Horizon fluff

A semi-classical approach to (BTZ) black hole microstates

Hamid R. Afshar

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Institute for Research in Fundamental Sciences (IPM)

In collaboration with ...

- Stephane Detournay (ULB)
- Daniel Grumiller, Wout Merbis (TU Wien)
- Blagoje Oblak (ETH)
- Alfredo Perez, David Tempo, Ricardo Troncoso (CECS Valdivia)
- Shahin Sheikh-Jabbari (IPM Tehran)
- Hossein Yavartanoo (ITP Beijing)

Summary of results

- New boundary conditions in AdS_3 gravity leading to new symmetry algebra. (two copies of $u(1)_k$ current algebra)

$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,0}\,.$$

- Improving the semi-classical symmetry to a quantum version describing microstates of BTZ in terms of coherent states of particles on AdS_3.
- Counting these microstaes (horizon fluff) reproduces BH entropy and its log correction.

Horizon fluff proposal

- Black hole microstates = horizon fluff: subset of near horizon soft hairs not distinguishable by the observers away from the horizon.
- Black hole: a state in the Hilbert space of asymptotic symmetries:

$$[L_n,\, L_m] = (n-m)L_{m+n} + \frac{k}{2}\, n^3\, \delta_{n+m,0}\,.$$

- Soft hairs: states in the Hilbert space of 'near horizon' algebra:

$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,0}\,.$$

- Duality map between asymptotic Hilbert space (BTZ) to the near horizon Hilbert space provides the required degeneracy (entropy).

Outline

1. Motivation
2. Brown-Henneaux boundary conditions
3. Near horizon boundary conditions
4. Quantization of (coherent) conical defects
5. Black hole microstates
6. Logarithmic correction to entropy
7. Conclusion

Motivation

Motivation

The **Bekenstein-Hawking** area law for black hole entropy is observer independent and is accessible through semiclassical considerations.

```
S = A/(4G) - q ln A/(4G) +  O(1)

  = ln ( # microstates )
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Universality of this result suggests that a statistical description of microstates in the thermodynamic limit does not need full knowledge of the underlying quantum theory.

Soft hair

- **Soft hair**: Zero-energy excitation with non-trivial charges. This notion was first introduced by [Hawking Perry Strominger '15]
- Diffeomorphic geometries which differ by their boundary behavior can be physically distinct, **asymptotic soft hairs**. The conserved charges associated with the diffeomorphisms relating them is non-zero and form an infinite dimensional algebra.

  `They do not appear in the S-matrix and are shaved off!`

  [Mirbabayi Porrati '16 Bousso Porrati 17']
- A black hole spacetime in particular can carry low-energy quantum excitations, **near horizon soft hairs**, providing a huge degeneracy to their vacuum.

  `They can amount for microstates of black holes!?`

  [HA Grumiller Sheikh-Jabbari Yavartanoo 16' 17']

Brown-Henneaux boundary conditions

Brown-Henneaux symmetries

All locally AdS_3 geometries obeying Brown–Henneaux b.c. fall into representation of asymptotic (simplectic) symmetry algebra which is two copies of Virasoro at Brown-Henneaux central charge $c^{\pm} = 6k = \frac{3\ell}{2G}$.
The Virasoro symmetries act on the phase space as;

$$\delta_{\epsilon_\pm} L_\pm = 2L_\pm \epsilon'_\pm + \epsilon'_\pm L_\pm - \epsilon'''_\pm/2\,.$$

The corresponding geometries with $L_\pm$ (BTZ black holes, conic spaces and global AdS_3 and their conformal descendants) are in a one-to-one correspondence with the coadjoint orbits of these symmetries:

$$\mathcal{H}_{\rm Vir} = \mathcal{H}_{\rm BTZ} \cup \underbrace{\mathcal{H}_{\rm Conic} \cup \mathcal{H}_{\rm gAdS}}_{\mathcal{H}_{\rm CG}}\,.$$

Near horizon boundary conditions

Near horizon boundary conditions

All locally AdS_3 geometries with **horizon** at $r = 0$ are parametrized by 4 **real** functions

$$\mathrm{d}s^2 \;\; = \;\; \mathrm{d}r^2 - \ell^2 \sinh^2 \frac{r}{\ell} \left[a\, dt - \omega d\varphi\right]^2 + \cosh^2 \frac{r}{\ell} \left[\Omega\, dt + \gamma d\varphi\right]^2$$

$$\partial_t J^\pm = \pm \partial_\varphi \zeta^\pm \, ; \qquad 2\zeta^\pm \equiv -a \pm \tfrac{\Omega}{\ell} \quad \text{and} \quad 2J^\pm \equiv \tfrac{\gamma}{\ell} \pm \omega \, .$$

Constant family:

$$J_+ = J_- = \pm J_0 \, , \quad L_0 = {J_0}^2$$

`BTZ black holes:` $J_0 \geq 0$

`Global AdS`$_3$`:` $J_0 = \frac{i}{2}$

`Conical defects:` $J_0 = \frac{i\nu}{2} \, , \quad \nu \in (0, 1)$

Comparison to Brown-Henneaux boundary conditions

For this set of boundary conditions as $r \to \infty$;

$$\begin{aligned} \delta g_{tt} &= \mathcal{O}\left(\frac{1}{r^2}\right) \\ \delta g_{\varphi\varphi} &= \mathcal{O}\left(r^2\right) \\ \delta g_{t\varphi} &= \mathcal{O}\left(1\right) . \end{aligned}$$

while for the Brown-Henneaux boundary conditions;

$$\begin{aligned} \delta g_{tt} &= \mathcal{O}\left(1\right) \\ \delta g_{\varphi\varphi} &= \mathcal{O}\left(1\right) \\ \delta g_{t\varphi} &= \mathcal{O}\left(1\right) . \end{aligned}$$

Near horizon boundary conditions

$$\begin{aligned} ds^2 &= dr^2 - \ell^2 \sinh^2 \frac{r}{\ell} \left[a\,dt - \omega d\varphi\right]^2 + \cosh^2 \frac{r}{\ell} \left[\Omega\,dt + \gamma d\varphi\right]^2 \\ &= dr^2 - (ar)^2\,dt^2 + \gamma^2\,d\varphi^2 + \mathcal{O}(r^2). \qquad \varphi \sim \varphi + 2\pi \end{aligned}$$

- Rindler space: Universal near horizon to any non-extremal horizon.

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- Rindler space: Universal near horizon to any non-extremal horizon.
- In the canonical description the Rindler acceleration a is fixed.
- AdS radius ℓ drops out of the near horizon line-element.
- All solutions have a regular horizon, regardless of the value of γ, ω as long as $a/(2\pi)$ is identified with the Unruh temperature.

Symmetries of the Near-Horizon

The most general transformation that preserves this boundary condition and also preserves the field equation, with $\delta\zeta = 0$, transforms J's as;

$$\delta_\eta J = \eta' \,.$$

Two copies of $u(1)_k$-algebra

$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,0} \,.$$

`How are these symmetries related to Brown-Henneaux?`

Check how J is transformed under conformal transformaition:

$$\delta_\epsilon J = (\epsilon J)' - \epsilon''/2 \quad \to \quad L = J^2 + J' \,.$$

$$\boldsymbol{L}_n \equiv \frac{6}{c} \sum_{p \in \mathbb{Z}} \boldsymbol{J}_{n-p} \boldsymbol{J}_p + in\boldsymbol{J}_n$$

Entropy and energy of BTZ black hole

From the asymptotic point of view;

$$S_{\mathrm{BH}} = \frac{A}{4G} = 2\pi\left(\sqrt{\frac{cL_0^+}{6}} + \sqrt{\frac{cL_0^-}{6}}\right), \qquad \mathtt{H}_{\mathrm{Asym}} = L_0^+ + L_0^-$$

From the near horizon point of view;

$$S_{\mathrm{BH}} = \frac{A}{4G} = 2\pi(\boldsymbol{J}_0^+ + \boldsymbol{J}_0^-) = \left(\mathtt{T}_{\mathrm{Rindler}}^{-1}\right)\mathtt{H}_{\mathrm{NH}}, \qquad \mathtt{T}_{\mathrm{Rindler}} = \frac{a}{2\pi}$$

The near horizon Hamiltonian $\mathtt{H}_{\mathrm{NH}} = a(\boldsymbol{J}_0^+ + \boldsymbol{J}_0^-)$ is the center of the near horizon algebra and assigns a same energy to all descendents (**soft hair**). The asymptotic Hamiltonian is not the center of the asymptotic (Virasoro) algebra. So the asymptotic observer only sees hard states.

Quantization of (coherent) conical defects

Wilson lines as primary fields

$$\boxed{L = J' + J^2 \quad \rightarrow \quad \delta_\epsilon J = (\epsilon J)' - \epsilon''/2}$$

- The fields $J(\phi)$ are primary fields if the anomalous term ϵ'' can be ignored that is for **black hole** sector ($J_0^\star = J_0$) with J being **large**;
- We can construct a new primary field $\mathcal{W}$;

$$\boxed{\mathcal{W}(\phi) = e^{-2\int^\phi J} \quad \rightarrow \quad \delta_\epsilon \mathcal{W} = (\epsilon \mathcal{W})'}$$

which is a good description for **conic spaces** ($J_0^\star = -J_0$) as the **periodicity** property suggests:

$$\mathcal{W}^\pm(\phi + 2\pi) = e^{\mp 4\pi J_0} \mathcal{W}^\pm(\phi)$$

Quantization of $\mathcal{W}$ fields

$$\mathcal{W} \equiv e^{-2\int^{\phi} J} = e^{-2\Phi_0 - 2J_0\phi + \cdots}$$

Quantization;

$$[\boldsymbol{\Phi}_0, \boldsymbol{J}_0] = i\frac{c}{12}\,, \qquad \langle \boldsymbol{\Phi_0} \rangle = \Phi_0$$

Using the appropriate mode expansion we get;

$$[\boldsymbol{J}_n, \boldsymbol{\mathcal{W}}^{\pm}_m] = -\,i\boldsymbol{\mathcal{W}}^{\pm}_{n+m}\,, (\forall n \neq 0), \qquad [\boldsymbol{J}_0, \boldsymbol{\mathcal{W}}^{\pm}_n] = \mp\, i\tfrac{c}{6}\boldsymbol{\mathcal{W}}^{\pm}_n \delta_{n,0}\,.$$

The $\boldsymbol{\mathcal{W}}$ operators are like coherent operators.

Free field realization of $\mathcal{W}$-fields

The"interaction terms" $\langle \mathcal{W}\mathcal{W}J\rangle$ and $\langle \mathcal{W}JJ\rangle$ are suppressed by factors of $1/c$. In the large c regime, the algebra can be closed as:

$$\begin{aligned}[\boldsymbol{\mathcal{W}}_n^{\pm\nu}, \boldsymbol{\mathcal{W}}_m^{\mp\nu}] &= \left(\frac{c}{12} n \mp \boldsymbol{J}_0\right) \delta_{n,-m} \\ &= \frac{c}{12}(n \pm \nu)\delta_{n,-m}\ , \qquad \nu \in (0,1)\end{aligned}$$

This gives a free field realization for $\mathcal{W}$-fields; In the large c limit, $\boldsymbol{\mathcal{W}}$-fields are a weekly coupled description of $\mathcal{H}_{\rm CG}$ as **gas of coherent particles on AdS$_3$.**

Near horizon soft hairs

- The commutation relation of $\mathcal{W}_n$'s takes a very simple form

$$[\mathcal{J}_n, \mathcal{J}_m] = \frac{n}{2}\delta_{n,-m}\,,$$

- The Fourier modes $\mathcal{J}_n$:

$$\mathcal{J}_{c(n+\nu)} \sim \mathcal{W}_n^{\nu}$$

- The vacuum state is

$$\mathcal{J}_n|0\rangle = 0, \qquad \forall n \geq 0.$$

Bohr-type quantization conditions

- ○ $c \in \mathbb{N} \;\rightarrow\;$ Chern-Simons level
- ○ $\nu = \frac{1}{c}, \frac{2}{c}, \cdots, 1 \;\rightarrow\;$ D1-D5 realization [Maldacena Maoz '00]

Virasoro algebra of near horizon

In parallel to the black hole sector we can have a Virasoro generator;

$$\boldsymbol{L}_n^r = \frac{6}{c}\sum_{p\in\mathbb{Z}} :\mathcal{W}_{n-p}^{-r}\mathcal{W}_p^r: + f_r\delta_{n,0}\,, \quad r = \nu c$$

The generators $L_n = \sum_{r=1}^{c} L_n^r$, can be written in terms of $\mathcal{J}_n$ modes;

$$\boldsymbol{L}_n = \frac{1}{c}\sum_{p\in\mathbb{Z}} :\mathcal{J}_{nc-p}\mathcal{J}_p: - \frac{1}{24c}\delta_{n,0}$$

They satisfy a Virasoro algebra at Brown-Henneaux central charge c.

Black hole microstates

State of a BTZ

Duality ($\frac{1}{c}\mathcal{L}_{nc} = \boldsymbol{L}_n$)

$$\frac{1}{c}\sum_{p\in\mathbb{Z}} :\mathcal{J}_{nc-p}\mathcal{J}_p: \; = in\boldsymbol{J}_n + \frac{6}{c}\sum_{p\in\mathbb{Z}} :\boldsymbol{J}_{n-p}\boldsymbol{J}_p: \,.$$

The two Hilbert spaces in $\mathcal{H}_{\mathrm{Vir}} = \mathcal{H}_{\mathrm{BTZ}} \cup \mathcal{H}_{\mathrm{CG}}$ are related.

$$\mathcal{H}_{\mathrm{CG}} \longleftrightarrow \mathcal{H}_{\mathrm{BTZ}}$$

A given AdS_3 black hole state:

$$|\mathrm{BTZ}\rangle \in \mathcal{H}_{\mathrm{BTZ}} \longleftrightarrow \texttt{micro-states} = \left| J_0^{\pm}; \{n_i^{\pm}\} \right\rangle \in \mathcal{H}_{\mathrm{CG}}$$

$$\langle \boldsymbol{L}_0^{\pm} \rangle_{\mathrm{BTZ}} = \tfrac{c}{6}(J_0^{\pm})^2 = \tfrac{1}{2}(\ell M \pm J)\,.$$

Horizon fluffs = Microstates $\in \mathcal{H}_{\text{CG}}$

$$\boxed{[\mathcal{J}_m, \mathcal{J}_n] = \frac{m}{2}\delta_{m+n,0}}$$

$$\boxed{|J_0; \{n_i\}\rangle = \mathcal{J}_{-n_i} \cdots \mathcal{J}_{-n_2}\mathcal{J}_{-n_1} |0; \{n_i\}\rangle \ , \quad \forall n_i > 0}$$

So $\left|J_0^{\pm}; \{n_i^{\pm}\}\right\rangle$ describes a black hole state if;

$$\boxed{L_0^{\pm} = \tfrac{1}{2}(\ell M \pm J) = \tfrac{1}{c}\sum_i n_i^{\pm}}$$

Mathematically, this reduces to **Hardy and Ramanujan** combinatorial problem: the number of ways a positive integer N can be partitioned into non-negative integers in the limit of large N;

$$p(N) \simeq \frac{1}{4N\sqrt{3}} exp\left(2\pi\sqrt{\frac{N}{6}}\right) \ , \qquad N \gg 1 \, .$$

Entropy

Microcanonical entropy as logarithm of the number of states:

$$S_0 = \ln p(cL_0^+) + \ln p(cL_0^-) = 2\pi \left(\sqrt{\frac{cL_0^+}{6}} + \sqrt{\frac{cL_0^-}{6}} \right)$$

Reminding $L_0^{\pm} = \frac{6}{c}(\boldsymbol{J}_0^{\pm})^2$, the entropy is;

$$S_0 = 2\pi \left(\boldsymbol{J}_0^+ + \boldsymbol{J}_0^- \right) = \frac{A}{4G}$$

Logarithmic correction to entropy

Micro-canonical entropy

$$\boxed{S_0 = 2\pi\left(\boldsymbol{J}_0^+ + \boldsymbol{J}_0^-\right) = \frac{A}{4G}}$$

The logarithmic correction:

$$S = S_0 - 2\,\ln S_0 + \ldots,$$

The microcanonical entropy is obtained through replacing J_0 with

$$J_0 = \frac{k}{2\pi}\Big\langle \int_0^{2\pi} \mathrm{d}\phi\, J(\phi)\Big\rangle_{\text{mic}} \to J_0 + \frac{1}{2}\ln J_0\,.$$

Consequently we find the exact match for the log correction in the mic-canonical ensemble for BTZ black holes;

$$S_{\text{mic}} = = S_{\text{BH}} - \tfrac{3}{2}\,\ln S_{\text{BH}} + \ldots.$$

Conclusion

Summary

- The horizon fluff proposal is a semi-classical proposal to construct BTZ microstates. It reproduces the **Bekenstein-Hawking** entropy and also the **logarithmic corrections** to it.
- We assumed some basic "Bohr-type" quantization on central charge and the dificit angle.
- We proposed a black-hole/particle correspondence; states in $\mathcal{H}_{\mathrm{BTZ}}$ are certain coherent staes in $\mathcal{H}_{\mathrm{CG}}$.

$\varepsilon\upsilon\chi\alpha\iota\sigma\tau\omega$

Backup slides

All Locally AdS_3 geometries obeying Brown–Henneaux b.c.:

$$ds^2 = \ell^2 \frac{dr^2}{r^2} - r^2 \Big(dx^+ - \frac{\ell^2 L_-(x^-)}{r^2} dx^- \Big) \Big(dx^- - \frac{\ell^2 L_+(x^+)}{r^2} dx^+ \Big)$$

$$L_\pm(x^\pm + 2\pi) = L_\pm(x^\pm) , \qquad x^\pm = t/\ell \pm \phi, \qquad \phi \in [0, 2\pi].$$

Constant family:

$$L_+ = L_- = L_0$$

`BTZ black holes:` $L_0 \geq 0$

`Global AdS`$_3$`:` $L_0 = -1/4$

`Conical defects:` $-1/4 < L_0 < 0$