Out of equilibrium dynamics and Robinson-Trautman spacetimes

Kostas Skenderis

STAG RESEARCH CENTRE

University of Southampton

NINTH CRETE REGIONAL MEETING IN STRING THEORY
In memoriam: Ioannis Bakas
Kolymbari, July 13, 2017

Outline

Robinson-Trautmann spacetimes

- The Robinson-Trautmann (RT) spacetimes are solutions of Einstein equations in $d = 4$ [I. Robinson and A. Trautman, (1960)]
- They describe the gravitational field due to a compact star which relaxes to equilibrium by radiating away its excess energy and asymmetry.
- In recent times they have been used to model qualitative features of mergers of black holes.

RT spacetimes and SUSY quantum mechanics

- At late times the solution tends to the Schwarzschild solution.
- The leading deviation from Schwarzschild is a very special linearized perturbation of Schwarzschild: the so-called algebraically special mode [Chandrasekhar].
- Perturbations of Schwarzschild are organized according to an underlying supersymmetric quantum mechanics.
- The algebraically special modes are the corresponding supersymmetric ground states (zero energy states).

➠ The Robinson-Trautman solution is a non-linear version of the algebraically special perturbations of Schwarzschild.

Robinson-Trautmann spacetimes and holography

- The solution exists for any value of the cosmological constant.
- With negative cosmological constant, the solution is a (rare example of a) time-dependent asympotically locally AdS solution.
- At late times, it approaches the AdS-Schwarzschild solution.
- It provides a laboratory for exploring out-of-equilibrium dynamics and the approach to equilibrium using gauge/gravity duality.

References

- This talk is based on work done with Ioannis Bakas and Ben Withers:
  - I. Bakas and K. Skenderis, Non-equilibrium dynamics and AdS4 Robinson-Trautman, JHEP 1408 (2014) 056
  - I. Bakas, K. Skenderis and B. Withers, Self-similar equilibration of strongly interacting systems from holography, Phys.Rev. D93 (2016) no.10, 101902.
  - K. Skenderis and B. Withers, Robinson-Trautman spacetimes and gauge/gravity duality, 1703.10865.
- Related work appeared in [G. de Freitas, H. Reall, 1403.3537]

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Robinson-Trautman spacetimes

- The metric is given by

$$ds^2_{RT} = -F du^2 - 2du dr + \frac{r^2}{\sigma^2} d\Sigma_k^2$$

with $d\Sigma_k^2$ a constant curvature metric ($R_k = 2k$, $k = 0, \pm 1$) .

- The function F is uniquely determined in terms of σ,

$$F \equiv -\frac{\Lambda}{3} r^2 - 2r \frac{\partial_u \sigma}{\sigma} + \frac{R_g}{2} - \frac{2m}{r}.$$

where Λ is related to the cosmological constant and R_g is the curvature of $d\Sigma_k^2/\sigma^2$.

- $\sigma(x^a, u)$ should solve the *Robinson-Trautman equation*,

$$12 m \partial_u \sigma^2 + 2\sigma^4 \nabla^2_{\Sigma_k} \sigma^2 + \sigma^4 \nabla^2_{\Sigma_k} \left(\sigma^2 \nabla^2_{\Sigma_k} \log \sigma^2 \right) = 0.$$

Robinson-Trautman equation and the Calabi flow

- The Robinson-Trautman equation coincides with the Calabi equation,

$$\partial_u g_{ab} = \frac{1}{12m} \nabla_g^2 R_g \, g_{ab}.$$

which describes a class of deformations of the $2d$ metric

$$ds_2^2 = \frac{1}{\sigma^2} d\Sigma_k^2$$

- The Calabi flow is defined more generally for a metric $g_{a\bar{b}}$ on a Kähler manifold M by the Calabi equation

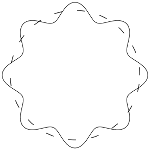
$$\partial_u g_{a\bar{b}} = \frac{\partial^2 R}{\partial z^a \partial z^{\bar{b}}}$$

where R is the curvature scalar of g.

- It provides volume preserving deformations within a given Kähler class of the metric.

Calabi flow on M^2

- The Calabi flow can be regarded as a non-linear diffusion process.
- The steady state solutions are constant curvature metrics (up to conformal transformations).
- For compact M^2 the flow monotonically deforms the metric to the steady state solution.
- For non-compact M^2 it seems there is no full classification of the late time behaviour.



AdS Schwarzschild as Robinson-Trautman

- Using the fixed point solution for the case of S^2

$$\frac{1}{\sigma_0^2} = \frac{1}{(1 + z\bar{z}/2)^2} \; .$$

the metric becomes

$$ds^2 = \frac{2r^2}{(1 + z\bar{z}/2)^2} dz d\bar{z} - 2dudr - \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right) du^2$$

which is the Schwarzschild metric in the Eddington - Finlkenstein coordinates.

- Similarly, one obtains the AdS brane brane and the hyperbolic AdS black hole in the other two cases.

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RT as an Asymptotically locally AdS solution

- When $\Lambda < 0$ the solution is asymptotically locally AdS. This means that near the conformal boundary the metric takes the Fefferman-Graham form:

$$ds^2 = \frac{d\varrho^2}{\varrho^2} + \frac{1}{\varrho^2}\left(g_{(0)ab}(x) + \varrho^2 g_{(2)ab}(x) + \varrho^3 g_{(3)ab}(x) + \cdots\right) dx^a dx^b$$

- One can reach this gauge by a coordinate transformation $(r^* = u - t, t, z, \bar{z}) \to (\varrho, t, z, \bar{z})$:

$$\begin{aligned} r_\star &\to \varrho + \qquad\qquad (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \\ t &\to t + (\quad)\varrho^2 + (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \\ z &\to z + \qquad\qquad (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \\ \bar{z} &\to \bar{z} + \qquad\qquad (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \end{aligned}$$

Asymptotic structure

- The boundary metric is time-dependent and it is not conformally flat

$$ds_0^2 = -dt^2 + \frac{L^2}{\hat{\sigma}^2} d\Sigma_k^2$$

where $\hat{\sigma}(x^a, t) = \sigma(x^a, u = t - r^\star)|_{r^\star=0}$ and $L^2 = -3/\Lambda$.

- $g_{(2)ab} = -\mathcal{R}_{ab} + \frac{1}{4}\mathcal{R} g_{(0)ab}$, where $\mathcal{R}_{ab}$ is the Ricci tensor of $g_{(0)}$, as expected [de Haro, Solodukhin, KS].

Energy-momentum tensor

- Asymptotically locally AdS spacetime come equipped with a conserved T_{ab} which in even dimension is traceless,

$$\nabla^b T_{ab} = 0, \qquad T^a_a = 0$$

- The tensor can be extracted from the asymptotics of the solution [Henningson, KS][de Haro, Solodukhin, KS]

$$T_{ab} = -\frac{3}{2\kappa^2}\left(-\frac{3}{\Lambda}\right) g_{(3)ab}$$

- For example, for the case of S^2 we obtain,

$$\kappa^2 T_{tt} = -\frac{2m\Lambda}{3}, \qquad \kappa^2 T_{tz} = -\frac{1}{2}\partial_z(\hat{\Delta}\hat{\Phi})$$
$$\kappa^2 T_{z\bar{z}} = m e^{\hat{\Phi}}, \qquad \kappa^2 T_{zz} = -\frac{3}{4\Lambda}\partial_t\left((\partial_z\hat{\Phi})^2 - 2\partial_z^2\hat{\Phi}\right),$$

where $\hat{\Phi} = -\log\hat{\sigma}$

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Hydrodynamics

- On general grounds, sufficiently close to equilibrium one expects a hydrodynamic description.
- At sufficient late times, the holographic energy momentum tensor of RT spacetimes should take a hydrodynamic form,

$$T^{ab} = \rho u^a u^b + p\Delta^{ab} - \eta\sigma^{ab}$$

... and indeed it does.

Out of equilibrium

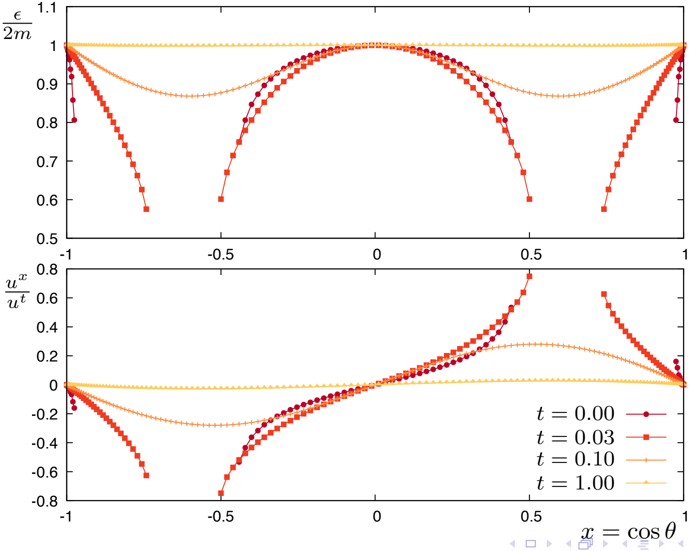
- Is there always a notion of local energy density?
- How does the system equilibrate?

Local energy density

- The local energy ϵ is defined as the eigenvalue of the energy momentum tensor corresponding to a timelike eigenvector:

$$T^{\mu}{}_{\nu} u^{\nu} = -\epsilon u^{\mu} \qquad \text{with} \quad u^{\mu} u_{\mu} = -1$$

  This defines also the local velocity field u^{μ}.
- One can show analytically that for RT solution, a positive local energy exist **to all orders** in the late time expansion of the bulk metric.
- What about early times?
- If the non-equilibrium state is sufficiently close to the thermal state, then there exist a local energy density also at early times.
- Otherwise, a local energy density may not exist.


$\frac{\epsilon}{2m}$
1.1
1
0.9
0.8
0.7
0.6
0.5
-1
-0.5
0
0.5
1
$\frac{u^x}{u^t}$
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
$t = 0.00$
$t = 0.03$
$t = 0.10$
$t = 1.00$
$x = \cos\theta$

Pressures and strain

If u^μ exists, we construct two vectors, $n_I^\mu (I = 1, 2)$,

$$n_I \cdot n_J = \delta_{IJ} \qquad u \cdot n_I = 0,$$

and use them to define two pressures and the strain

$$p_1 = n_1 \cdot T \cdot n_1 \qquad p_2 = n_2 \cdot T \cdot n_2, \qquad t = n_1 \cdot T \cdot n_2$$

For axially symmetric solutions $t = 0$.

Approach to equilibrium

- ‘Hydrodynamisation’ time, t_{hyd}:

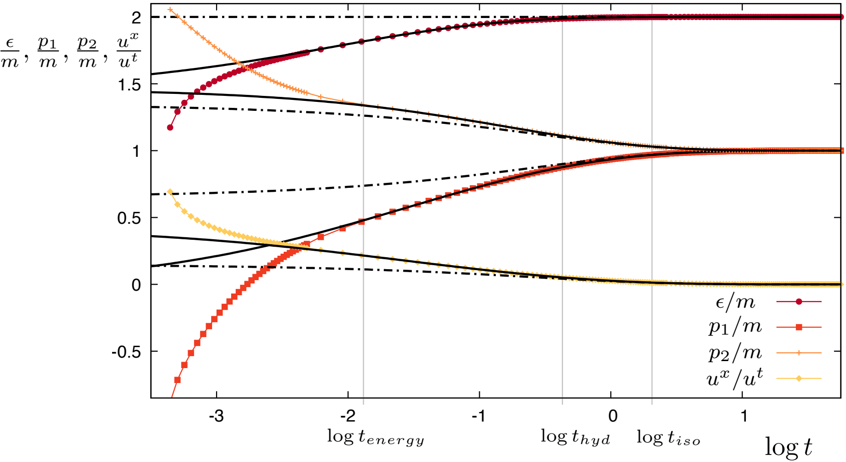
$$\max_x \left| \frac{p_I - p_I^{(1)}}{\bar{p}} \right| < \frac{1}{10}, \qquad (t_{hyd})$$

- Energy equilibration time, t_{energy}:

$$\max_x \left| \frac{\epsilon - \epsilon_{eq}}{\epsilon} \right| < \frac{1}{10}. \qquad (t_{energy})$$

- Isotropisation time, t_{iso},

$$\max_x \left| \frac{p_1 - p_2}{\bar{p}} \right| < \frac{1}{10}. \qquad (t_{iso})$$



Black dashed lines: leading late time contribution.

Solid black lines: nonlinear completion to order 15 in the late time expansion.

Outline

Heat transport

- Suppose that we have two semi-infinite rods, each at different temperature $T_\pm$ and we join them at $t = 0$.
- How does this system will evolve?

Conventional material

- In conventional materials the evolution will be governed by the heat equation,

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}.$$

- This equation is invariant under the scaling, $x \to \lambda^2 x, t \to \lambda t$.
- There is a self-similar solution $h(\mu(t,x))$, where $\mu = x/t^{1/2}$,

$$h = a + b\,\text{Erf}\left(\frac{\mu}{2\sqrt{D}}\right)$$

(a, b are integration constants), which is a late-time attractor.

Strongly interacting systems

- The RT solution can be used to study the same problem for strongly interacting system via gauge/gravity duality.
- We will work with RT solution on R^2, with coordinates x, y.
- We impose translational invariance in y.
- We impose that the RT solution tends to the AdS black branes with different temperatures, $T_{\pm}$, as we go to $x \to \pm\infty$ in the other direction.

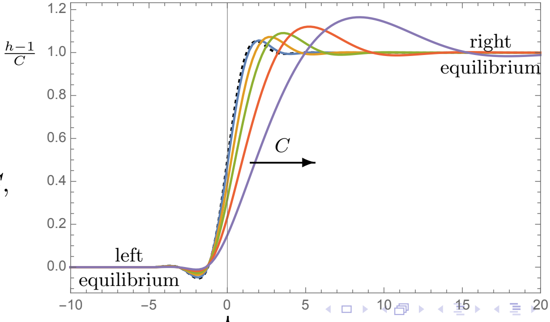
Scaling solutions

- The RT metric is invariant under the following scaling

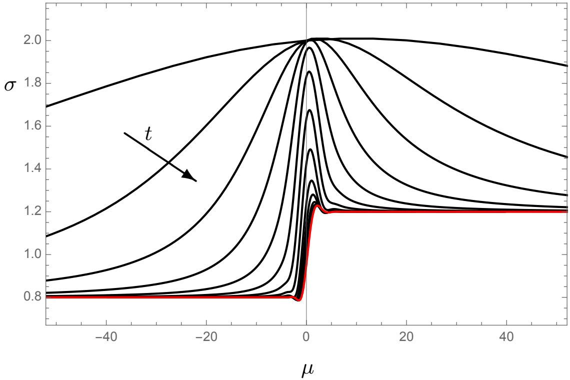
$$\begin{aligned} u &\to \lambda_u u, \quad x \to \lambda_x x, \quad y \to \lambda_x y \\ r &\to \lambda_u^{-1} r, \quad m \to \lambda_u^{-3} m, \quad \sigma \to \lambda_x \lambda_u^{-1} \sigma. \end{aligned}$$

- There is a scaling solution $h(\mu(x,t))$, where $\mu(t,x) = x/t^{1/4}$.

$$h(R=-1)=1,$$
$$h(R=1)=1+C,$$
$$R=\tanh\mu$$



Attractor behaviour: general initial conditions



Evolution of the initial data showing convergence to the planar similarity solution in red. Each curve shows a different time in the evolution.

Comments

- We holographically engineered 2+1 dimensional systems which at $t = 0$ are described by two different thermal states infinitely separated in one direction.
- The final state is a self-similar solution which only depends on the left and right temperatures and not on the details of the initial conditions.
- The self-similar solutions are Lifshitz invariant and perturbations around them are governed by the spectrum of operators of an underlying Lifshitz critical theory.
- Our discussion should thus be applicable to all systems which are in the same universality class with this critical point.

Outline

Conclusions

- The Robinson-Trautman solution is an interesting laboratory to study out-of-equilibrium dynamics.

- We discussed two studies:
  1. existence of local rest frame and approach to equilibrium
  2. heat transport

- These solutions exhibit many other interesting properties:
  1. One can formulate and prove a Penrose inequality and a version of the hoop conjecture [Bakas, KS].
  2. They represent the only example, where a 3d geometric flow is embedded in 4d Einstein gravity.
  3. ....