

9th Crete Regional Meeting in String Theory 2017

# Self-Tuning Of The Cosmological Constant And Holographic RG Flows

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with **Jewel Kumar Ghosh, Elias Kiritsis & Francesco Nitti**

# Motivation

**Braneworlds** offer a possible solution to the **Cosmological Constant (CC) problem**:

- Possible to reconcile a **flat** or **weakly curved** 4d space-time despite a large 4d vacuum energy  $\Lambda_4$ .
- Vacuum energy  $\Lambda_4$  curves the higher dimensional bulk, while the 4d brane world volume may be flat or weakly curved.
- The setup allows for **self-tuning**: a flat or weakly curved brane world volume may emerge for generic values of  $\Lambda_4$ .

# Motivation

**Braneworlds** offer a possible solution to the **Cosmological Constant (CC) problem:**

- **Previous work:**

[Arkani-Hamed, Dimopoulos, Kaloper and Sundrum '00]

[Kachru, Schulz, Silverstein '00]

[Csaki, Erlich, Grojean, Hollowood '00]

- **Here:**

Work with a semi-holographic model by Kiritsis, Nitti.

The model has been discussed in the **talk by Francesco Nitti**.

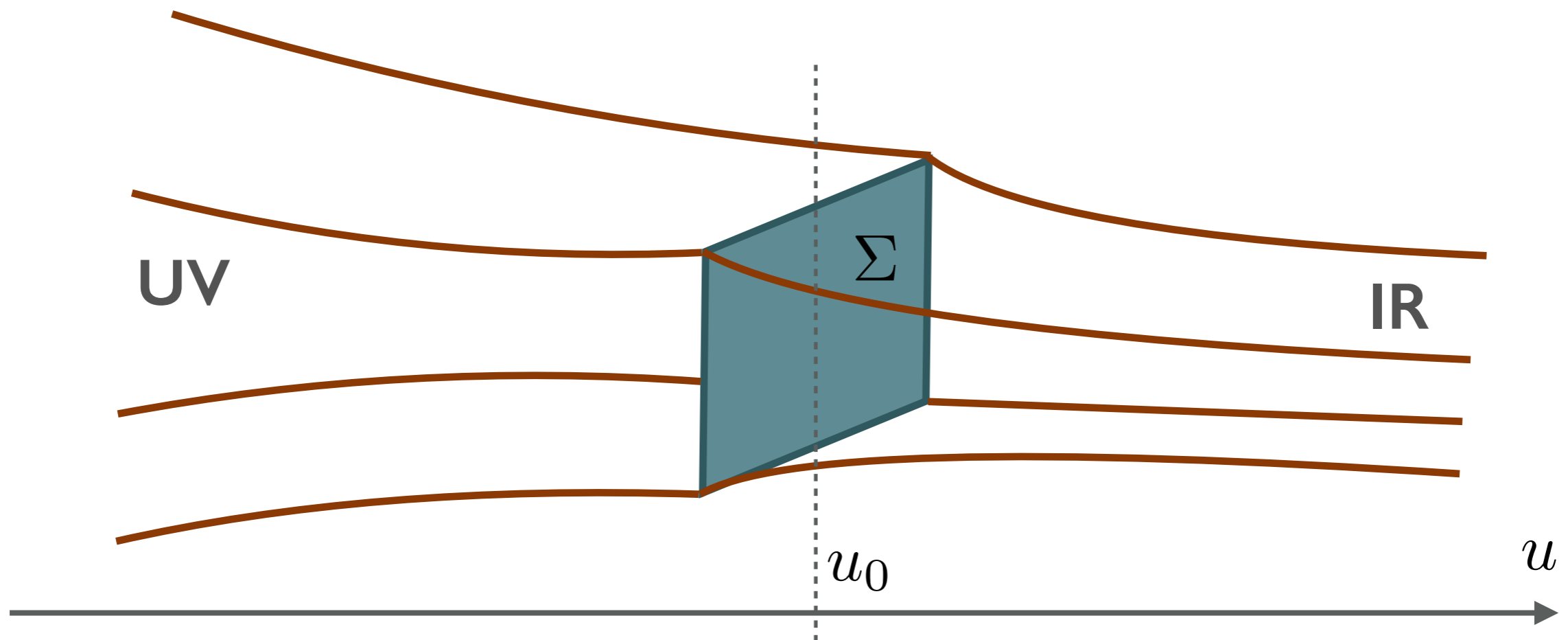
Self-tuning has been explored in [Charmousis, Kiritsis, Nitti '17]

# Motivation

## 5d gravity theory with a bulk scalar

$$S = M^3 \int du \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right] + S_{GH}$$
$$+ M^3 \int_{\Sigma} d^4\sigma \sqrt{-\gamma} \left[ -W_B(\varphi) - \frac{1}{2} Z(\varphi) \gamma^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi + U(\varphi) R^{(\gamma)} + \mathcal{L}_{SM} \right]$$

## 4d QFT on a brane including SM



# Motivation

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Address self-tuning in **increasingly realistic** setups:

- 1.) Establish self-tuning for a **flat** brane.
- 2.) Reinststate Higgs sector and study **EW symmetry breaking**.
- 3.) Establish self tuning for a **curved** brane.

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Address self-tuning in **increasingly realistic** setups:

1.) Establish self-tuning for a **flat** brane. ✓

See [Charmousis, Kiritsis, Nitti arXiv:1704.05075] and **talk** by Francesco Nitti

2.) Reinststate Higgs sector and st

**EW symmetry breaking**

3.) Establish self-tuning for a **curved** brane.

**Work in progress**

# Outline

## 1.) Recap:

Review of the setup and self-tuning for a flat brane

## 2.) Self-tuning of the CC and EW symmetry breaking:

Reinstate the Higgs sector

## 3.) Self-tuning for a curved brane:

- Allow brane worldvolume to be maximally symmetric curved spacetime (dS, AdS)
- Finding bulk solutions equivalent to studying **holographic RG flows** for QFTs on curved spacetime.

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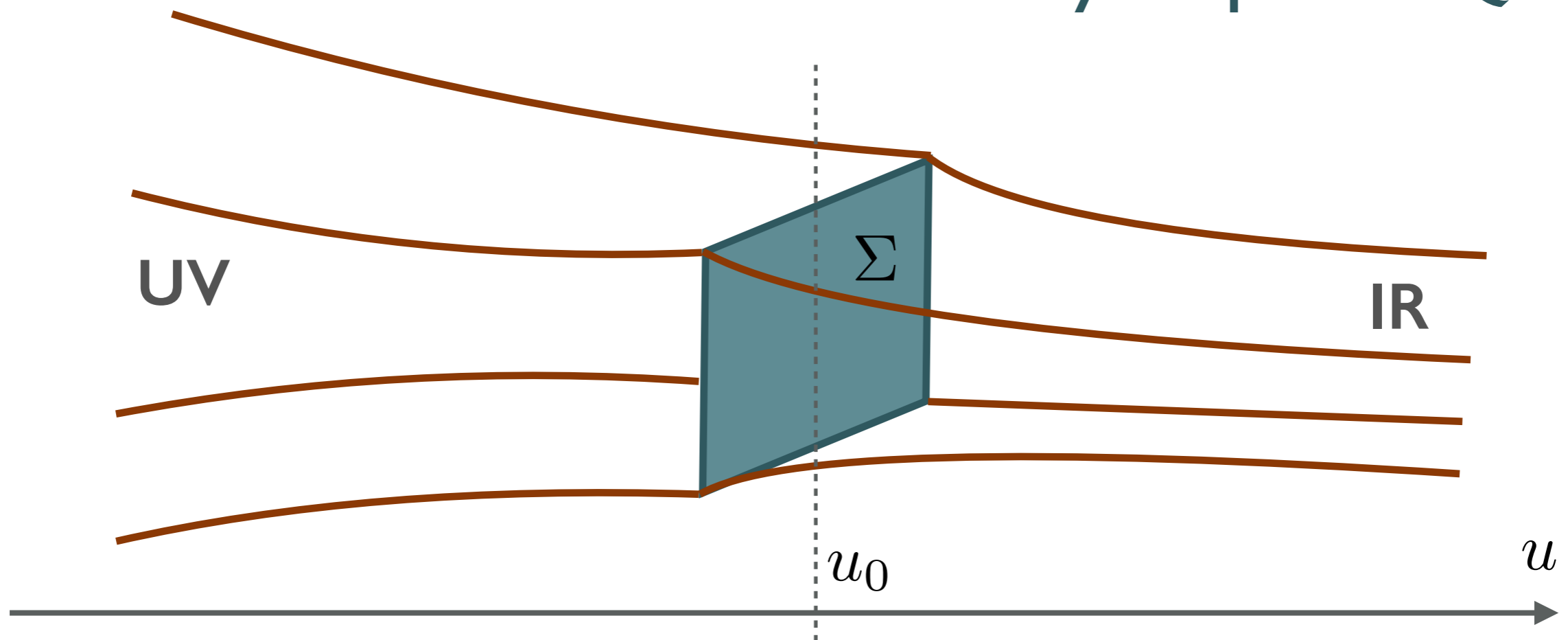


# Self-tuning: Flat Brane

## 5d gravity dual of 4d CFT

$$S = M^3 \int du \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right] + S_{GH}$$
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Weakly coupled 4d QFT

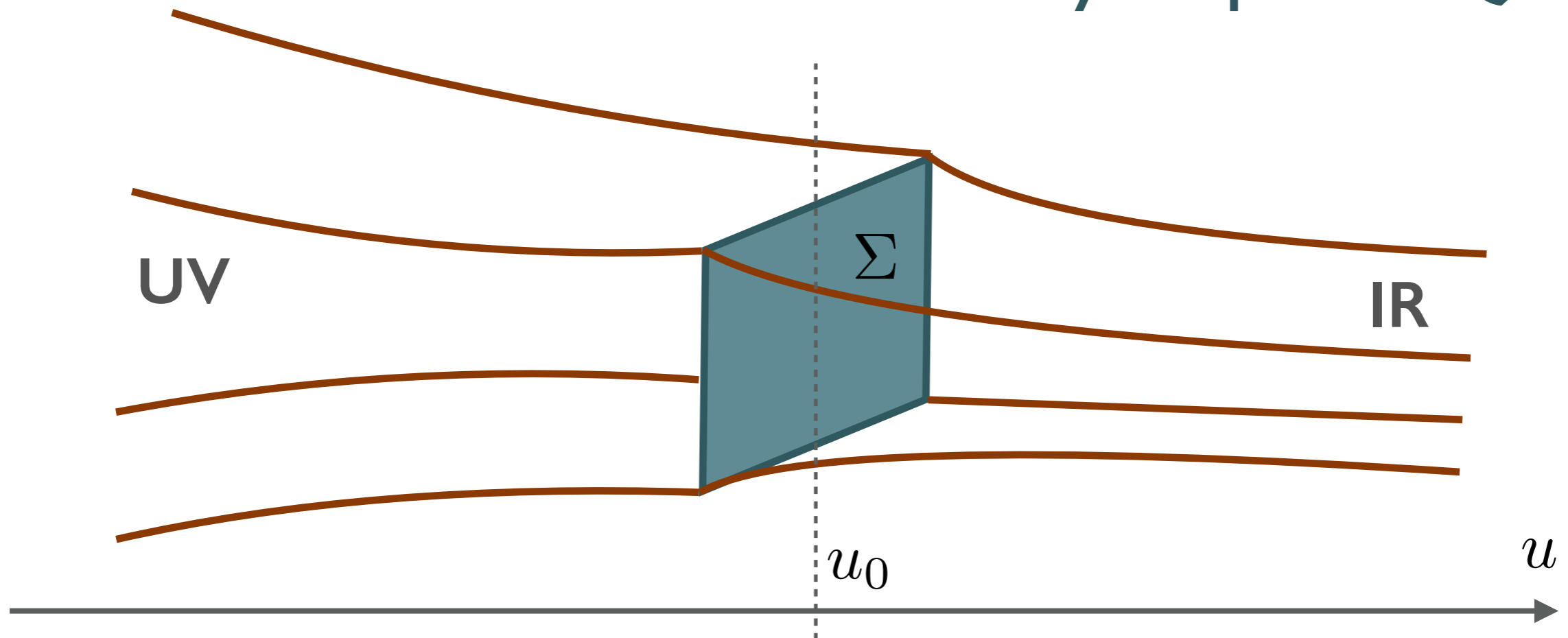


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## Ansatz for a flat brane:

- Domain wall metric with flat slices:

$$ds^2 = du^2 + e^{2A(u)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} .$$

- Bulk scalar:  $\varphi = \varphi(u)$  .

The brane is located at a fixed  $u_0$  which corresponds to a fixed  $\varphi_0 = \varphi(u_0)$  .

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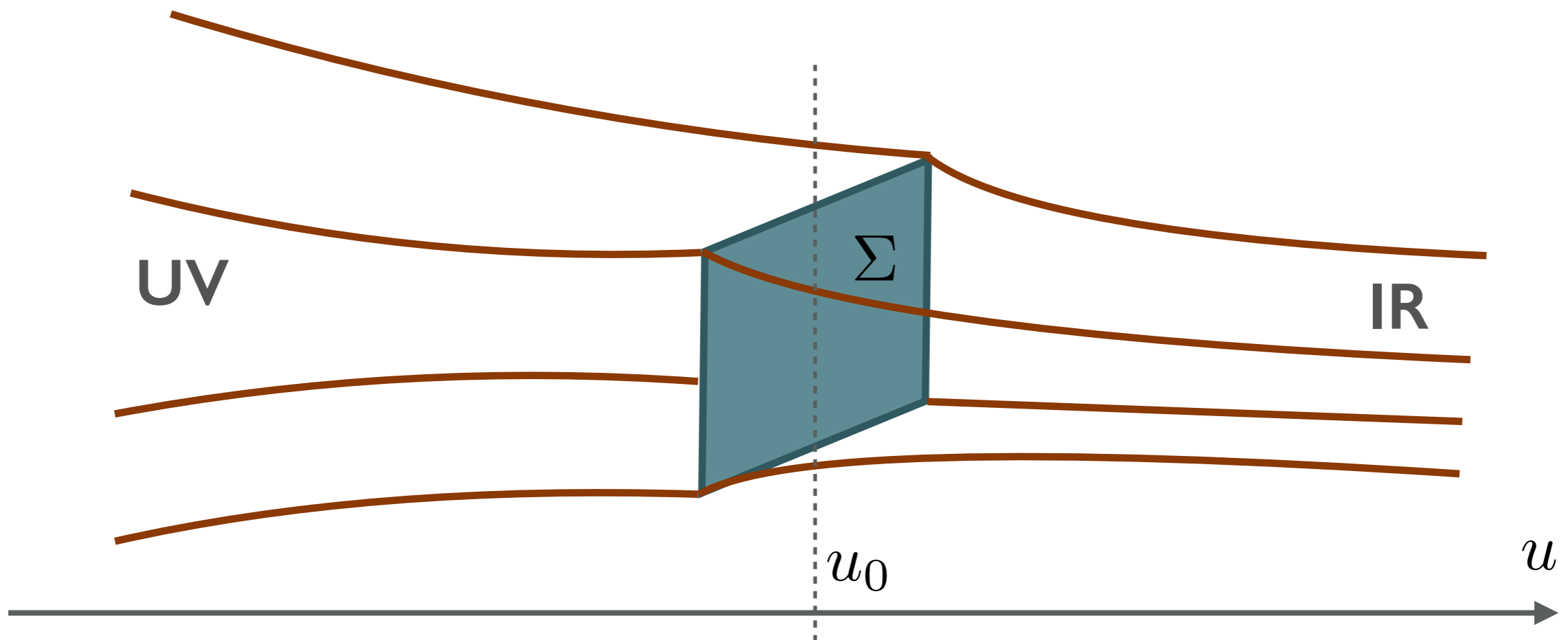
The brane is located at a fixed  $u_0$  which corresponds to a fixed  $\varphi_0 = \varphi(u_0)$  .

**Q:** Is there are solution despite  $W_B \neq 0$  ?

# Self-tuning: Flat Brane

## Strategy:

- Solve for scale factor and bulk scalar in **UV** region.
- Solve for scale factor and bulk scalar in **IR** region.
- Match solutions at brane position via **Israel junction conditions**.



# Self-tuning: Flat Brane

## Strategy:

- Convenient to define a new variable: the superpotential  $W(\varphi)$ .

$$W(\varphi(u)) = -2(d-1) \frac{dA}{du},$$
$$\frac{dW}{d\varphi}(\varphi(u)) = \frac{d\varphi}{du}.$$

Convention: Bulk spacetime  $(d+1)$ -dimensional.

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- Einstein equations: above definitions together with eq. below:

$$-\frac{d}{4(d-1)} W^2 + \frac{1}{2} \left( \frac{dW}{d\varphi} \right)^2 = V.$$

Convention: Bulk spacetime  $(d+1)$ -dimensional.

# Self-tuning: Flat Brane

## Self-tuning solution:

- Solve for  $W_{IR}$  in IR region. Regularity fixes solution uniquely.
- Solve for  $W_{UV}$  in UV region up to an integration constant  $C_{UV}$ .
- Apply junction conditions:

$$W_{UV}(\varphi_0) - W_{IR}(\varphi_0) = -W_B(\varphi_0),$$

$$\frac{dW_{UV}}{d\varphi}(\varphi_0) - \frac{dW_{IR}}{d\varphi}(\varphi_0) = -\frac{dW_B}{d\varphi}(\varphi_0).$$

Solving these fixes  $C_{UV}$  and the brane position  $\varphi_0$ .



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**Self-tuning:** both  $C_{UV}$  and  $\varphi_0$  adjust to allow for a **flat brane solution** despite  $W_B \neq 0$ .

**Self-tuning:**

**Flat Brane + Higgs Sector**

# Self-tuning: Flat Brane + Higgs Sector

## 5d gravity dual of 4d CFT

$$S = M^3 \int du \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right] + S_{GH}$$
$$+ M^3 \int_{\Sigma} d^4\sigma \sqrt{-\gamma} \mathcal{L}(\gamma_{\mu\nu}, \varphi, \text{SM fields})$$

**Weakly coupled 4d QFT**

Let  $H$  be the Higgs doublet.

Write down the Higgs sector explicitly.

# Self-tuning: Flat Brane + Higgs Sector

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Let  $H$  be the Higgs doublet.

Write down the Higgs sector explicitly.

Study how this impacts the self-tuning solution.

# Self-tuning: Flat Brane + Higgs Sector

$$\begin{aligned}
 S = & M^3 \int du \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right] + S_{GH} \\
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 & \quad \left. - X(\varphi) |H|^2 - Y(\varphi) |H|^4 + P(\varphi) R^{(\gamma)} |H|^2 + \dots \right]
 \end{aligned}$$

The brane action can be written more compactly as:

$$S_b = M^3 \int_{\Sigma} d^4\sigma \sqrt{-\gamma} \left[ -\hat{W}_B(\varphi, |H|) - \frac{1}{2} Z(\varphi) \gamma^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi + \hat{U}(\varphi, |H|) R^{(\gamma)} + \mathcal{L}_{\text{SM}} \right],$$

$$\text{with } \hat{W}_B(\varphi, |H|) = W_B(\varphi) + X(\varphi) |H|^2 + Y(\varphi) |H|^4,$$

$$\text{and } \hat{U}(\varphi, |H|) = U(\varphi) + P(\varphi) |H|^2.$$

# Self-tuning: Flat Brane + Higgs Sector

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Again, use ansatz for a **flat** brane:

- Domain wall metric with flat slices:

$$ds^2 = du^2 + e^{2A(u)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} .$$

- Bulk scalar:  $\varphi = \varphi(u)$  .

# Self-tuning: Flat Brane + Higgs Sector

## Solution:

- Solve for  $W_{IR}$  and  $W_{UV}$  in the bulk as before.
- The junction conditions now become:

$$W_{UV}(\varphi_0) - W_{IR}(\varphi_0) = -\hat{W}_B(\varphi_0, |H|),$$

$$\frac{dW_{UV}}{d\varphi}(\varphi_0) - \frac{dW_{IR}}{d\varphi}(\varphi_0) = -\frac{d\hat{W}_B}{d\varphi}(\varphi_0, |H|).$$

- In addition, varying w.r.t. the Higgs gives:

$$[X(\varphi_0) + 2Y(\varphi_0)|H|^2] H = 0.$$

# Self-tuning: Flat Brane + Higgs Sector

## EW symmetry breaking:

- Solve for  $C_{UV}$ ,  $\varphi_0$ ,  $|H|$ .
- Have successful EW symmetry breaking if  $X(\varphi_0) < 0$ :

$$\Rightarrow |H|^2 = -\frac{X(\varphi_0)}{2Y(\varphi_0)}.$$

- The self-tuning mechanism neutralises any contribution to the vacuum energy due to the Higgs sector.
- Interestingly, in this setup the physics of EW symmetry breaking and the self-tuning of the CC are intertwined.



# Self-tuning: Flat Brane + Higgs Sector

## Open questions:

- For EW symmetry breaking it is important that  $X(\varphi)$  can become negative. Need to understand the microscopic origin of  $X(\varphi)$  to check whether this can occur.
- What about the EW hierarchy problem?  
Can the solution reproduce the observed value of the Higgs vev? I.e. does this setup generate a hierarchy between the Higgs vev and the Planck scale and / or the cutoff?

$$|H|^2 = -\frac{X(\varphi_0)}{2Y(\varphi_0)}.$$

**Self-tuning:**  
**Curved Brane**  
**&**  
**Holographic RG Flows**

# Self-tuning: Curved Brane

**So far:** only considered metric ansatz with **flat slicing**:

$$\varphi = \varphi(u) \quad \text{and} \quad ds^2 = du^2 + e^{2A(u)} \eta_{\mu\nu} dx^\mu dx^\nu .$$

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**Now:** allow for **curved slices** with **maximal symmetry**:

$$\varphi = \varphi(u) \quad \text{and} \quad ds^2 = du^2 + e^{2A(u)} \zeta_{\mu\nu} dx^\mu dx^\nu ,$$

i.e.  $\zeta_{\mu\nu}$  describes a  $d$ -dimensional maximally symmetric spacetime:

$$R_{\mu\nu}^{(\zeta)} = \kappa \zeta_{\mu\nu} , \quad R^{(\zeta)} = d\kappa , \quad \text{and} \quad \kappa = \begin{cases} \frac{(d-1)}{\alpha^2} & \text{dS}_d \\ 0 & \text{Minkowski} \\ -\frac{(d-1)}{\alpha^2} & \text{AdS}_d \end{cases}$$

# Self-tuning: Curved Slicing

## Strategy as before:

- Solve the bulk e.o.m. in the UV and the IR.
- Apply matching conditions at brane locus.

## However:

- Note that the bulk e.o.m. are modified compared to the case with flat slicing
- **Ignore** the presence of the **brane** at first and study solutions to the bulk e.o.m. first.

# Self-tuning: Curved Slicing

$$S = M^3 \int du \int d^d x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right] + S_{GH} ,$$

subject to  $\varphi = \varphi(u)$  and  $ds^2 = du^2 + e^{2A(u)} \zeta_{\mu\nu} dx^\mu dx^\nu$  ,

**Closely related to study of Holographic RG flows** for field theories on **curved manifolds**.

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**Closely related to study of Holographic RG flows** for field theories on **curved manifolds**.

- The bulk scalar  $\varphi$  will be dual to a scalar operator  $\mathcal{O}$  .
- The solution  $\varphi(u)$  is interpreted as the running coupling.
- Given a solution  $A(u), \varphi(u)$  the beta function is:  $\beta = \frac{d\varphi}{dA}$  .
- The space of solutions to  $A(u), \varphi(u)$  will be in one-to-one correspondence with the space of possible RG-flows.

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**Closely related to study of Holographic RG flows** for field theories on **curved manifolds**.

**Here:**

- Consider negative potentials only  $V < 0$ .
- We will consider asymptotically  $\text{AdS}_{(d+1)}$  solutions.
- Hence we will study RG flows for CFTs deformed by a relevant operator, defined on a maximally symmetric curved manifold.



# Holographic RG Flows

Use an example to illustrate our results:

- Let 
$$V = -\frac{d(d-1)}{\ell^2} - \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4}\varphi^4.$$

- Restrict focus on **dS<sub>d</sub> slicings only**, i.e.  $\zeta_{\mu\nu}$  is a metric on dS<sub>d</sub>.
- WLOG we choose  $u$  to increase along a flow.  
Can also choose  $A(u)$  to be monotonically decreasing along the flow.

# Holographic RG Flows

To make contact with literature on holographic RG flows rewrite the e.o.m. as coupled 1st order differential equations:

- Define “superpotentials”:  
$$W(\varphi(u)) = -2(d-1)\dot{A}(u),$$
$$S(\varphi(u)) = \dot{\varphi}(u),$$
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$$\cdot \equiv \frac{d}{du}, \quad ' \equiv \frac{d}{d\varphi}.$$

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- E.o.m.:
$$2(d-1)\ddot{A} + \dot{\varphi}^2 + \frac{2}{d}e^{-2A}R^{(\zeta)} = 0,$$
$$d(d-1)\dot{A}^2 - \frac{1}{2}\dot{\varphi}^2 + V - e^{-2A}R^{(\zeta)} = 0,$$
$$\ddot{\varphi} + d\dot{A}\dot{\varphi} - V' = 0.$$

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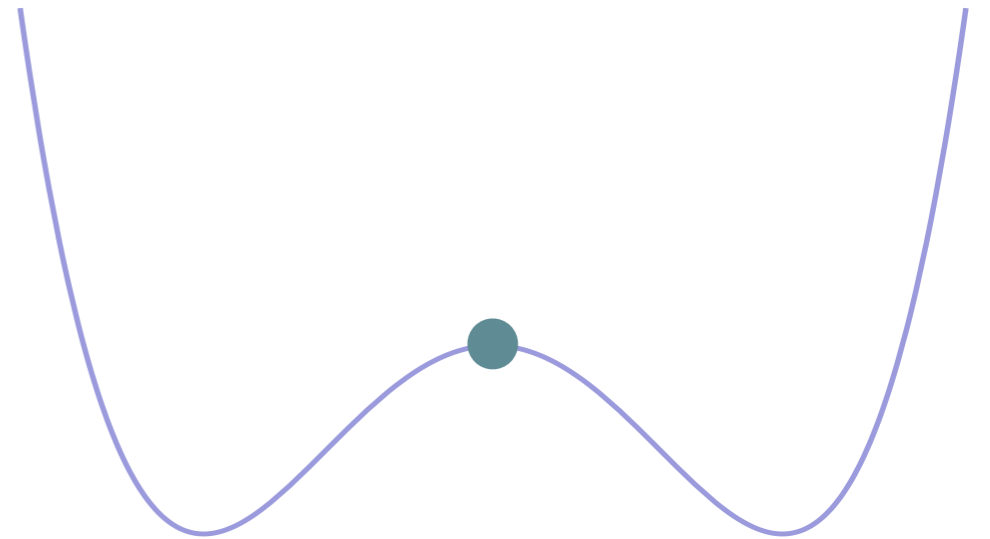
- E.o.m.:  
$$S^2 - SW' + \frac{2}{d}T = 0,$$
$$\frac{d}{2(d-1)}W^2 - S^2 - 2T + 2V = 0,$$
$$S' - \frac{d}{2(d-1)}SW - V' = 0.$$

$$\dot{\phantom{x}} \equiv \frac{d}{du}, \quad ' \equiv \frac{d}{d\varphi}.$$

# Holographic RG Flows

**Extrema** of the potential: **Maxima**

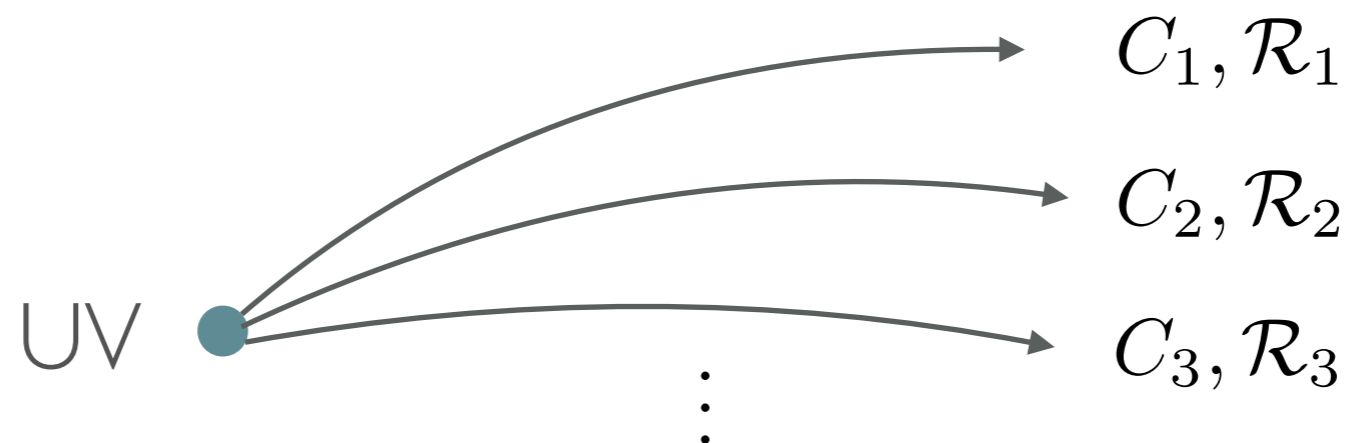
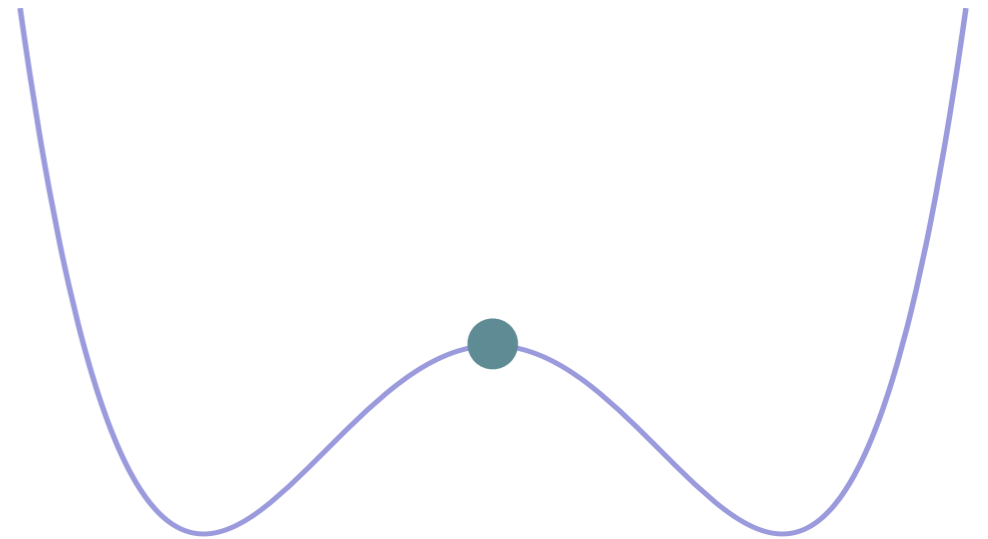
- **Maxima** will correspond to **UV fixed** points.
- Scale factor  $e^{A(u)}$  diverges there.
- The bulk geometry will asymptote to **AdS<sub>(d+1)</sub>**.
- Each maximum allows for a family of solutions, parameterised by two constants  $C$  and  $\mathcal{R}$ .



# Holographic RG Flows

**Extrema** of the potential: **Maxima**

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# Holographic RG Flows

**Extrema** of the potential: **Maxima**

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - m^2 \ell^2}$$

- In the vicinity of the UV fixed point  $u \rightarrow -\infty$  the solutions for  $\varphi(u)$  and the metric become:

$$\varphi(u) \underset{u \rightarrow -\infty}{=} \varphi_- e^{\Delta_- u/\ell} + \frac{\langle \mathcal{O} \rangle}{2\Delta_+ - d} e^{\Delta_+ u/\ell},$$

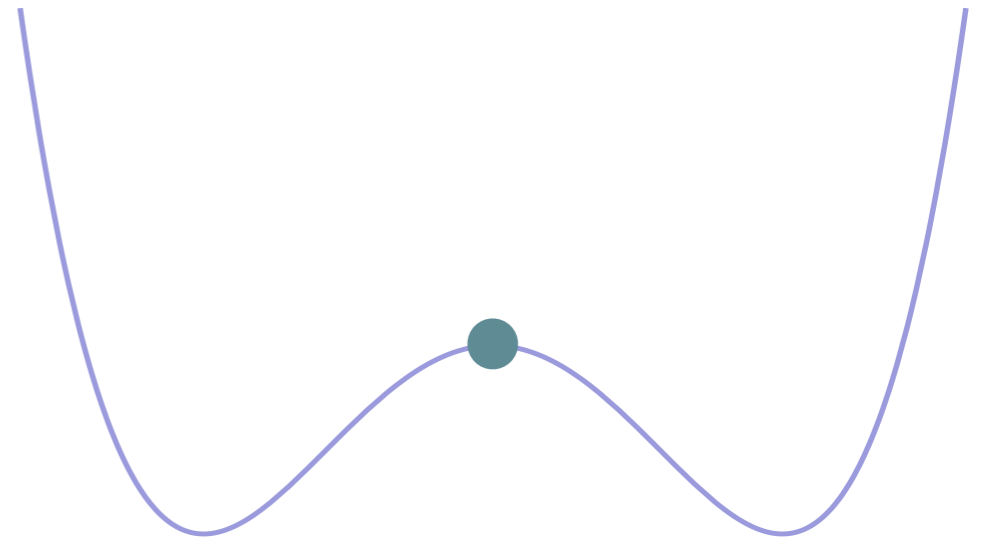
$$ds^2 \underset{u \rightarrow -\infty}{=} du^2 + e^{-2u/\ell} \tilde{\gamma}_{\mu\nu} dx^\mu dx^\nu.$$

- Given a source  $\varphi_-$ :

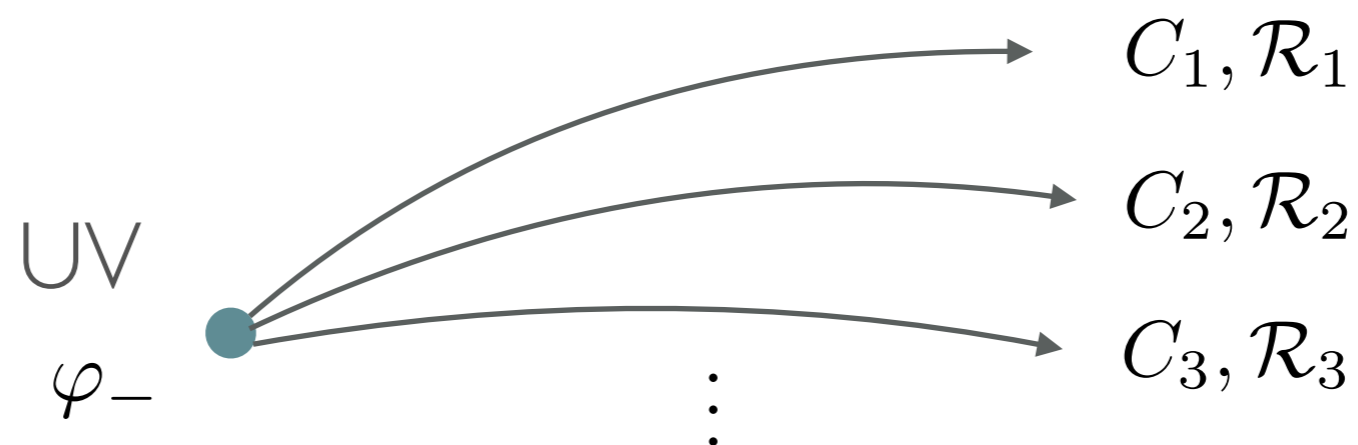
$$\text{VEV: } \langle \mathcal{O} \rangle = \frac{C d \ell}{\Delta_-} \varphi_-^{\Delta_+/\Delta_-} \quad \text{Boundary curvature: } R^{\tilde{\gamma}} = \mathcal{R} \varphi_-^{2/\Delta_-}$$

# Holographic RG Flows

## Maxima = UV fixed points



- Scale factor  $e^{A(u)}$  diverges there.
- The bulk geometry will asymptote to **AdS**<sub>(d+1)</sub>.
- Each maximum allows for a family of solutions, parameterised by two constants  $C$  and  $\mathcal{R}$ .





# Holographic RG Flows

Where do flows end? What is the IR?

- IR: the scale factor  $e^{A(u)}$  vanishes and the flow stops:  $\dot{\varphi} = 0$ .

I.) Flat slicing:  $R^{(\zeta)} = 0$

**IR fixed points = minima** of the potential.

(Flows to infinity are also allowed if sufficiently well-behaved.)

Only have a finite number of solutions at most.

# Holographic RG Flows

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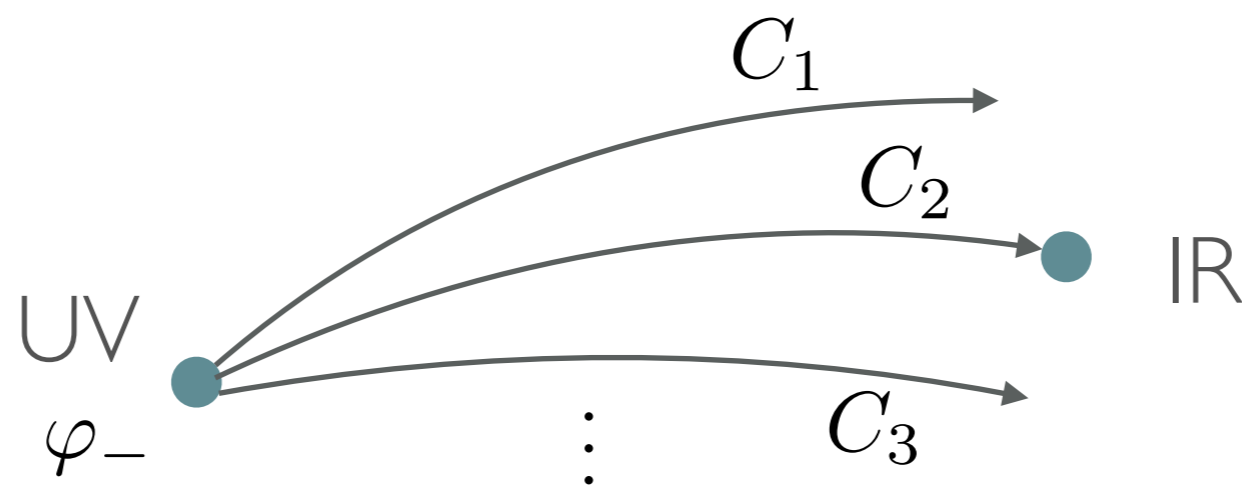
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**IR fixed points = minima** of the potential.

(Flows to infinity are also allowed if sufficiently well-behaved.)

Have a unique solution in IR.



# Holographic RG Flows

## Where do flows end? What is the IR?

- IR: the scale factor  $e^{A(u)}$  vanishes and the flow stops:  $\dot{\varphi} = 0$ .

### I.) Curved slicing: $R^{(\zeta)} \neq 0$

A solution with  $e^A \rightarrow 0$ ,  $\dot{\varphi} \rightarrow 0$  exists for every value of  $\varphi$ , as long as  $V' \neq 0$  there. Every point  $\varphi$  except extrema of the potential can be an IR.

At an IR point at some  $\varphi_*$  the bulk geometry asymptotes to  $\text{AdS}_{(d+1)}$  with AdS length  $\ell_{IR}$  given by  $V(\varphi_*) = -\frac{d(d-1)}{\ell_{IR}^2}$ .

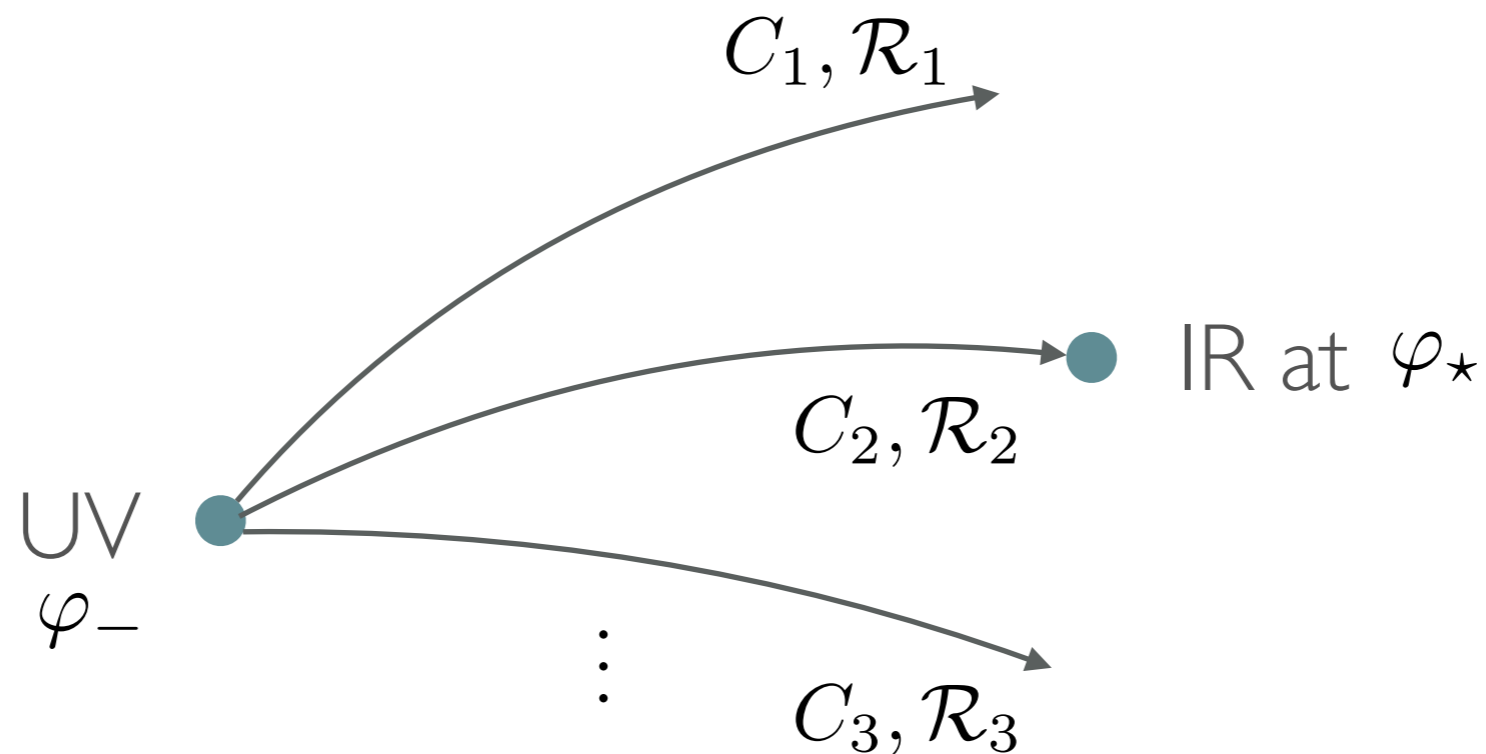
# Holographic RG Flows

Where do flows end? What is the IR?

- IR: the scale factor  $e^{A(u)}$  vanishes and the flow stops:  $\dot{\varphi} = 0$ .

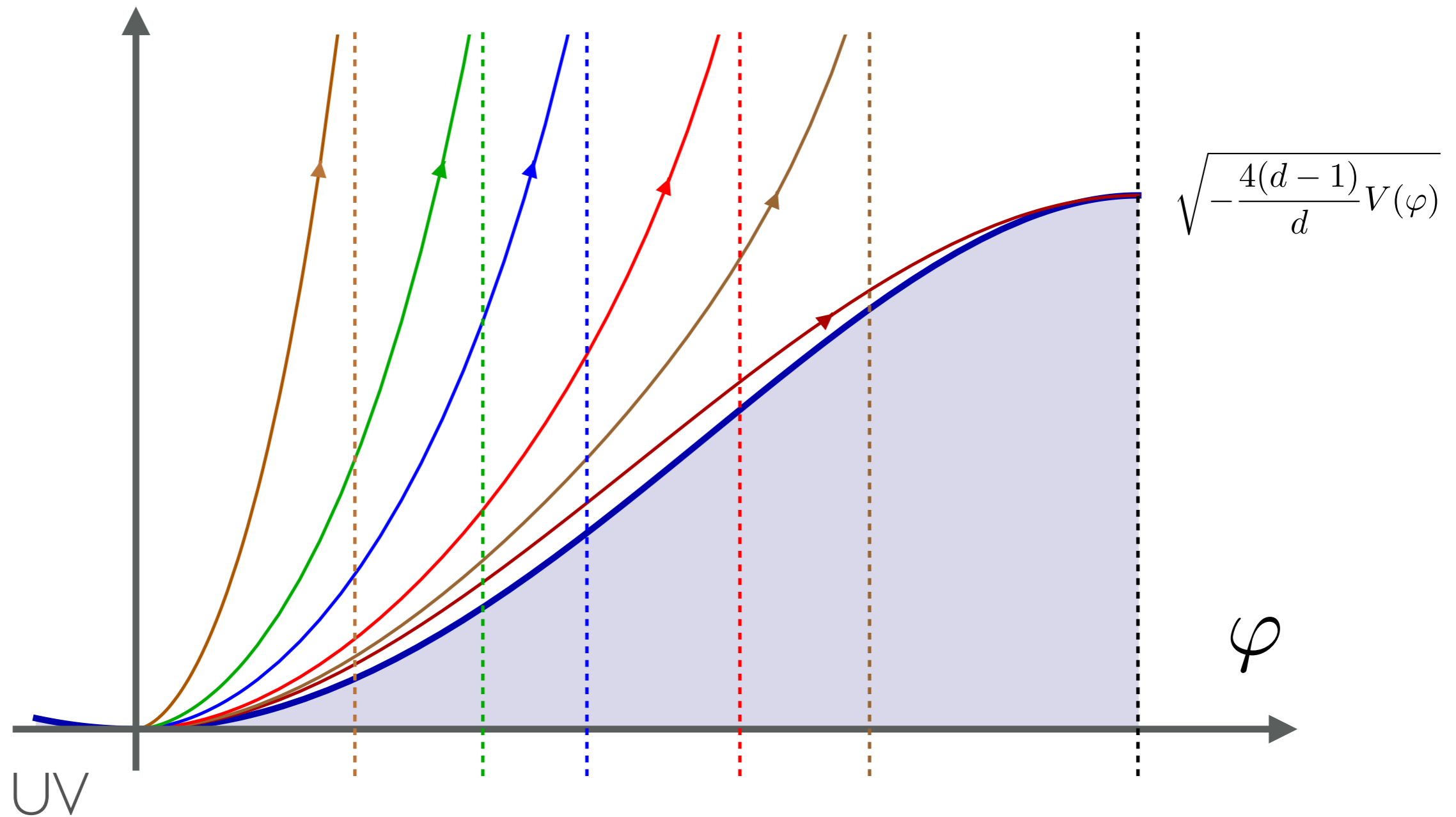
I.) Curved slicing:  $R^{(\zeta)} \neq 0$

Again, find a unique solutions per IR.



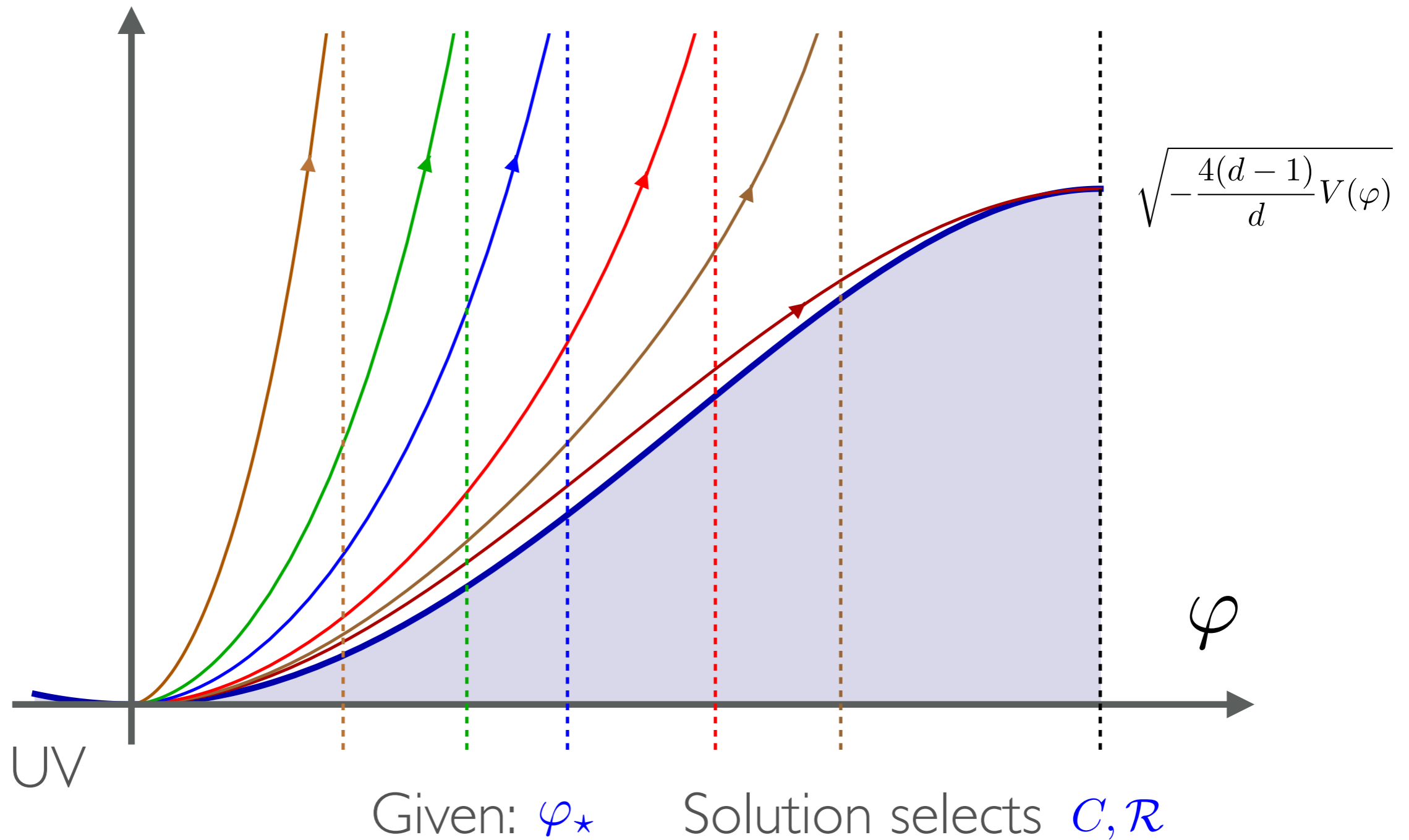
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**Summary:** plot  $W(\phi)$  vs.  $\phi$  for various IR points  $\phi_*$ .



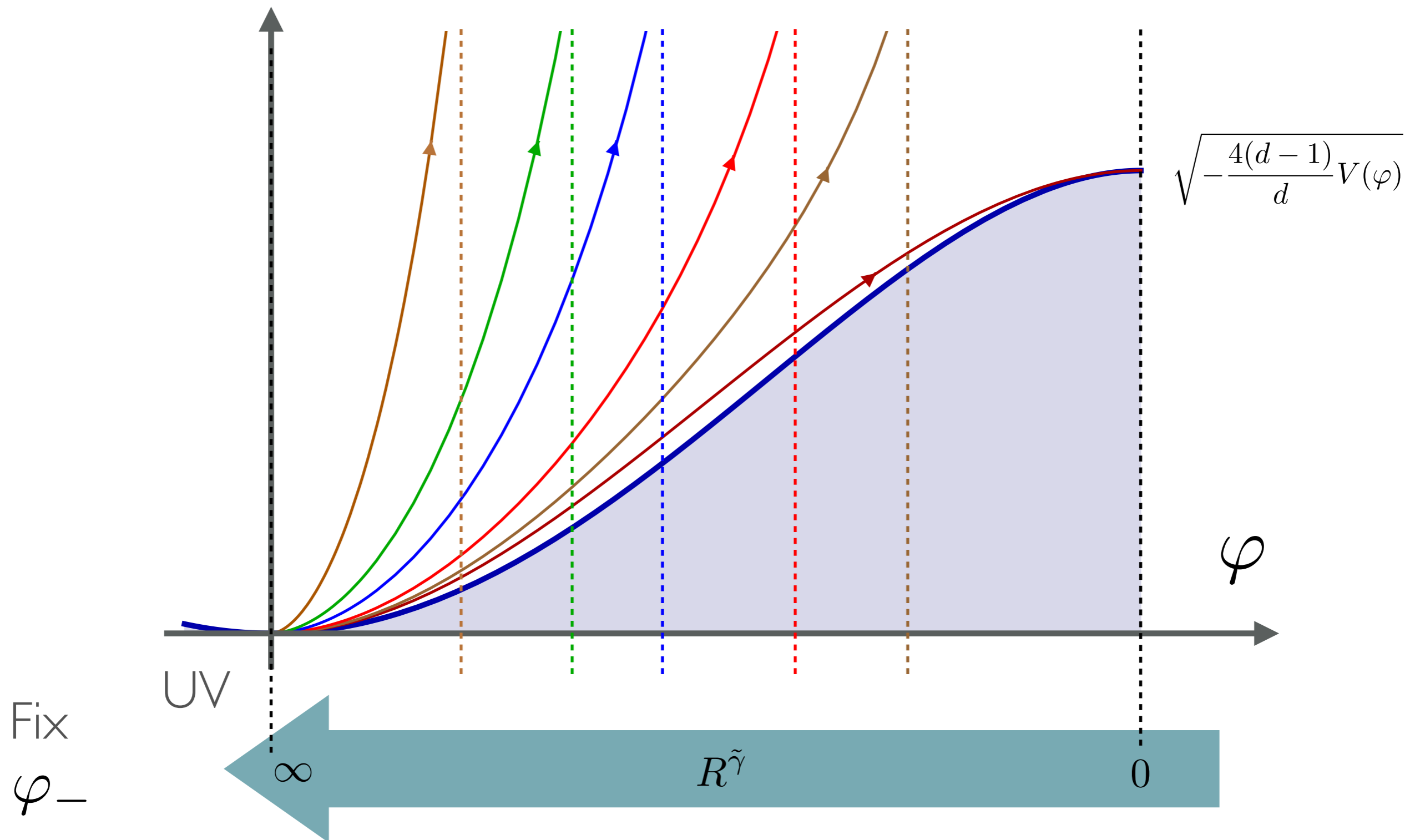
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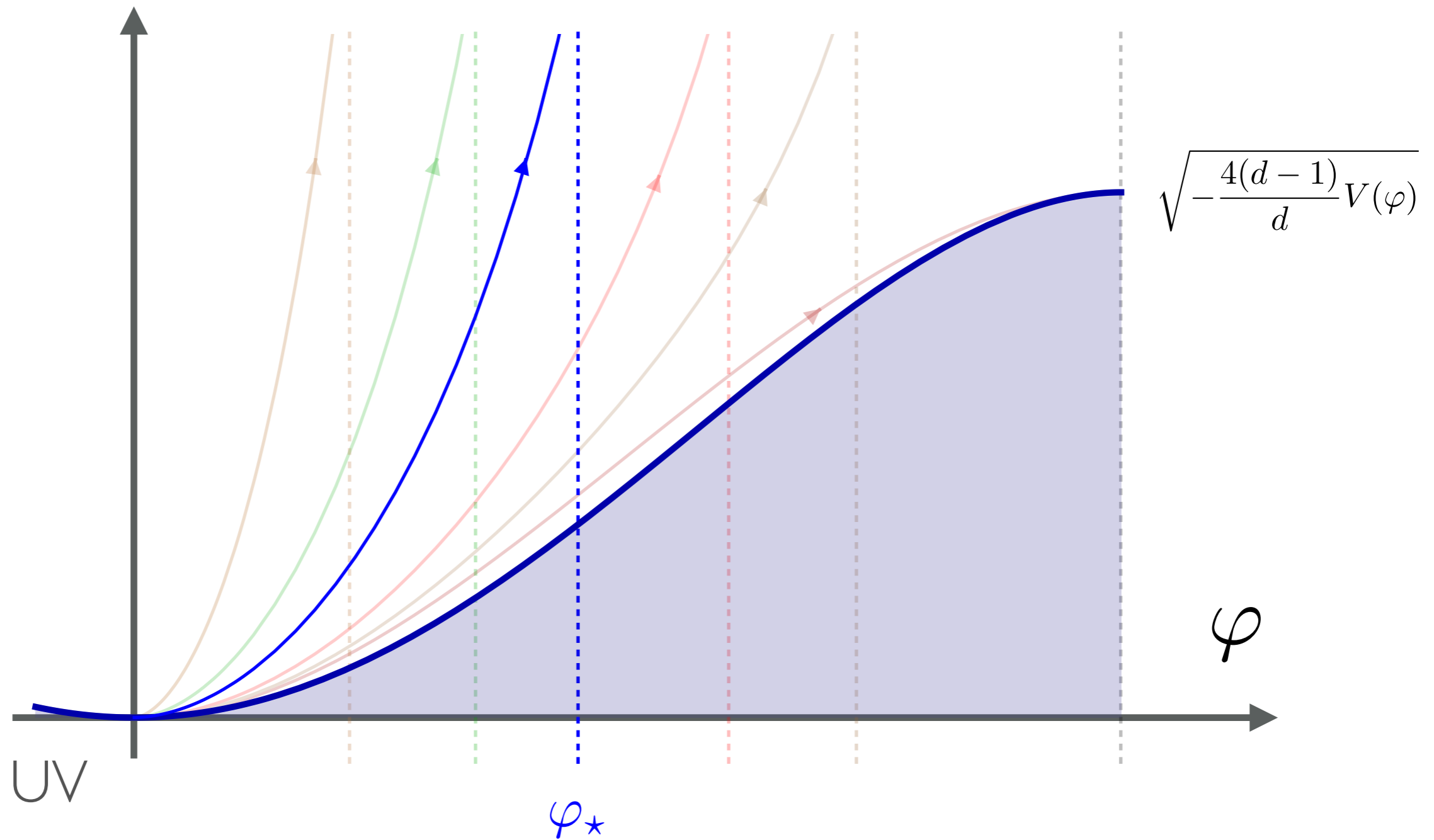
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Fix  $\phi_-$  and  $R^{\tilde{\gamma}}$

Solution selects  $C$ .



# Outlook

Holographic RG flows can exhibit **exotic** phenomena:

→ Talk by Leandro Silva Pimenta

- Fixed points can be skipped
- $\varphi$  can change direction along the flow can reverse (bounce)

These phenomena persist at finite curvature.

→ Talk by Jewel Kumar Ghosh

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The **F-theorem** is concerned with the change of an appropriately defined function along the RG flow for QFTs on a sphere in  $d=3$ .

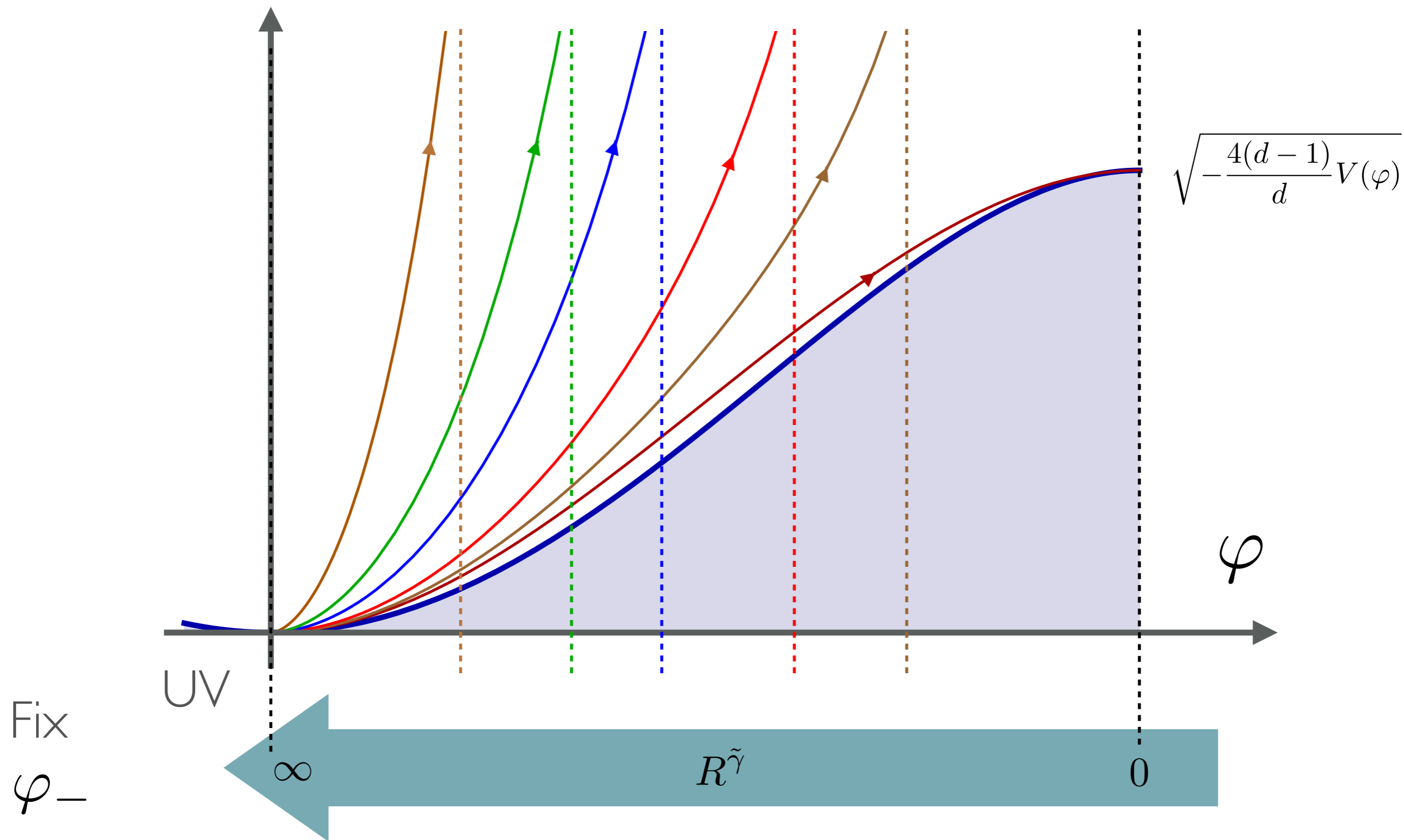
Test the F-theorem in a holographic setup.

[c.f. M.Taylor, W.Woodhead 2016]

**Many Thanks!**

# Many Thanks!

**Summary:** plot  $W(\phi)$  vs.  $\phi$  for various IR points  $\phi_*$ .



# Back-Up: Setup

## The model:

Consider a model with a **UV conformal fixed point** including

- a strongly coupled large- $N$  CFT, deformed by a relevant operator,
- the weakly coupled Standard Model fields, and
- some heavy messengers with mass scale  $\Lambda$ , coupling the first two.

Integrating out the messengers leaves as an EFT the (broken) CFT coupled to the SM, with some effective couplings set by  $\Lambda$ .

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## Semi-holographic description:

Strongly coupled  
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5d gravity with  
metric  $g_{ab}$  and  
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