\right)^2$$

$$u^\mu = \left(1, \vec{\beta}\right) + \mathcal{O}(d^{-1})$$

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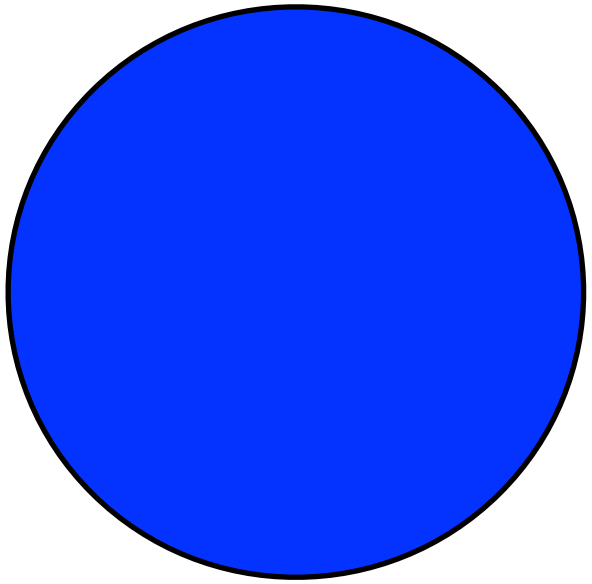
Large d Hydrodynamics



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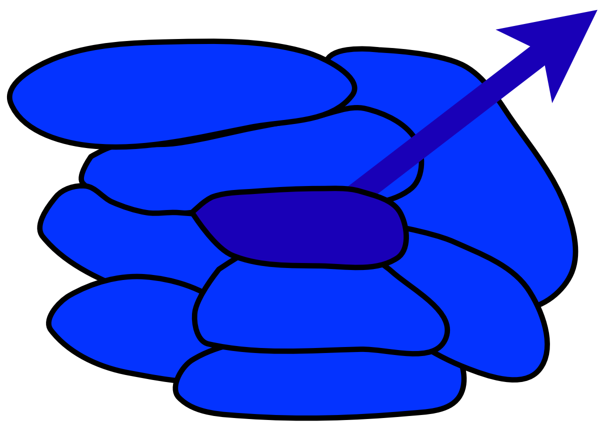
Large d Hydrodynamics

Equilibrium



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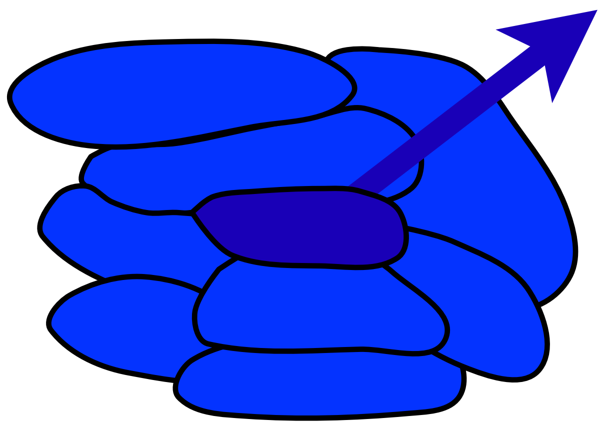
Large d Hydrodynamics

Constitutive relations



$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

Use:

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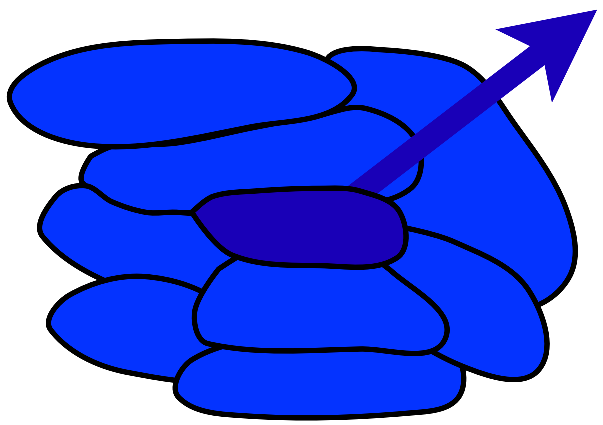
Large d Hydrodynamics

Constitutive relations



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Use:

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$$u^\mu = \left(1, \vec{\beta}\right) + \mathcal{O}(d^{-1})$$

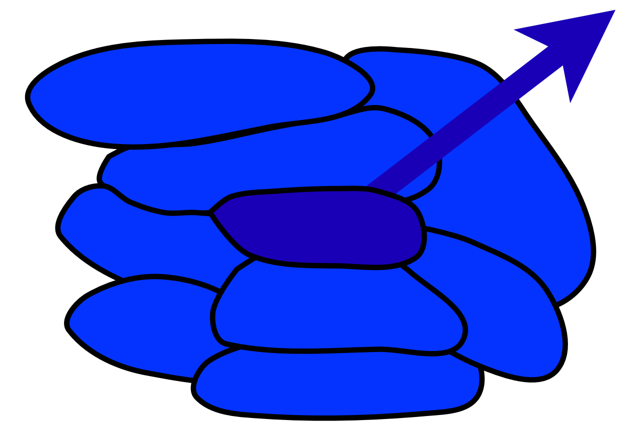
$$\epsilon = \mathcal{O}(d^0)$$

Dynamics:

$$\partial_\mu T^{\mu\nu} = 0$$

Large d Hydrodynamics

Constitutive relations



$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

Use:

$$ds^2 = -dt^2 + d\left(\frac{\vec{\chi}}{\sqrt{d}}\right)^2$$

$$u^\mu = \left(1, \vec{\beta}\right) + \mathcal{O}(d^{-1})$$

$$\epsilon = \mathcal{O}(d^0)$$

d equations for
d unknowns: $u^\mu(t,\vec{\chi})$
and $\epsilon(t,\vec{\chi})$

Dynamics:

$$\partial_\mu T^{\mu\nu} = 0$$

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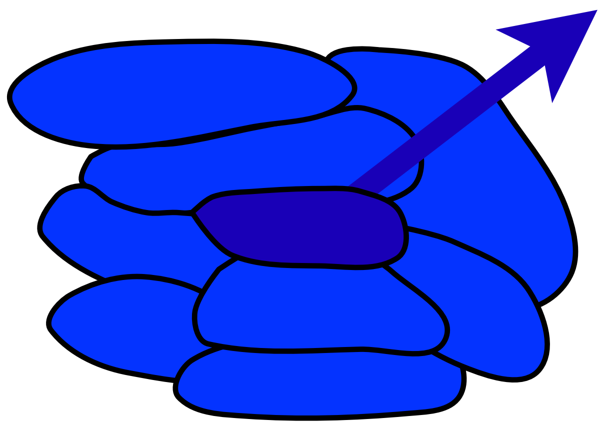

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

Use:

$$ds^2 = -dt^2 + \sum_{a=1}^{p} \frac{d\zeta^a d\zeta^a}{d} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{d}}\right)^2$$

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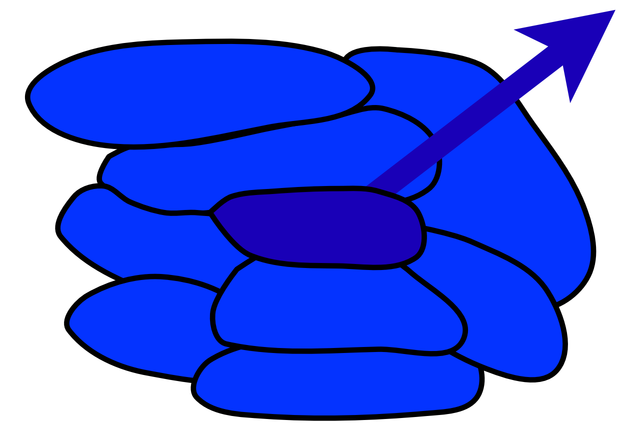

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

Use:

n

$$ds^2 = -dt^2 + \sum_{a=1}^{p} \frac{d\zeta^a d\zeta^a}{d} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{d}}\right)^2$$

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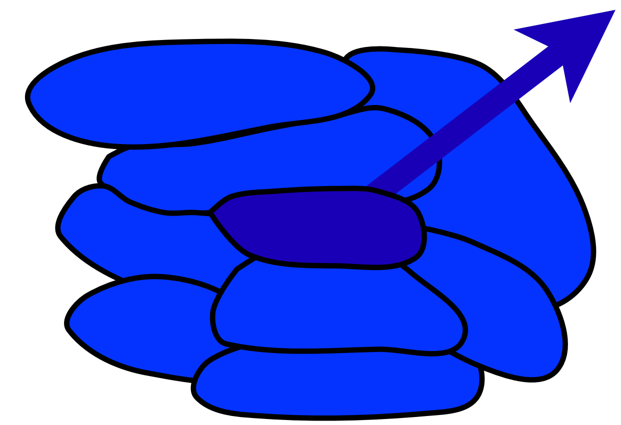

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

Use: 1 p n

$$ds^2 = -dt^2 + \sum_{a=1}^{p} \frac{d\zeta^a d\zeta^a}{d} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{d}}\right)^2$$

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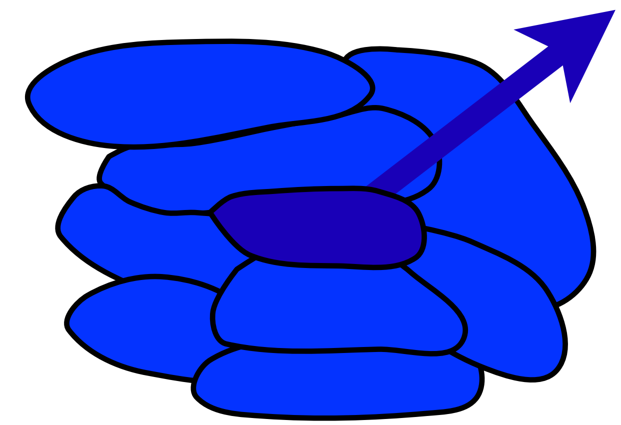

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

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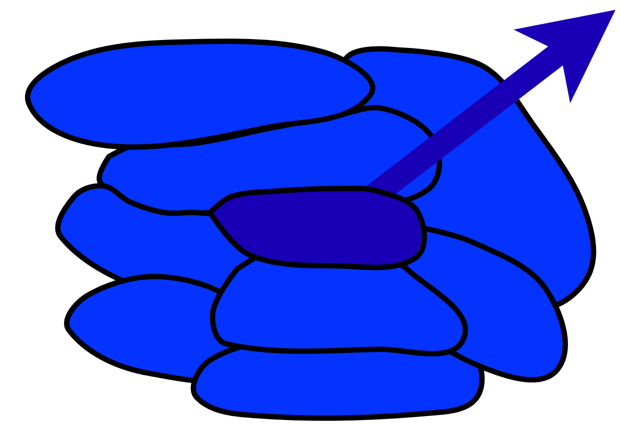

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

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$$ds^2 = -dt^2 + \sum_{a=1}^{p} \frac{d\zeta^a d\zeta^a}{n} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{n}}\right)^2$$

$$u^\mu = \left(1, \beta^b(t,\zeta^a), \vec{0}\right) + \mathcal{O}(n^{-1})$$

$$\epsilon = \epsilon(t,\zeta^a) + \mathcal{O}(n^{-1})$$

d equations for
d unknowns: $u^\mu(t,\vec{\chi})$
and $\epsilon(t,\vec{\chi})$

Dynamics:

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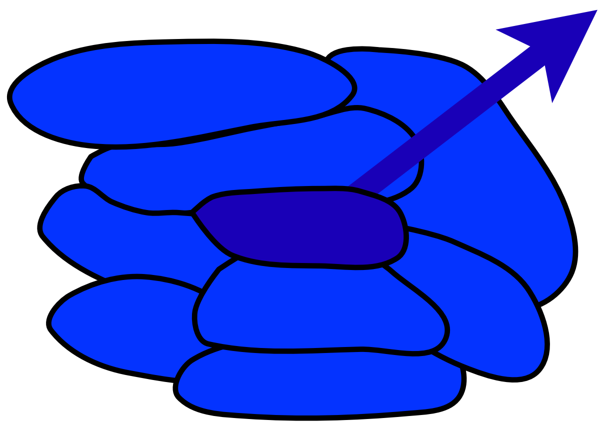
Large d Hydrodynamics

Constitutive relations



$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) + \mathcal{O}(\partial)$$

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1 p n

$$ds^2 = -dt^2 + \sum_{a=1}^{p} \frac{d\zeta^a d\zeta^a}{n} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{n}}\right)^2$$

$$u^\mu = \left(1, \beta^b(t,\zeta^a), \vec{0}\right) + \mathcal{O}(n^{-1})$$

$$\epsilon = \epsilon(t,\zeta^a) + \mathcal{O}(n^{-1})$$

p+1 equations for
p+1 unknowns: $\beta^b(t,\zeta^a)$
and $\epsilon(t,\zeta^a)$

Dynamics:

$$\partial_\mu T^{\mu\nu} = 0$$

Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right)$$

Use:

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$$\epsilon = \epsilon(t,\zeta^a) + \mathcal{O}(n^{-1})$$

Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(d u^\mu u^\nu + \eta^{\mu\nu}\right) - 2\eta\sigma^{\mu\nu}$$

$$+ \Big[\lambda_1 u^\lambda \mathcal{D}_\lambda \sigma^{\mu\nu}$$

$$+\lambda_2 \left(\sigma^{\lambda\mu}\sigma_\lambda{}^\nu - \frac{\sigma^{\alpha\beta}\sigma_{\alpha\beta}}{d-1}P^{\mu\nu}\right)$$

$$+\lambda_3 \left(\omega^{\mu\lambda}\sigma_\lambda{}^\nu + \omega^{\nu\lambda}\sigma_\lambda{}^\mu\right)$$

$$+\lambda_4 \left(\frac{1}{2}\left(\omega^{\mu\lambda}\omega^\nu{}_\lambda + \omega^{\nu\lambda}\omega^\mu{}_\lambda\right) - \frac{\omega^{\alpha\beta}\omega_{\alpha\beta}}{d-1}P^{\mu\nu}\right)\Big]$$

Use:

$$ds^2 = -dt^2 + \sum_{a=1}^{p}\frac{d\zeta^a d\zeta^a}{n} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{n}}\right)^2 \qquad u^\mu = \left(1, \beta^b(t,\zeta^a), \vec{0}\right) + \mathcal{O}(n^{-1})$$

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Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \frac{\epsilon}{d-1}\left(du^\mu u^\nu + \eta^{\mu\nu}\right) - 2\eta\sigma^{\mu\nu}$$

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$$ds^2 = -dt^2 + \sum_{a=1}^{p}\frac{d\zeta^a d\zeta^a}{n} + d\left(\frac{\vec{\chi}_\perp}{\sqrt{n}}\right)^2$$

$$\epsilon = \epsilon(t,\zeta^a) + \mathcal{O}(n^{-1})$$

$$u^\mu = \left(1, \beta^b(t,\zeta^a), \vec{0}\right) + \mathcal{O}(n^{-1})$$

$$\eta = \frac{h_0}{n}\epsilon + \mathcal{O}(n^0) \qquad \lambda_i = \frac{2\ell_i}{n}\epsilon + \mathcal{O}(n^0)$$

Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \epsilon \begin{pmatrix} 1 & \beta^b \\ \beta^a & \beta^a\beta^b + \delta^{ab} + T^{ab}_{(1)} + T^{ab}_{(2)} + \mathcal{O}\left(\partial^3\right) \end{pmatrix} + \mathcal{O}\left(n^{-1}\right)$$

$$T^{ab}_{(1)} = -h_0 \left(\partial^a\beta^b + \partial^b\beta^a\right)$$

$$\begin{aligned} T^{ab}_{(2)} =& \ell_1 \left(\partial_t + \beta^c\partial_c\right)\left(\partial^a\beta^b + \partial^b\beta^a\right) \\ &+ \frac{\ell_2}{2}\left(\partial^a\beta^c + \partial^c\beta^a\right)\left(\partial^b\beta_c + \partial_c\beta^b\right) \\ &+ \ell_3 \left(\left(\partial^a\beta^c - \partial^c\beta^a\right)\left(\partial^b\beta_c + \partial_c\beta^b\right) + (a \leftrightarrow b)\right) \\ &+ \frac{\ell_4}{2}\left(\partial^a\beta^c - \partial^c\beta^a\right)\left(\partial^b\beta_c - \partial_c\beta^b\right) \end{aligned}$$

Large d Hydrodynamics

Constitutive relations

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$$T^{ab}_{(1)} = -h_0\left(\partial^a\beta^b + \partial^b\beta^a\right)$$

$$\eta = \frac{h_0}{n}\epsilon$$

$$T^{ab}_{(2)} = \ell_1\left(\partial_t + \beta^c\partial_c\right)\left(\partial^a\beta^b + \partial^b\beta^a\right) + \frac{\ell_2}{2}\left(\partial^a\beta^c + \partial^c\beta^a\right)\left(\partial^b\beta_c + \partial_c\beta^b\right) + \ell_3\left(\left(\partial^a\beta^c - \partial^c\beta^a\right)\left(\partial^b\beta_c + \partial_c\beta^b\right) + (a \leftrightarrow b)\right) + \frac{\ell_4}{2}\left(\partial^a\beta^c - \partial^c\beta^a\right)\left(\partial^b\beta_c - \partial_c\beta^b\right)$$

$$\lambda_i = \frac{2\ell_i}{n}\epsilon$$

Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \epsilon \begin{pmatrix} 1 & \beta^b \\ \beta^a & \beta^a \beta^b + \delta^{ab} + T^{ab}_{(1)} + T^{ab}_{(2)} + \mathcal{O}\left(\partial^3\right) \end{pmatrix} + \mathcal{O}\left(n^{-1}\right)$$

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Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \epsilon \begin{pmatrix} 1 & \beta^b \\ \beta^a & \beta^a \beta^b + \delta^{ab} + T^{ab}_{(1)} + T^{ab}_{(2)} + \mathcal{O}\left(\partial^3\right) \end{pmatrix} + \mathcal{O}\left(n^{-1}\right)$$

Use a field redefinition to simplify the EOM:

$$\epsilon \rightarrow \epsilon + k_1 \epsilon \partial_b \beta^b$$

$$\beta^a \rightarrow \beta^a + c_1 \frac{\partial^a \epsilon}{\epsilon}$$

Large d Hydrodynamics

Constitutive relations

$$T^{\mu\nu} = \epsilon \begin{pmatrix} 1 & \beta^b \\ \beta^a & \beta^a\beta^b + \delta^{ab} + T^{ab}_{(1)} + T^{ab}_{(2)} + \mathcal{O}\left(\partial^3\right) \end{pmatrix} + \mathcal{O}\left(n^{-1}\right)$$

Use a field redefinition to simplify the EOM:

$$\epsilon \to \ \epsilon + k_1\epsilon\partial_b\beta^b + k_2\partial_b\partial^b\epsilon + k_3\epsilon\left(\partial_b\beta^b\right)^2 + k_4\epsilon\partial_b\beta^c\partial_c\beta^b + k_5\frac{\partial_b\epsilon\partial^b\epsilon}{\epsilon} + k_6\epsilon\partial_b\beta_c\partial^b\beta^c$$

$$\beta^a \to \ \beta^a + c_1\frac{\partial^a\epsilon}{\epsilon} + c_2\partial_b\beta^b\frac{\partial^a\epsilon}{\epsilon} + c_3\partial^a\beta^b\frac{\partial_b\epsilon}{\epsilon} + c_4\partial^a\partial_b\beta^b + c_5\frac{\partial_b\epsilon}{\epsilon}\partial^b\beta^a + c_6\partial_b\partial^b\beta^a$$

Large d Hydrodynamics

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$$\beta^a \rightarrow \beta^a + c_1 \frac{\partial^a \epsilon}{\epsilon} + c_2 \partial_b \beta^b \frac{\partial^a \epsilon}{\epsilon} + c_3 \partial^a \beta^b \frac{\partial_b \epsilon}{\epsilon} + c_4 \partial^a \partial_b \beta^b + c_5 \frac{\partial_b \epsilon}{\epsilon} \partial^b \beta^a + c_6 \partial_b \partial^b \beta^a$$

Compute $\partial_\mu T^{\mu\nu} = 0$ and tune the c_i's and k_i's so that the equations of motion are 2nd order in derivatives.

Large d Hydrodynamics

Use a field redefinition to simplify the EOM:

$$\epsilon \to \ \epsilon + k_1 \epsilon \partial_b \beta^b + k_2 \partial_b \partial^b \epsilon + k_3 \epsilon \left(\partial_b \beta^b\right)^2 + k_4 \epsilon \partial_b \beta^c \partial_c \beta^b + k_5 \frac{\partial_b \epsilon \partial^b \epsilon}{\epsilon} + k_6 \epsilon \partial_b \beta_c \partial^b \beta^c$$

$$\beta^a \to \ \beta^a + c_1 \frac{\partial^a \epsilon}{\epsilon} + c_2 \partial_b \beta^b \frac{\partial^a \epsilon}{\epsilon} + c_3 \partial^a \beta^b \frac{\partial_b \epsilon}{\epsilon} + c_4 \partial^a \partial_b \beta^b + c_5 \frac{\partial_b \epsilon}{\epsilon} \partial^b \beta^a + c_6 \partial_b \partial^b \beta^a$$

If we set all c_i's and k_i's to zero, except

$$k_2 = \ell_1 + \ell_4 \qquad -2c_1 h_0 = c_1^2 + 2\ell_1 \qquad c_5 = 2(\ell_1 + \ell_4) \qquad c_6 = \ell_1 + \ell_4$$

Large d Hydrodynamics

Use a field redefinition to simplify the EOM:

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and if

$$\ell_2 + 2\ell_3 + \ell_4 = 0 \qquad \ell_1 - \ell_2 - \ell_3 = 0$$

Large d Hydrodynamics

Use a field redefinition to simplify the EOM:

$$\epsilon \to \epsilon + k_1 \epsilon \partial_b \beta^b + k_2 \partial_b \partial^b \epsilon + k_3 \epsilon \left(\partial_b \beta^b\right)^2 + k_4 \epsilon \partial_b \beta^c \partial_c \beta^b + k_5 \frac{\partial_b \epsilon \partial^b \epsilon}{\epsilon} + k_6 \epsilon \partial_b \beta_c \partial^b \beta^c$$

$$\beta^a \to \beta^a + c_1 \frac{\partial^a \epsilon}{\epsilon} + c_2 \partial_b \beta^b \frac{\partial^a \epsilon}{\epsilon} + c_3 \partial^a \beta^b \frac{\partial_b \epsilon}{\epsilon} + c_4 \partial^a \partial_b \beta^b + c_5 \frac{\partial_b \epsilon}{\epsilon} \partial^b \beta^a + c_6 \partial_b \partial^b \beta^a$$

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and if

$$\ell_2 + 2\ell_3 + \ell_4 = 0 \qquad \ell_1 - \ell_2 - \ell_3 = 0$$

then the equations of motion become

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

$$\partial_t j^a - h_0 \, \partial_b \partial^b j^a = -\partial^a \epsilon - \partial_b \left(\frac{j^a j^b}{\epsilon}\right) - \left(\frac{c_1}{2} - \frac{\ell_1}{c_1}\right) \left(-\epsilon \partial^a \left(\frac{\partial_b j^b}{\epsilon}\right) + j^b \partial^a \left(\frac{\partial_b \epsilon}{\epsilon}\right) + \partial_b \left(\frac{j^a \partial^b \epsilon}{\epsilon}\right)\right)$$

Large d Hydrodynamics

Use a field redefinition to simplify the EOM:

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$$\beta^a \to \beta^a + c_1 \frac{\partial^a \epsilon}{\epsilon} + c_2 \partial_b \beta^b \frac{\partial^a \epsilon}{\epsilon} + c_3 \partial^a \beta^b \frac{\partial_b \epsilon}{\epsilon} + c_4 \partial^a \partial_b \beta^b + c_5 \frac{\partial_b \epsilon}{\epsilon} \partial^b \beta^a + c_6 \partial_b \partial^b \beta^a$$

If we set all c_i's and k_i's to zero, except

$$k_2 = \ell_1 + \ell_4 \qquad -2c_1 h_0 = c_1^2 + 2\ell_1 \qquad c_5 = 2(\ell_1 + \ell_4) \qquad c_6 = \ell_1 + \ell_4$$

and if

$$\ell_2 + 2\ell_3 + \ell_4 = 0 \qquad \ell_1 - \ell_2 - \ell_3 = 0$$

then the equations of motion become

$$j^a = \beta^a \epsilon$$

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

$$\partial_t j^a - h_0 \partial_b \partial^b j^a = -\partial^a \epsilon - \partial_b \left(\frac{j^a j^b}{\epsilon}\right) - \left(\frac{c_1}{2} - \frac{\ell_1}{c_1}\right)\left(-\epsilon \partial^a \left(\frac{\partial_b j^b}{\epsilon}\right) + j^b \partial^a \left(\frac{\partial_b \epsilon}{\epsilon}\right) + \partial_b \left(\frac{j^a \partial^b \epsilon}{\epsilon}\right)\right)$$

Large d Hydrodynamics

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$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

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Large d Hydrodynamics

The equations of motion:

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

$$\partial_t j^a - h_0\, \partial_b \partial^b j^a = -\partial^a \epsilon - \partial_b \left(\frac{j^a j^b}{\epsilon}\right) - \left(\frac{c_1}{2} - \frac{\ell_1}{c_1}\right) \left(-\epsilon \partial^a \left(\frac{\partial_b j^b}{\epsilon}\right) + j^b \partial^a \left(\frac{\partial_b \epsilon}{\epsilon}\right) + \partial_b \left(\frac{j^a \partial^b \epsilon}{\epsilon}\right)\right)$$

Large d Hydrodynamics

The equations of motion:

?

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a + \mathcal{O}(\partial^4)$$

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?

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Large d Hydrodynamics

The equations of motion:

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$$\lambda_2 + 2\lambda_3 + \lambda_4 = 0 \qquad \lambda_1 - \lambda_2 - \lambda_3 = 0$$

Large d AdS/CFT

Consider:

$$S = \int \sqrt{-g} \left(R + \frac{(d-1)(d-2)}{L^2} \right) d^d x$$

with

$$ds^2 = 2dt(-Adt + dr - F_a d\zeta^a) + G_{ab} d\zeta^a d\zeta^b + G_\perp d\vec{\chi}_\perp^2$$

Such that at large r the metric is given by

$$ds^2 \xrightarrow[r\to\infty]{} r^2 \left(-dt^2 + \frac{\delta_{ab}}{n} d\zeta^a d\zeta^b + \frac{d\vec{\chi}_\perp^2}{n} \right)$$

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$$A \xrightarrow[r\to\infty]{} r^2 + \mathcal{O}(r^{-n}) \qquad F_a \xrightarrow[r\to\infty]{} \mathcal{O}(r^{-n})$$

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Setting $n \to \infty$ and keeping $R = r^n$ fixed gives non trivial dynamics.

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Consider: (Emparan, Suzuki, Tanabe; Bhattacharyya et. al.)

$$ds^2 = 2dt\left(-Adt + \frac{dR}{nR} - F_a d\zeta^a\right) + G_{ab}d\zeta^a d\zeta^b + G_\perp d\vec{\chi}^2$$

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These match as long as:

$$h_0 = 1 \qquad c_1^2 = 2\ell_1$$

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$$\eta = \frac{h_0}{n}\epsilon \qquad \lambda_i = \frac{2\ell_i}{n}\epsilon \qquad -2c_1 h_0 = c_1^2 + 2\ell_1$$

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And indeed:

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$$p = a/E \qquad \vec{u} = \frac{\vec{f}}{a\,U} \qquad \vec{\nabla} \to L\vec{\nabla}$$

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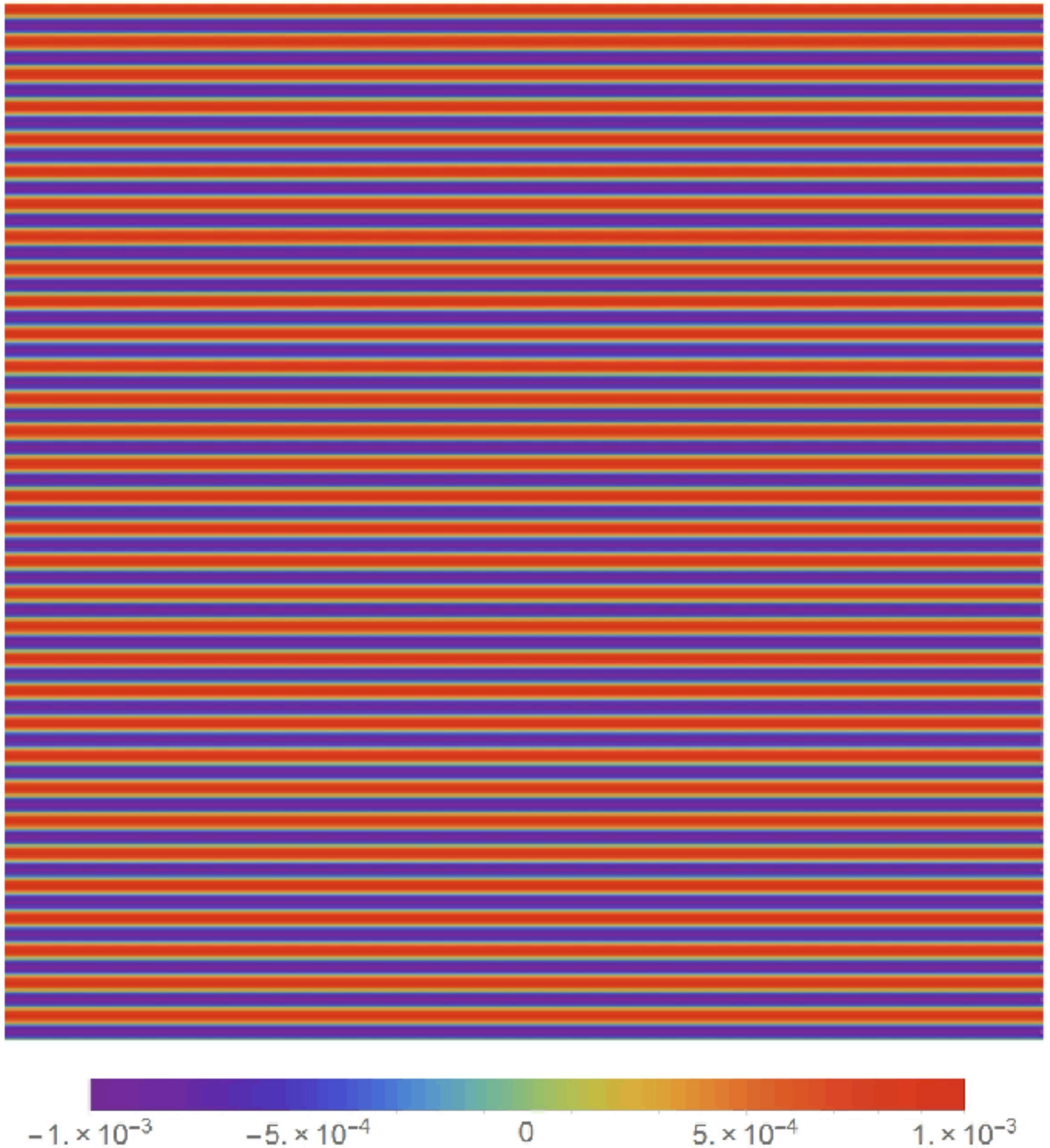
$$\omega = \partial_x u_y - \partial_y u_x$$



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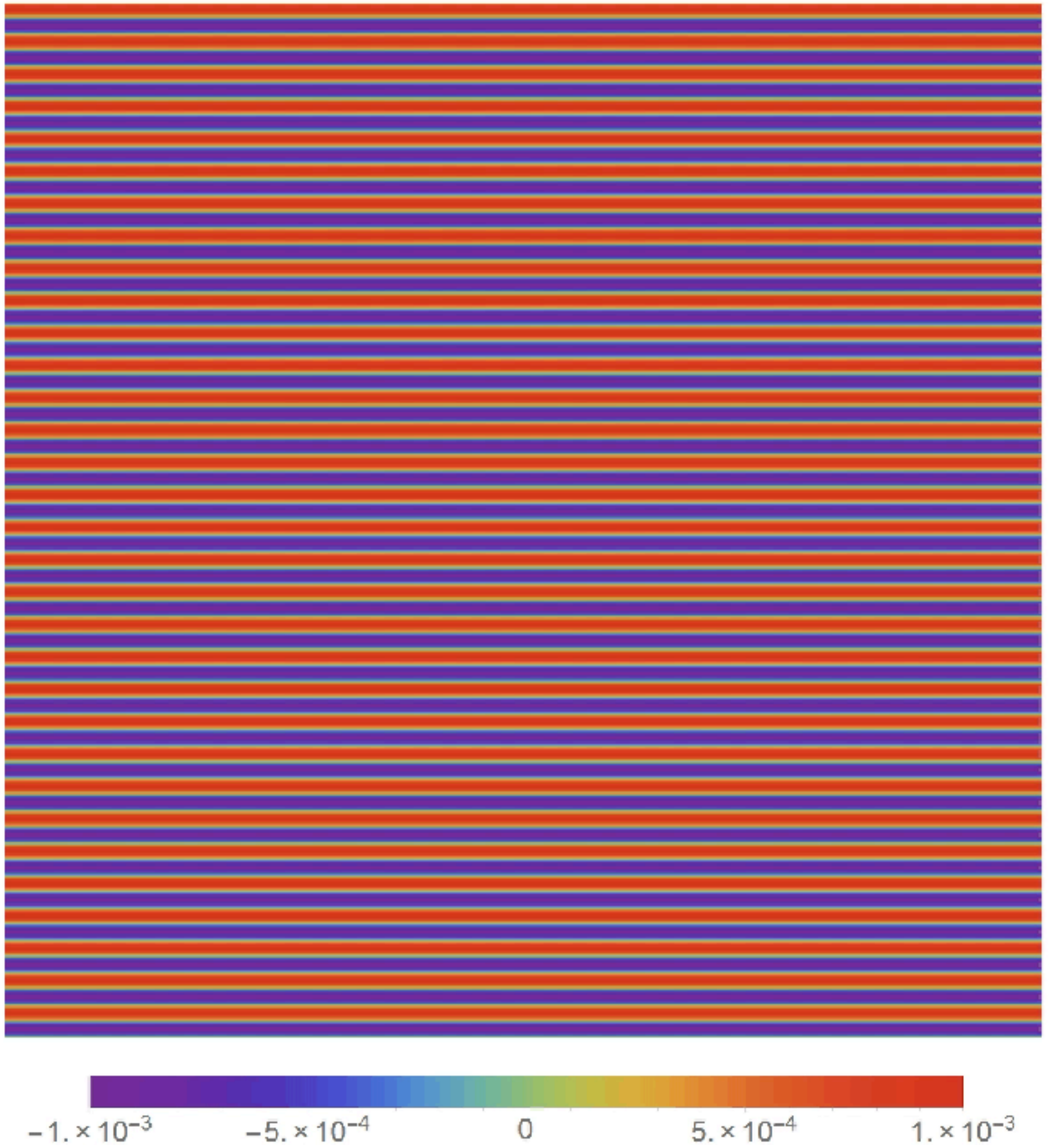

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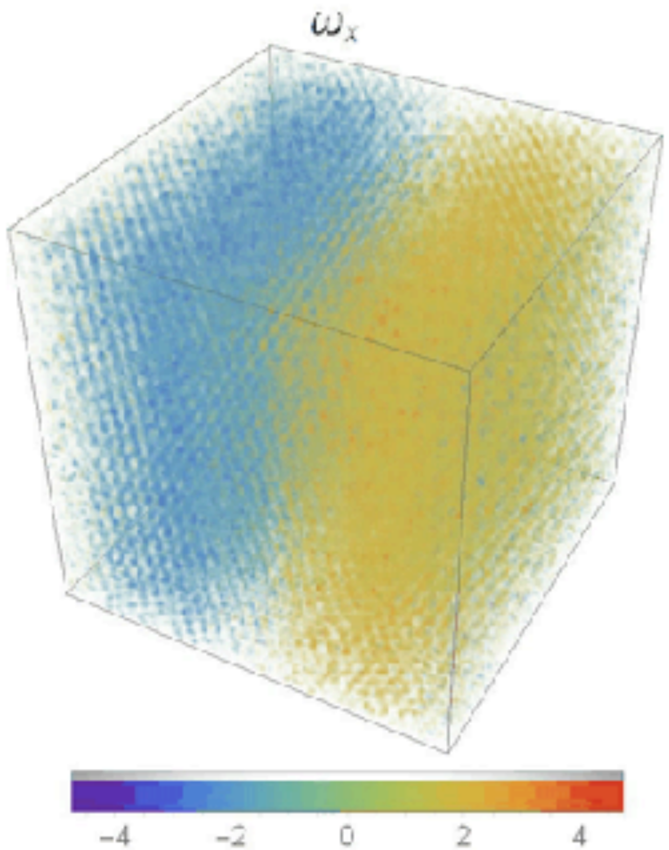

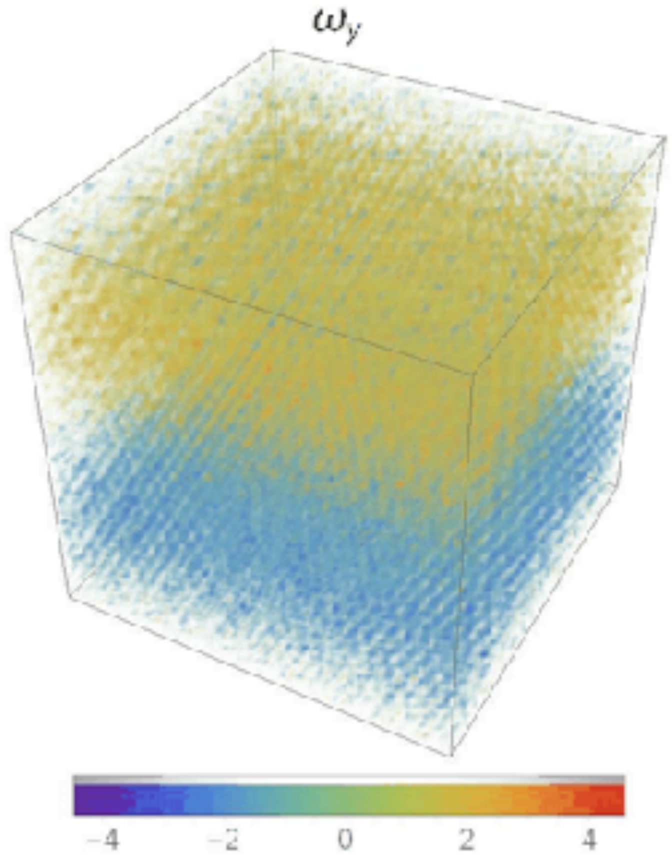

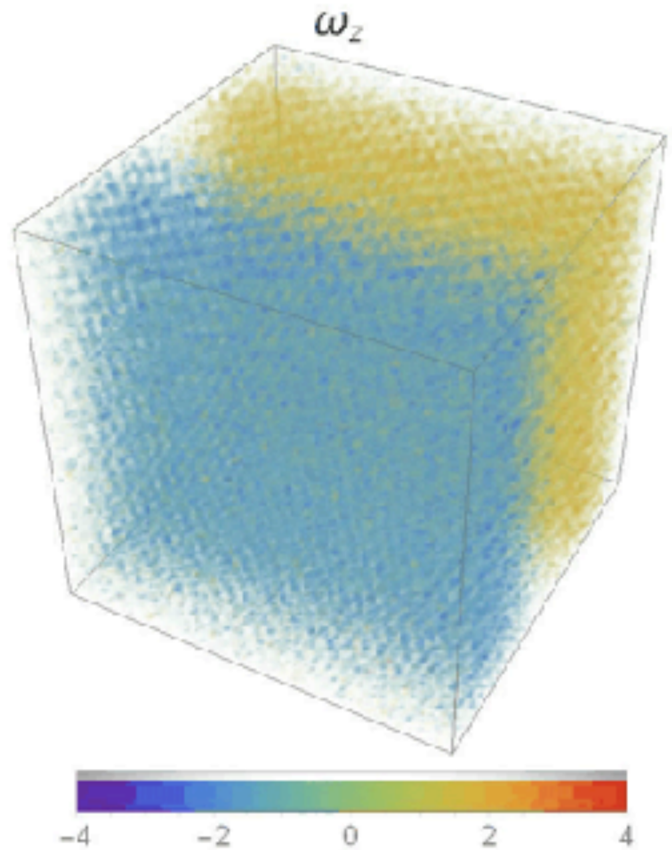

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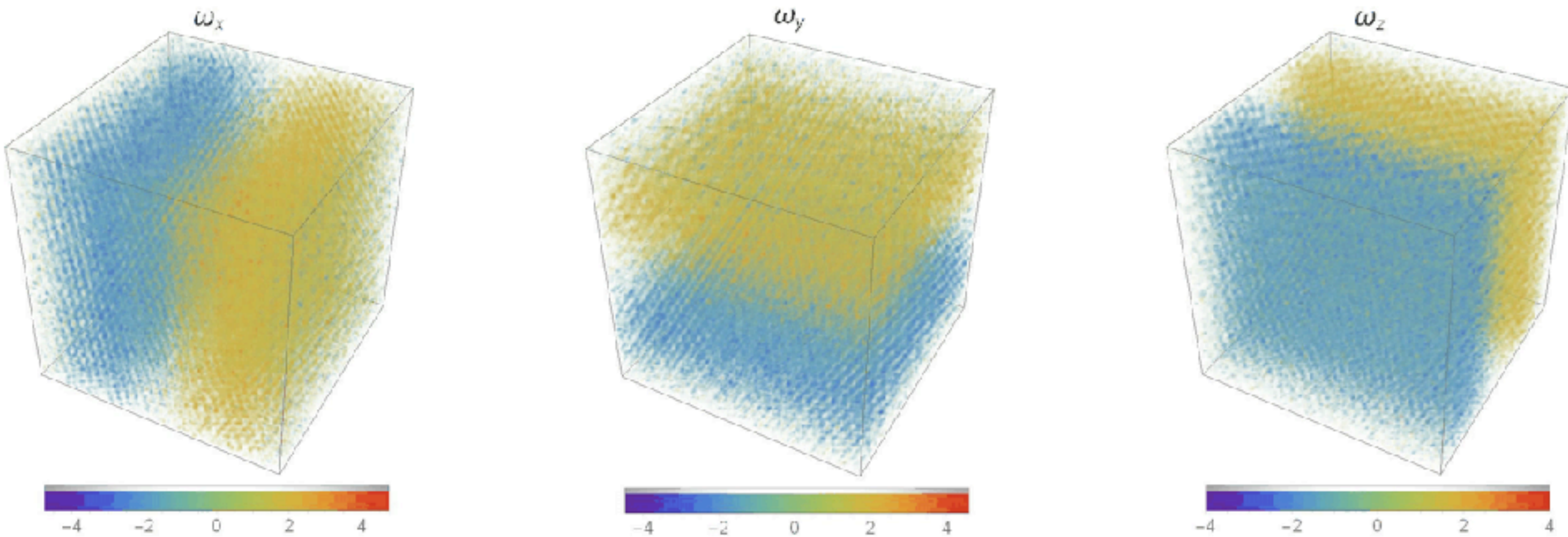

Quantifying turbulence

Starting from:

$$p\vec{\nabla} \cdot \vec{u} = 0$$

$$\partial_t \vec{u} + \left(\vec{u} \cdot \vec{\nabla}\right)\vec{u} + \frac{\vec{\nabla} p}{M^2 p} = \frac{1}{Re}\nabla^2 \vec{u}$$

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One defines:

$$\hat{u}(t, \vec{k}) = \int \vec{u}(t, \vec{\zeta}) e^{-i\vec{k}\cdot\vec{\zeta}} d^p \zeta$$

$$E(t, k) = \frac{\partial}{\partial k} \int_{|\vec{k}'| \leq k} |\hat{u}(t, \vec{k}')|^2 \frac{d^p k}{(2\pi)^k}$$

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Turbulent behavior is characterized by

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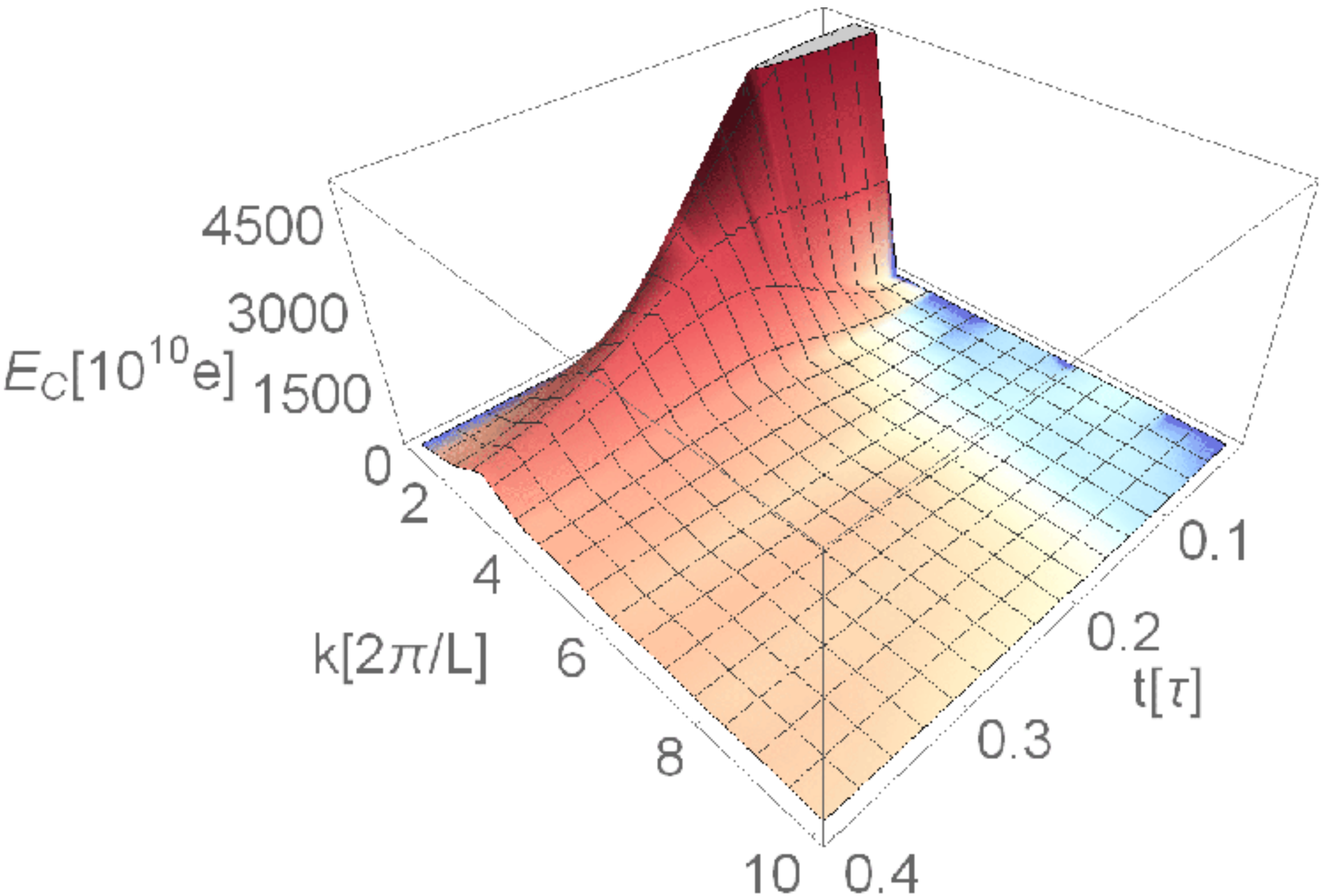

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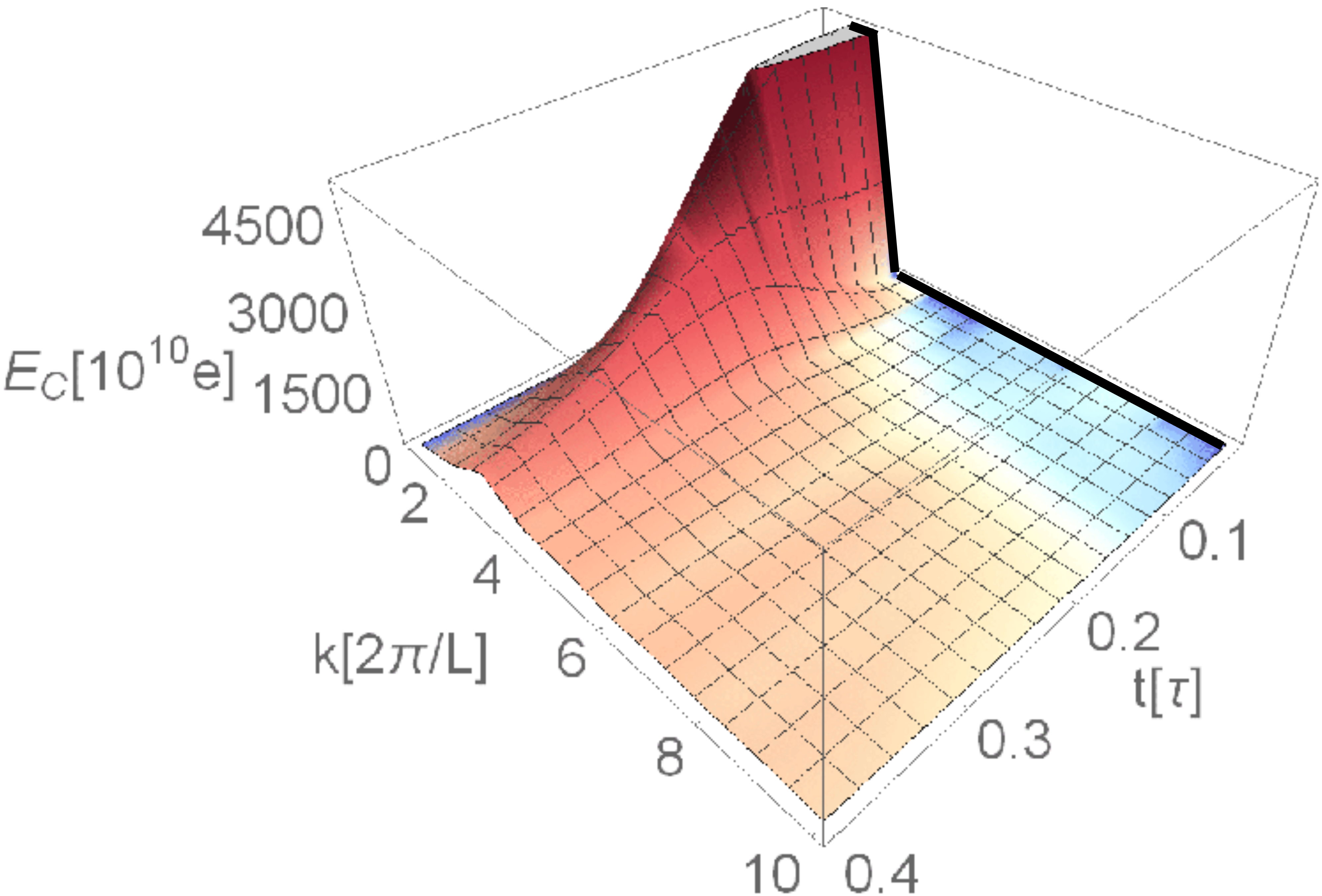

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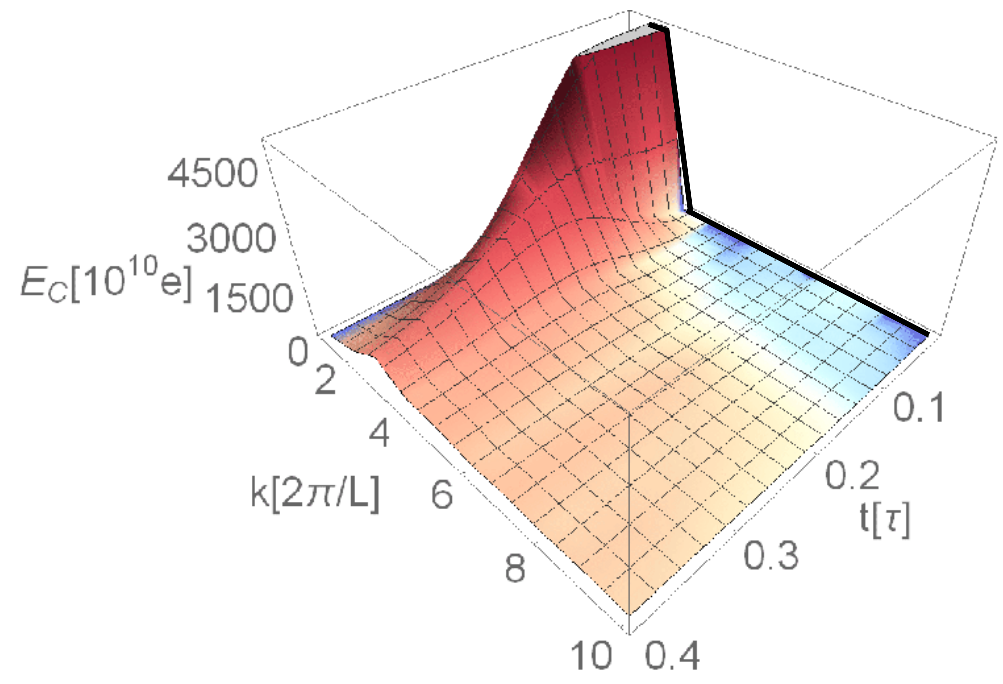
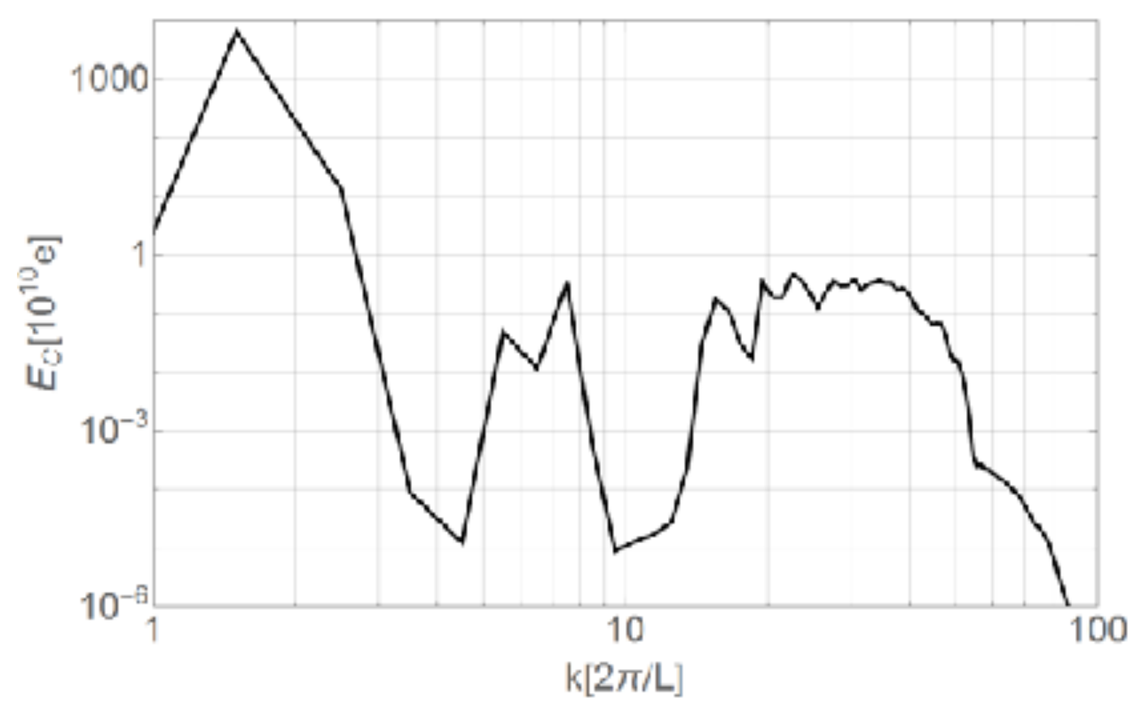
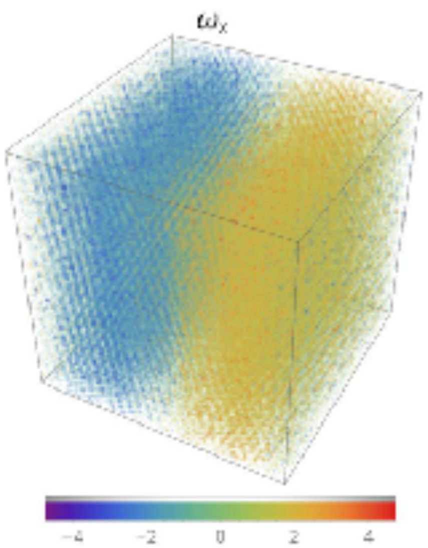
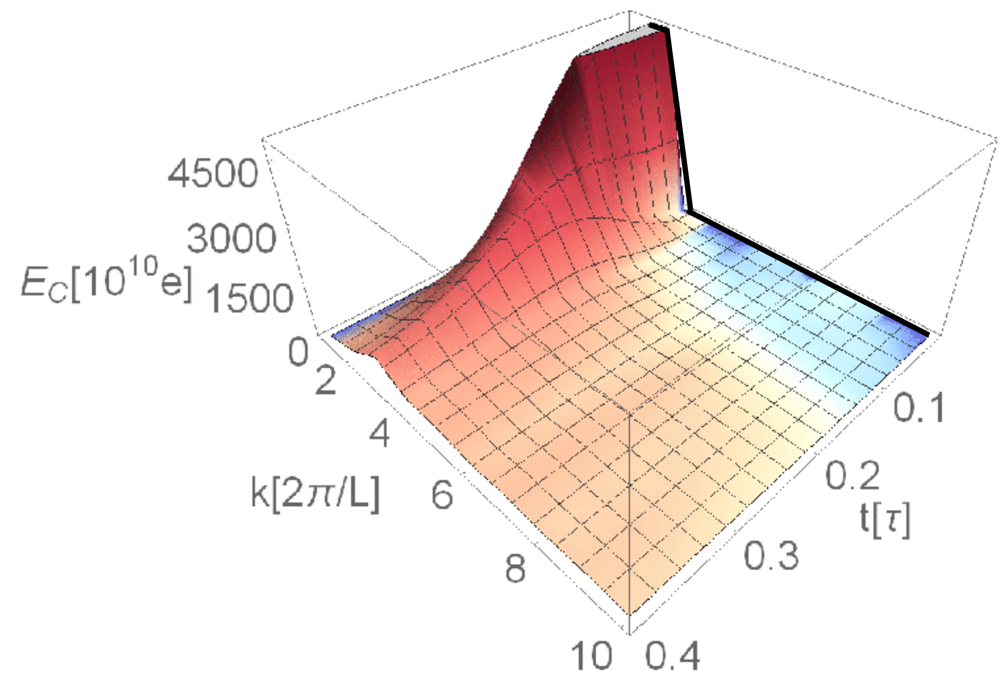
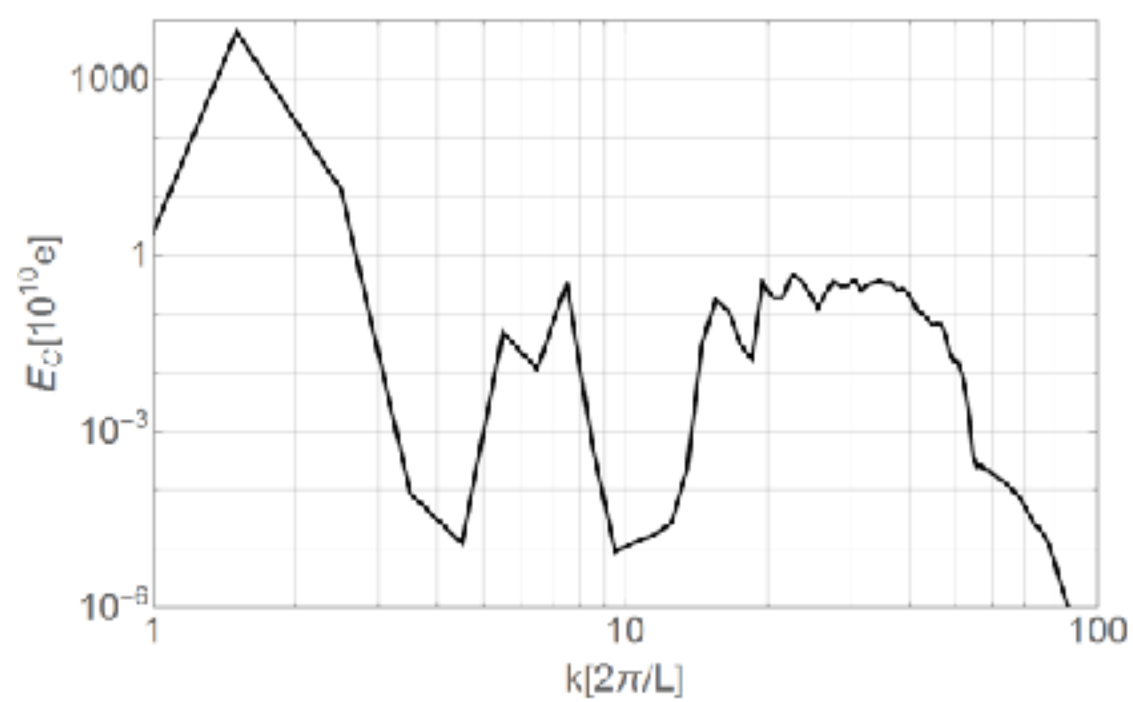




Quantifying turbulence







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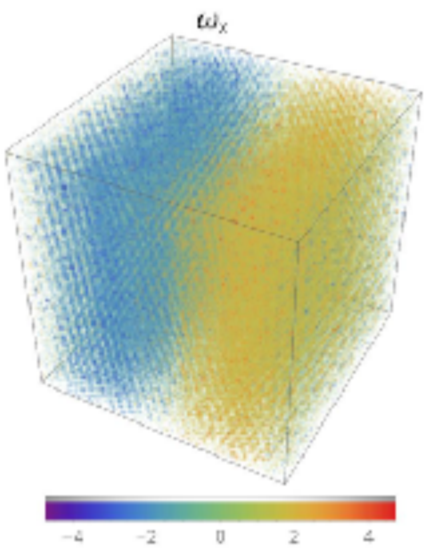

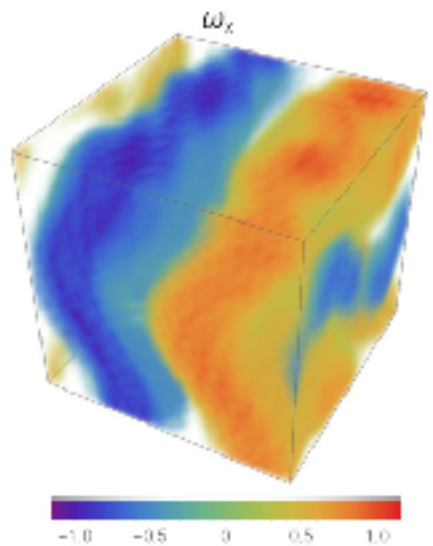

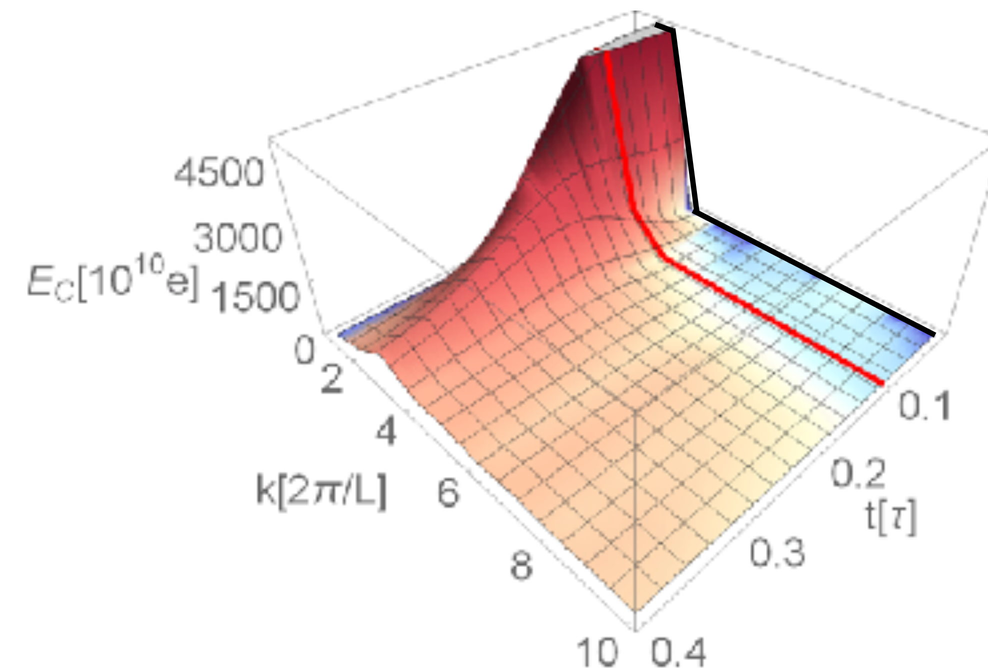

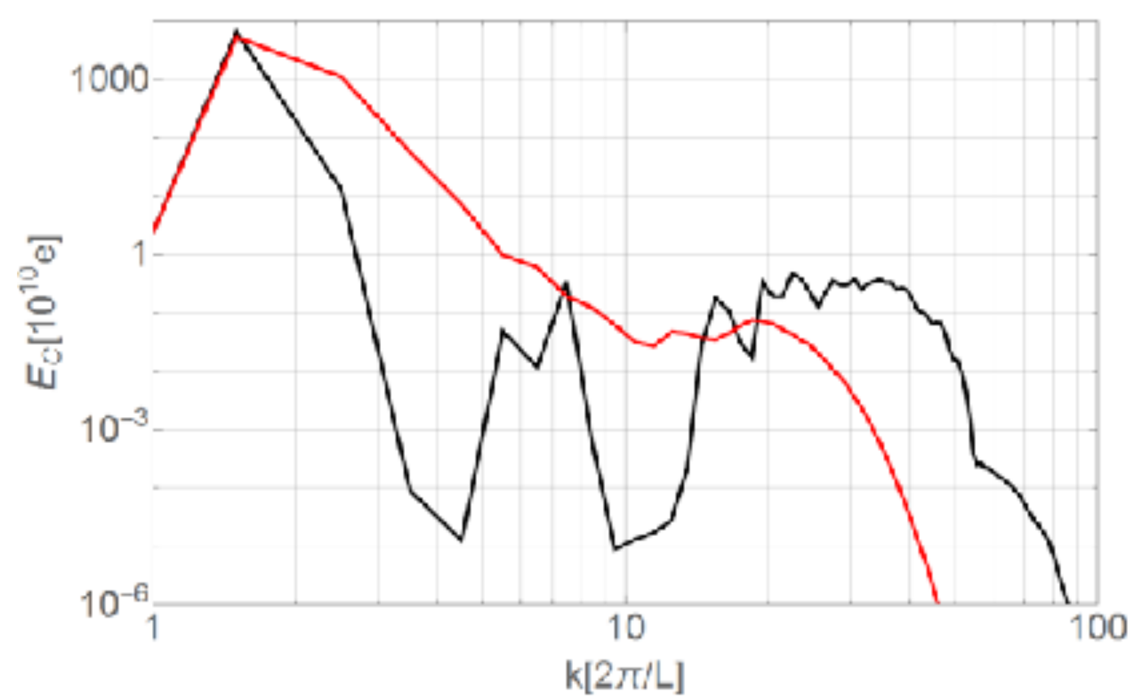

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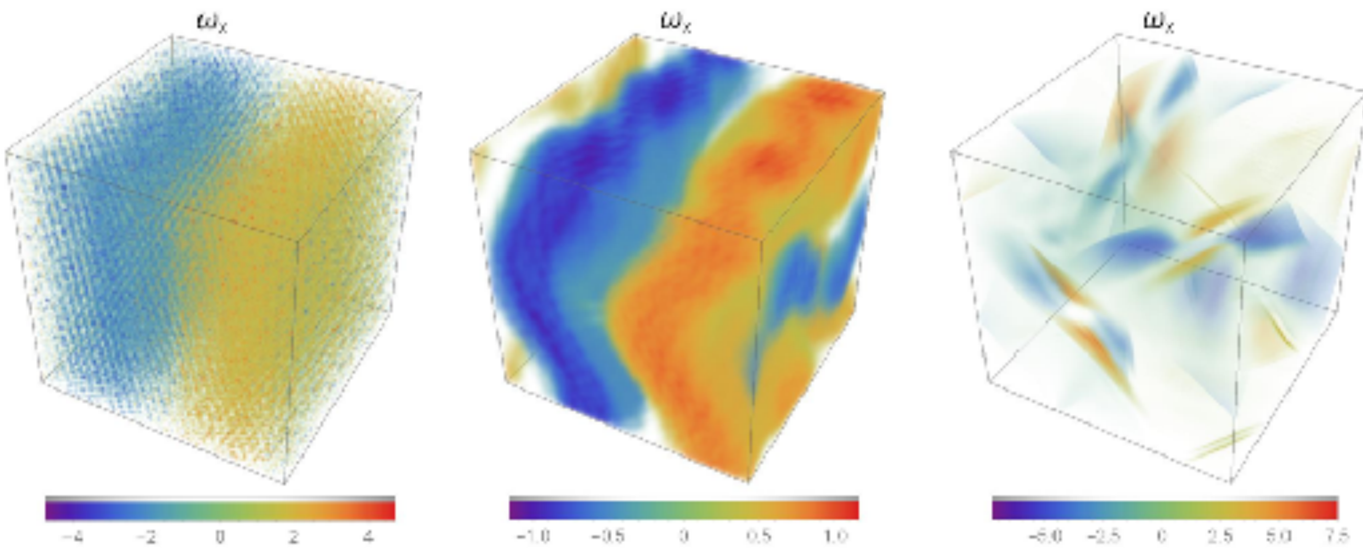

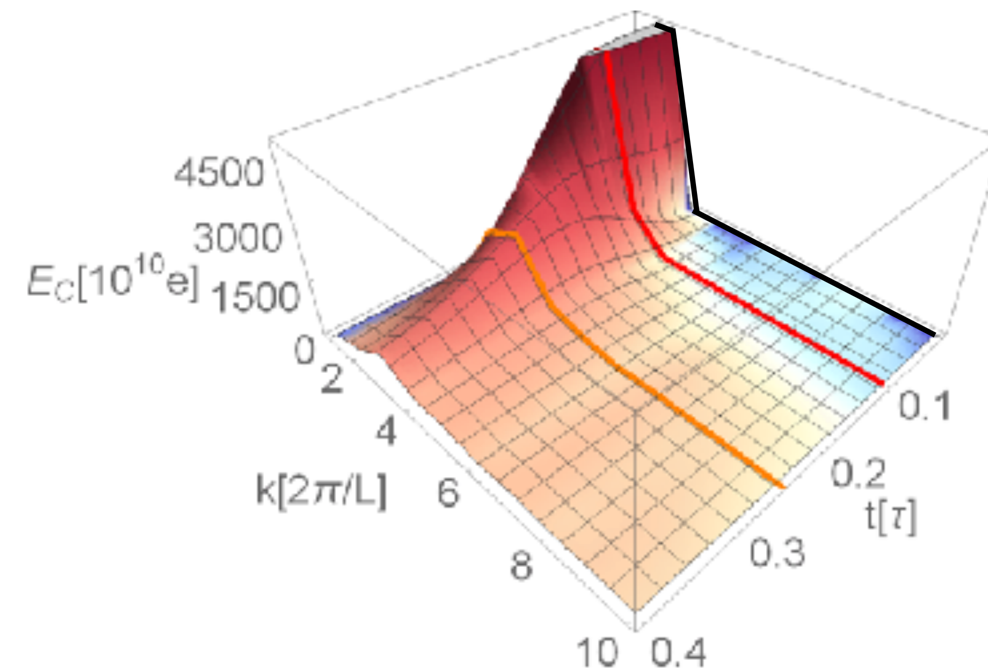

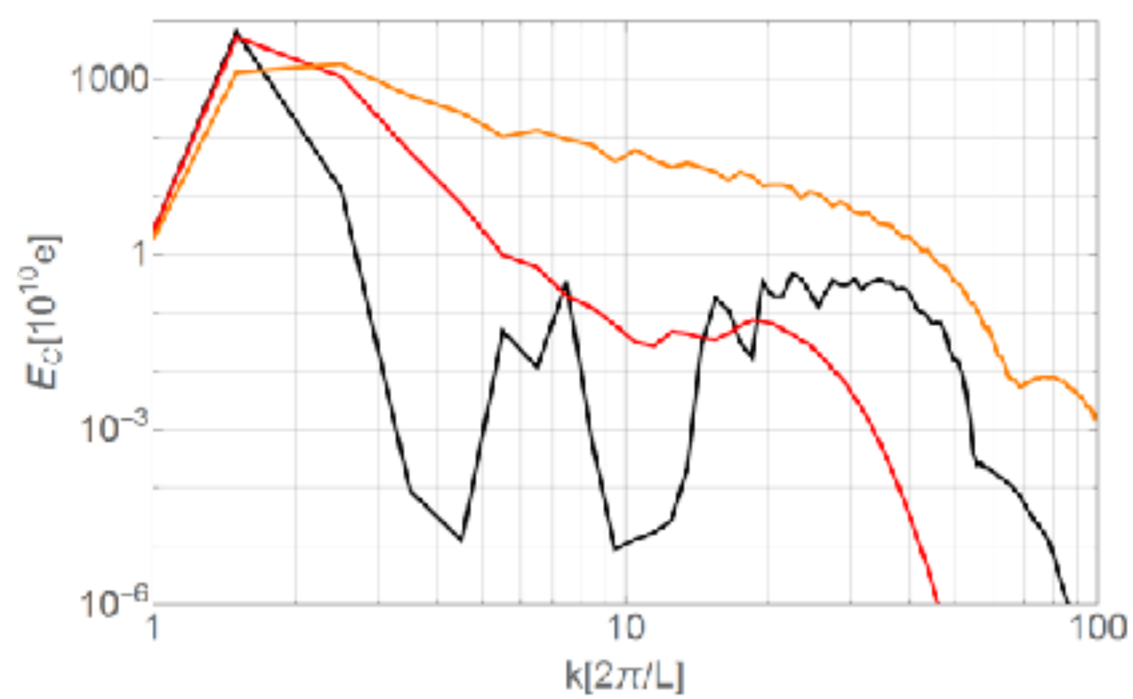

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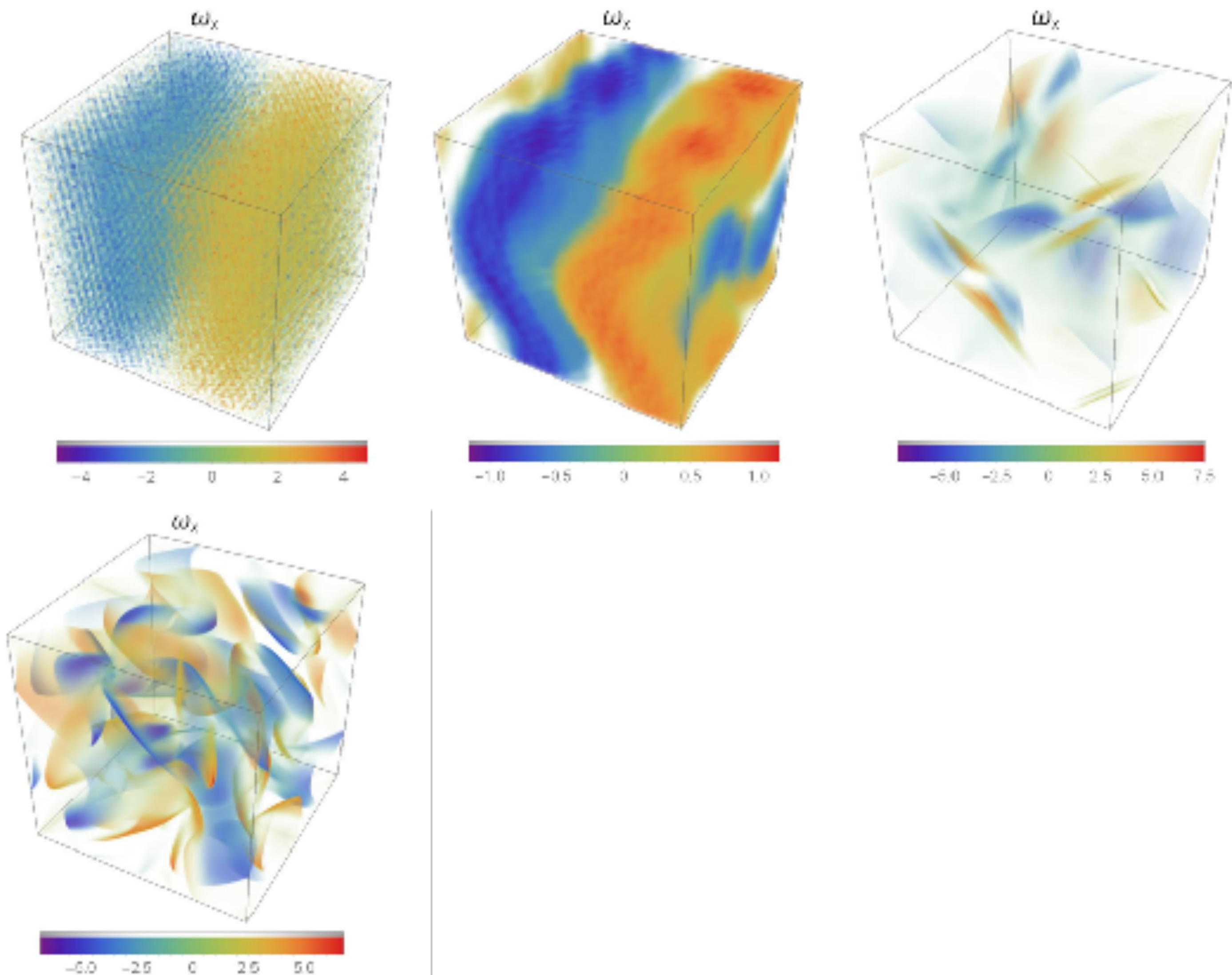

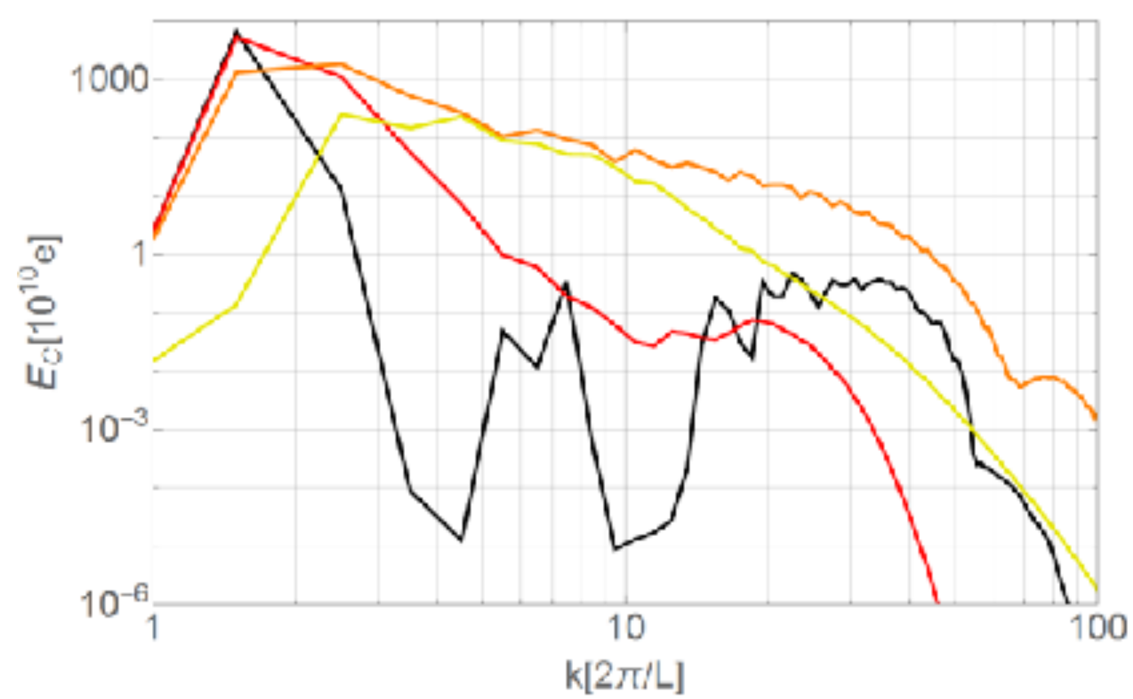

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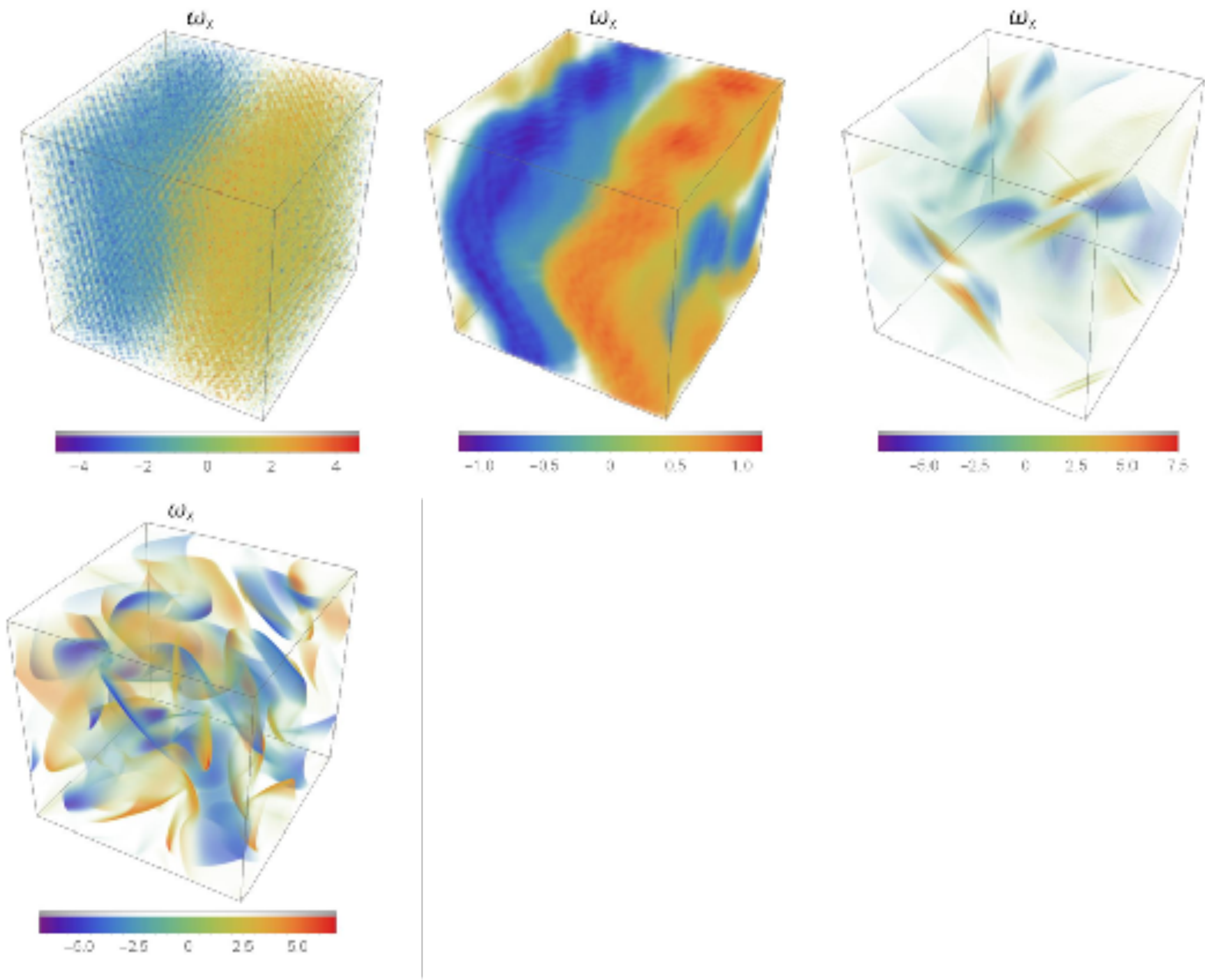

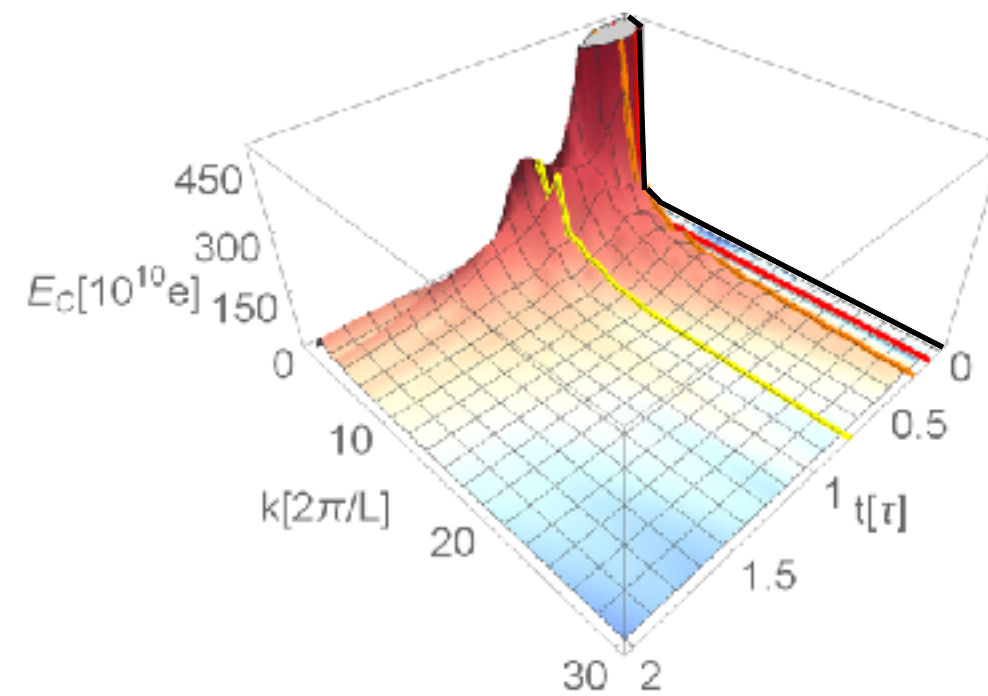

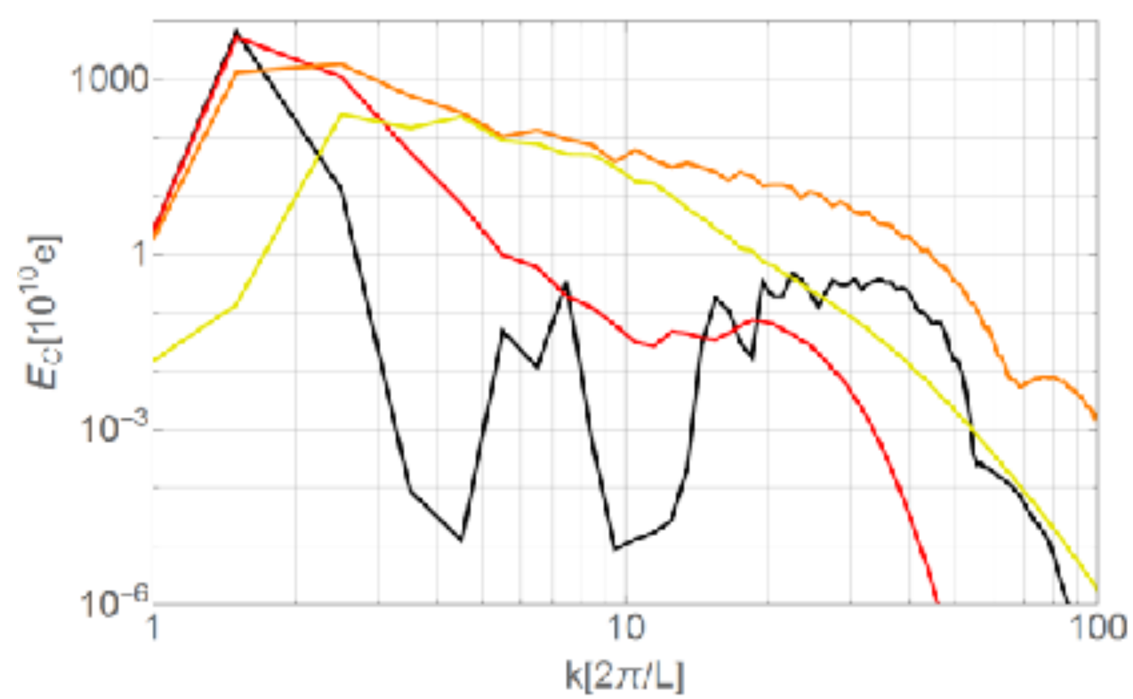

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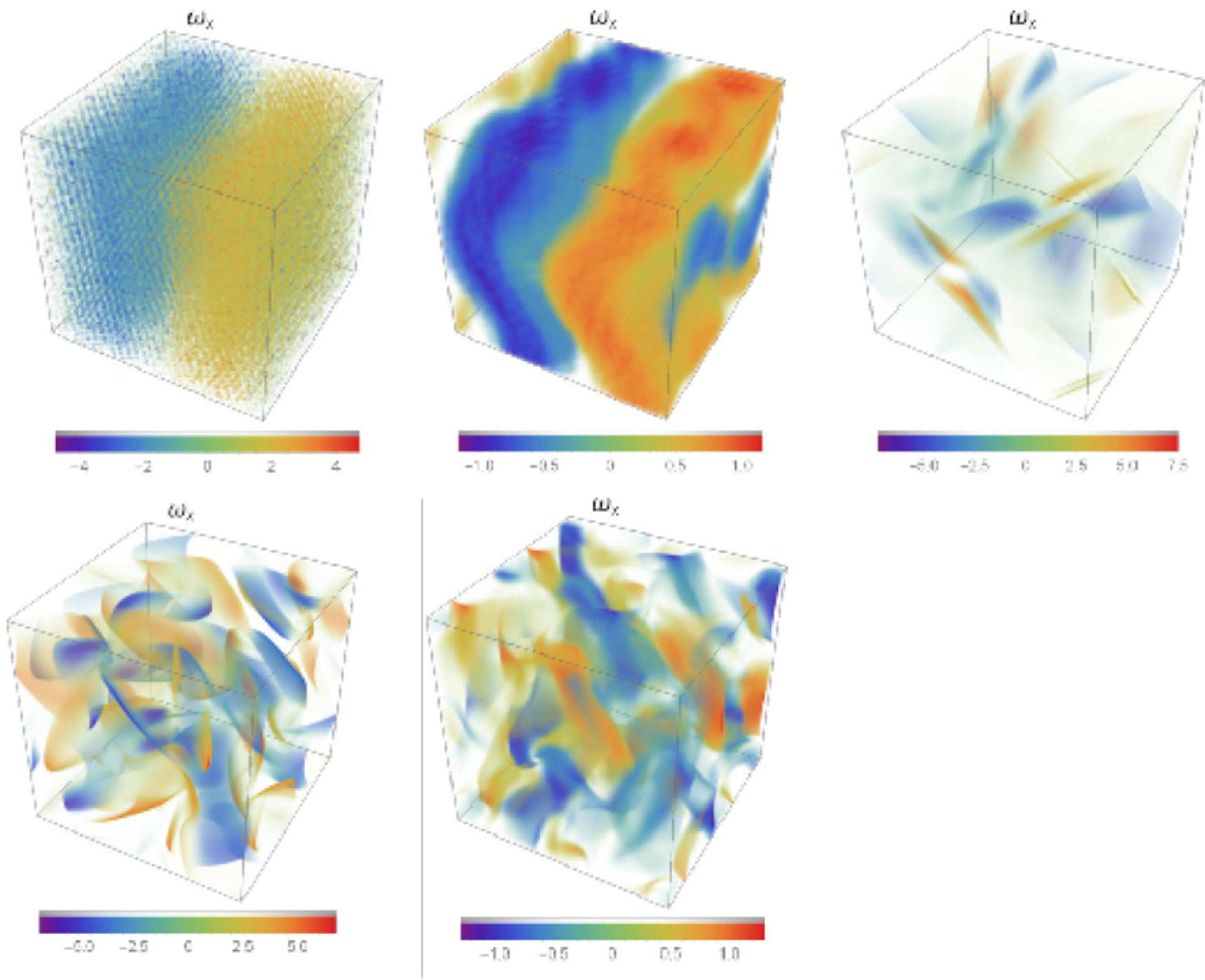

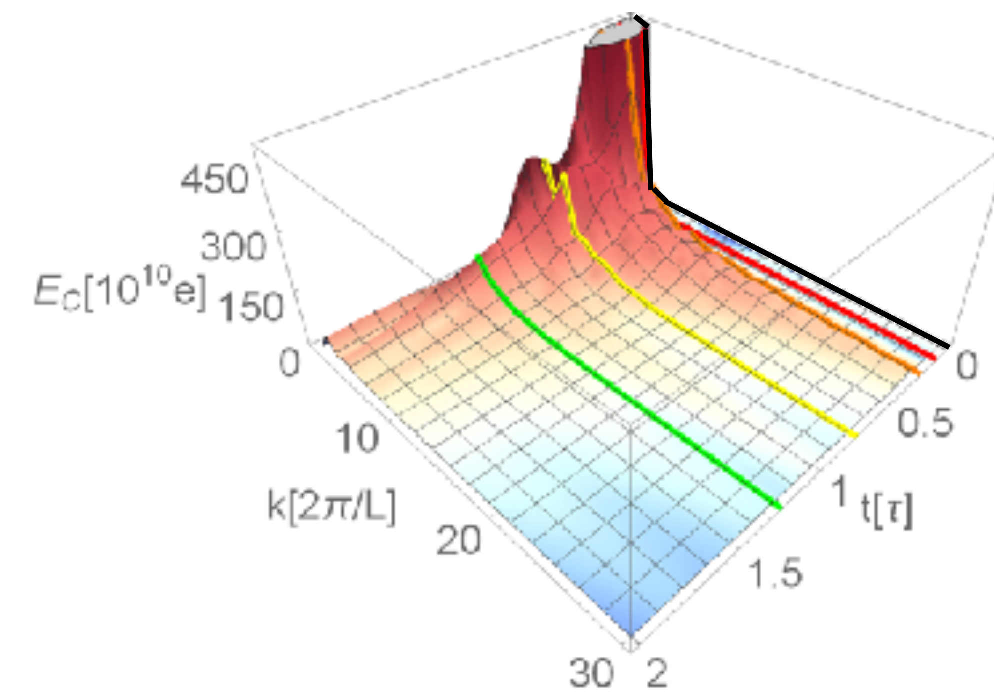

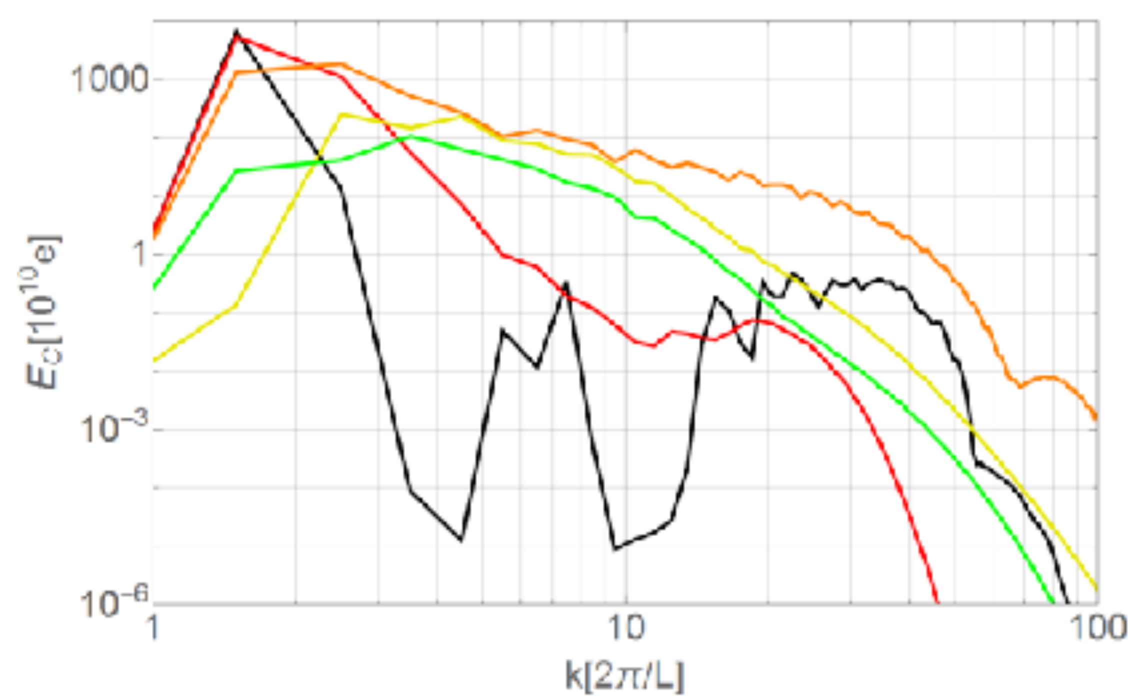

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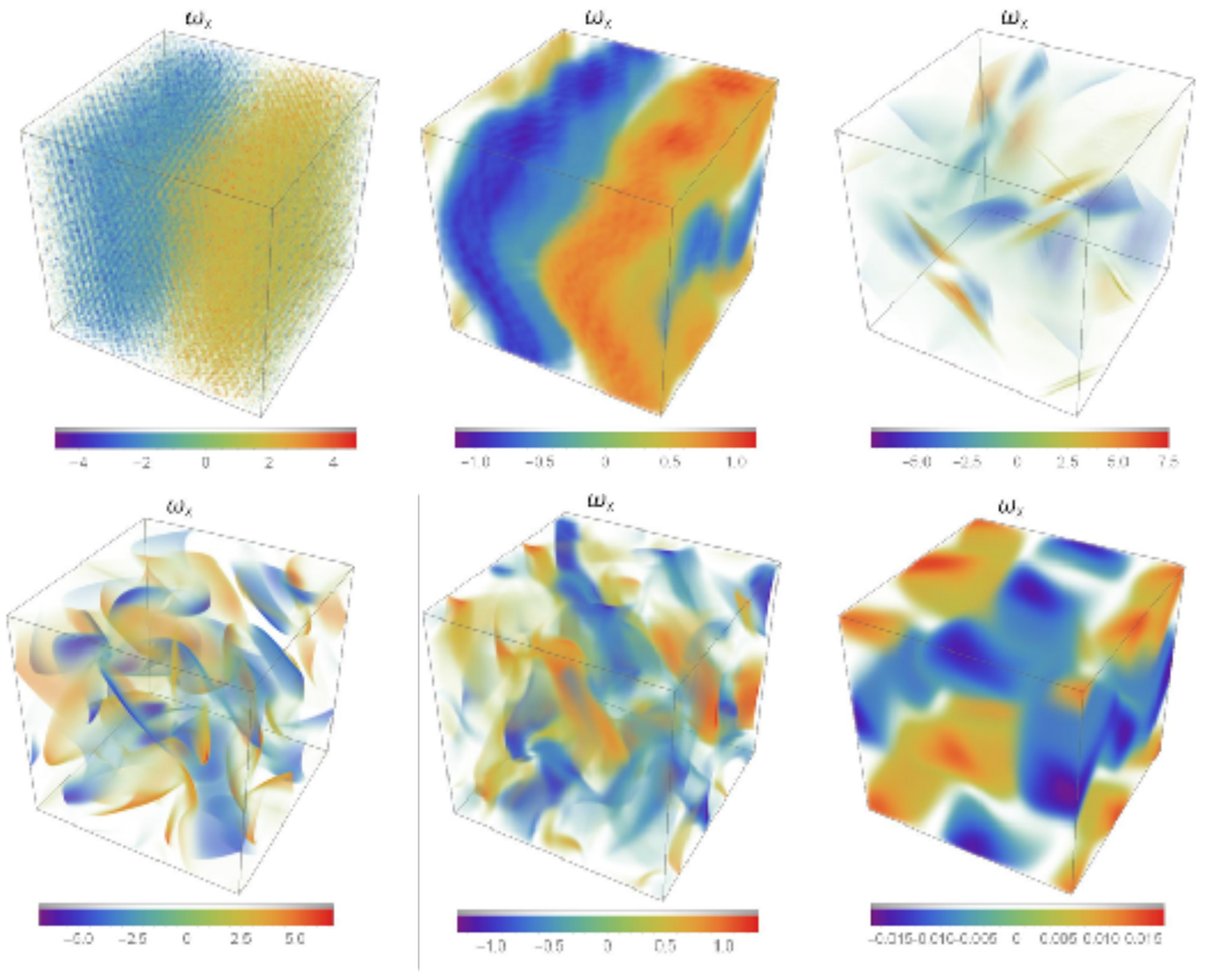

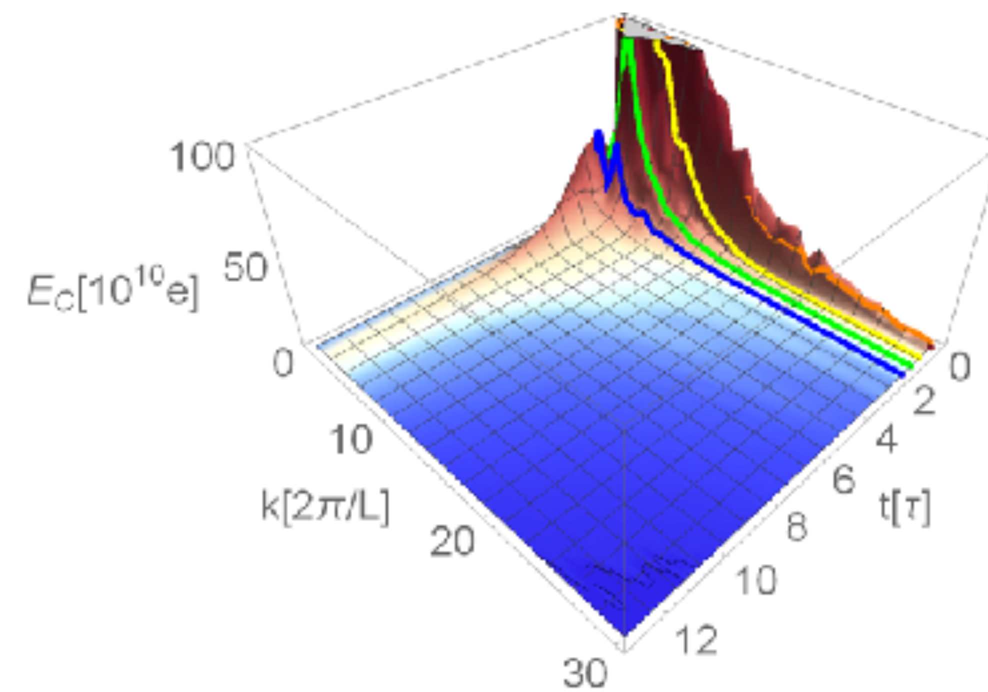

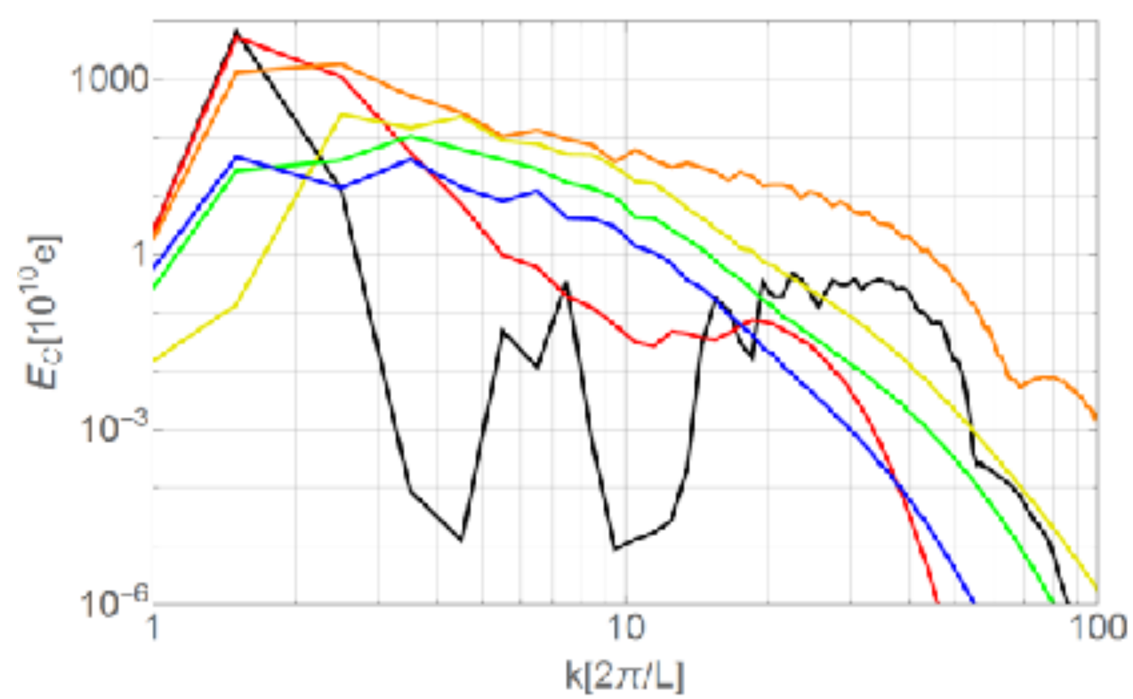

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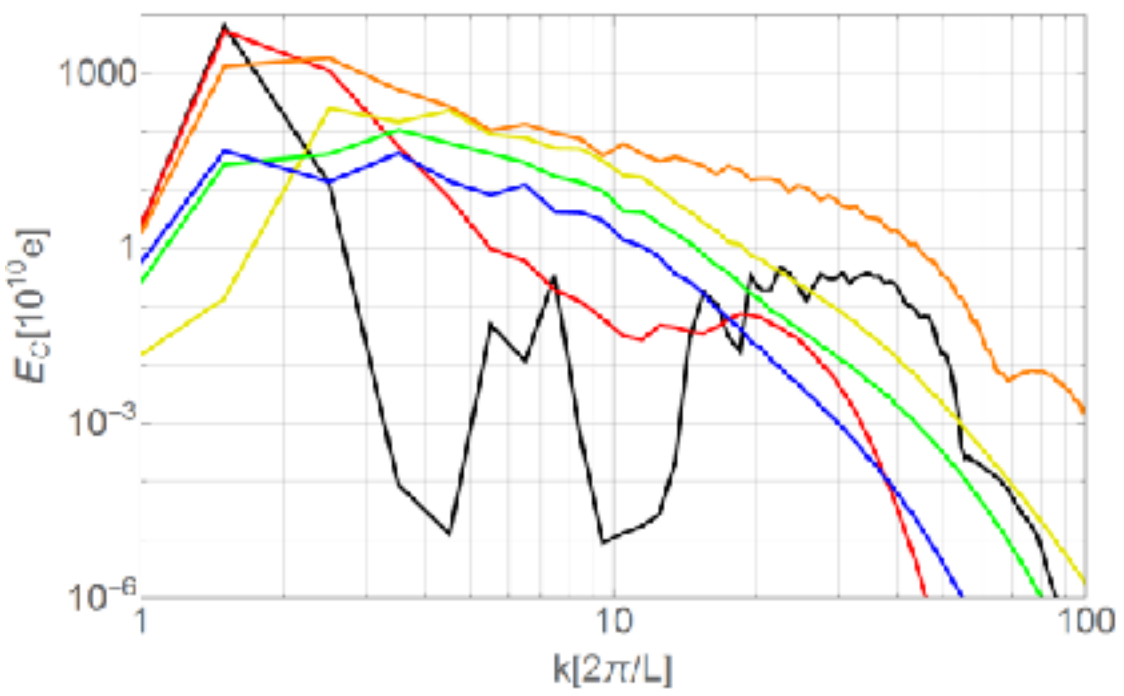

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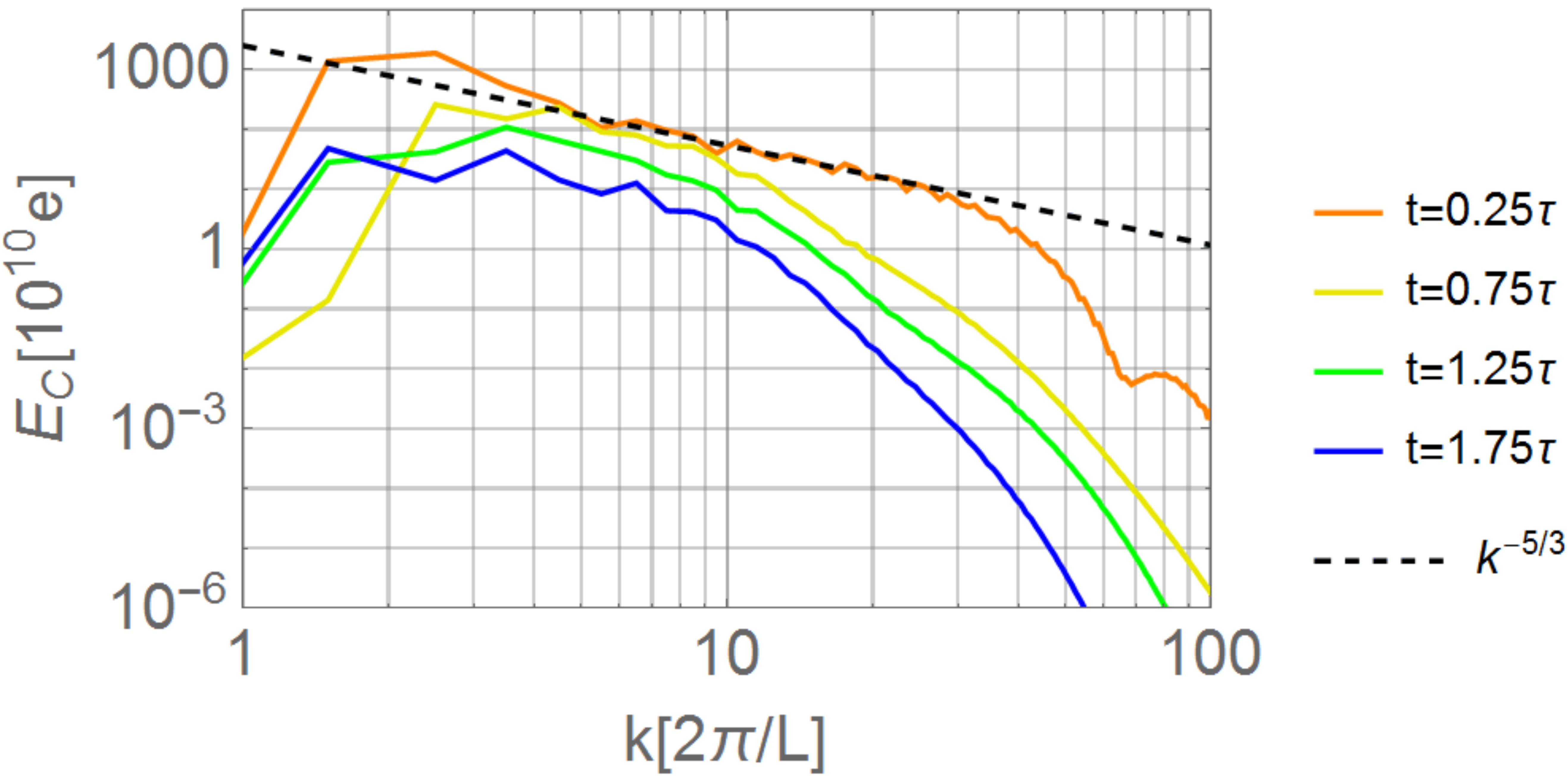

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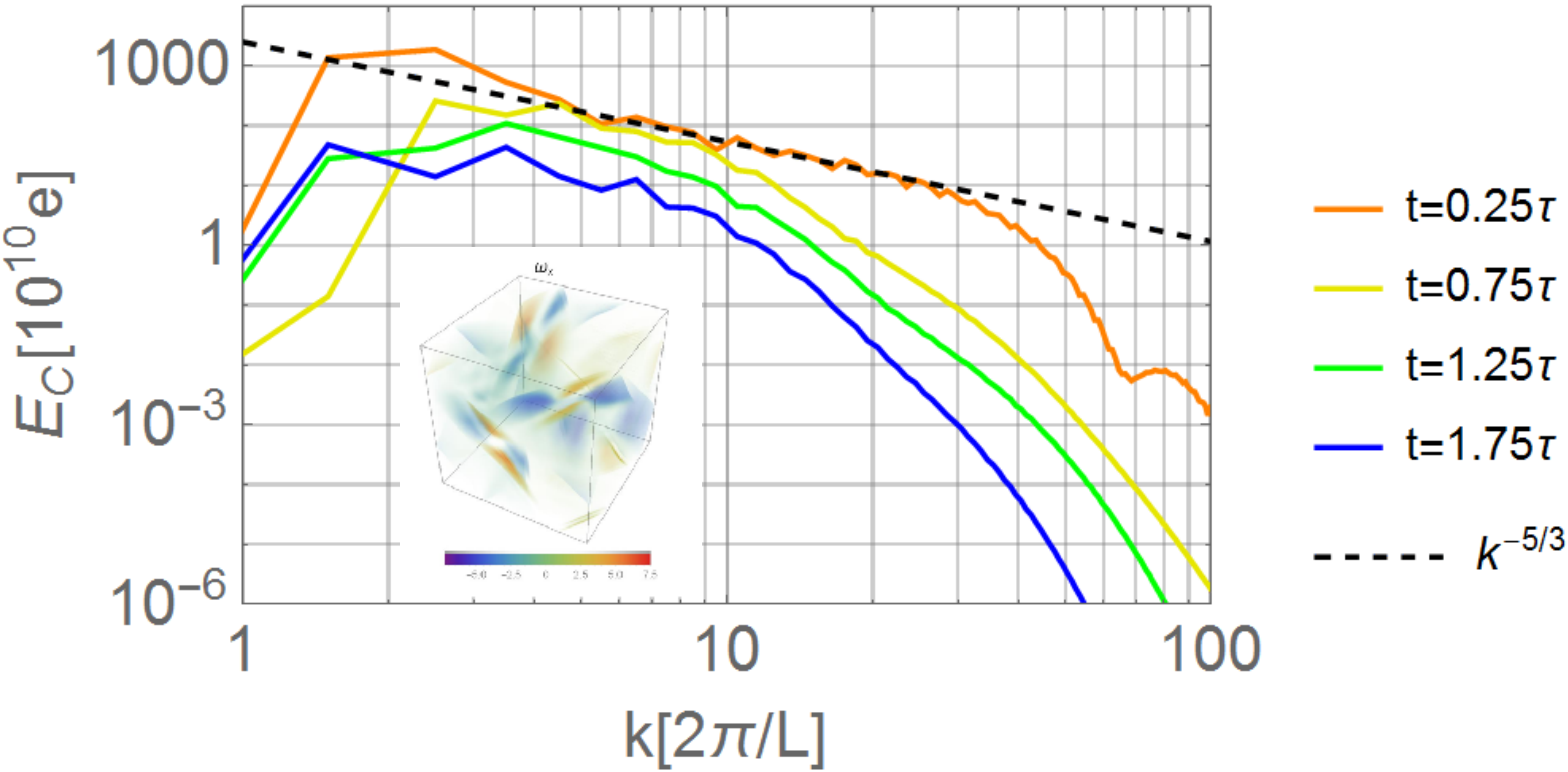

Kolmogorov scaling of A

(Adams, Chesler, Liu)

Starting from:

$$\partial_t p + \left(\vec{u}\cdot\vec{\nabla}\right) p + p\vec{\nabla}\cdot\vec{u} = \frac{1}{Re}\nabla^2 p$$

$$\partial_t \vec{u} + \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} + \frac{\vec{\nabla}p}{M^2 p} = \frac{1}{Re}\nabla^2\vec{u} + \frac{2}{Re}\left(\frac{\vec{\nabla}p}{p}\cdot\vec{\nabla}\right)\vec{u}$$

One defines:

$$\hat{u}_\epsilon(t,\vec{k}) = \int \sqrt{\epsilon}(t,\vec{\zeta})\vec{u}(t,\vec{\zeta})e^{-i\vec{k}\cdot\vec{\zeta}}d^p\zeta$$

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Turbulence is characterized by

$$E(t,k) \sim k^{-\frac{5}{3}}$$

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$$n^M\nabla_M n_Q = \kappa n_Q$$

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$$\mathcal{A}(t,k) \equiv \frac{\partial}{\partial k}\int_{|\vec{k}'|\leq k}\hat{\theta}^{*I}{}_J\hat{\theta}^J{}_I\frac{d^pk'}{(2\pi)^p}$$

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$$\mathcal{A}/E = k^2 \qquad E(t,k) \sim k^{-\frac{5}{3}}$$

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An explicit computation gives:

$$\theta^I{}_J = \frac{a\delta^{Il}\delta_{Jk}}{n^{(p+n)/2}}\left(\partial_k\left(\frac{f_l}{a} - \frac{\partial_l a}{a}\right) + \partial_l\left(\frac{f_k}{a} - \frac{\partial_k a}{a}\right)\right)$$

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so that (at best)

$$\mathcal{A}/E = C_0k^2 + \mathcal{O}(M^2)$$

Summary

- Hydrodynamics in large d simplifies

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

$$\partial_t j^a - h_0\, \partial_b \partial^b j^a = -\partial^a \epsilon - \partial_b \left(\frac{j^a j^b}{\epsilon} \right) - \left(\frac{c_1}{2} - \frac{\ell_1}{c_1} \right) \left(-\epsilon \partial^a \left(\frac{\partial_b j^b}{\epsilon} \right) + j^b \partial^a \left(\frac{\partial_b \epsilon}{\epsilon} \right) + \partial_b \left(\frac{j^a \partial^b \epsilon}{\epsilon} \right) \right)$$

Summary

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?

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a + \mathcal{O}(\partial^4)$$

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?

?

$$\lambda_i = \frac{2\ell_i}{n}\epsilon \qquad \eta = \frac{h_0}{n}\epsilon$$

?

$$\lambda_2 + 2\lambda_3 + \lambda_4 = 0 \qquad \lambda_1 - \lambda_2 - \lambda_3 = 0$$

Summary

- Hydrodynamics in large d simplifies (under certain assumptions)

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

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- Holographic large d theories are given exactly by large d hydro

$$\partial_t a - \partial_b \partial^b a = -\partial_b f^b$$

$$\partial_t f_a - \partial_b \partial^b f_a = -\partial_a a - \partial_b \left(\frac{f_a f^b}{a} \right)$$

Summary

- Hydrodynamics in large d simplifies (under certain assumptions)

$$\partial_t \epsilon + c_1 \partial_a \partial^a \epsilon = -\partial_a j^a$$

$$\partial_t j^a - h_0\, \partial_b \partial^b j^a = -\partial^a \epsilon - \partial_b \left(\frac{j^a j^b}{\epsilon} \right) - \left(\frac{c_1}{2} - \frac{\ell_1}{c_1} \right) \left(-\epsilon \partial^a \left(\frac{\partial_b j^b}{\epsilon} \right) + j^b \partial^a \left(\frac{\partial_b \epsilon}{\epsilon} \right) + \partial_b \left(\frac{j^a \partial^b \epsilon}{\epsilon} \right) \right)$$

- Holographic large d theories are given exactly by large d hydro

$$\partial_t a - \partial_b \partial^b a = -\partial_b f^b$$

$$\partial_t f_a - \partial_b \partial^b f_a = -\partial_a a - \partial_b \left(\frac{f_a f^b}{a} \right)$$

- The mapping simplifies and provides an analytic handle over geometric quantities

$$\mathcal{A}/E = C_0 k^2 + \mathcal{O}(M^2)$$