Interacting current algebra theories

Konstantinos Sfetsos
National and Kapodistrian U. of Athens

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Based on work with:

- G. Georgiou, G. Itsios, K. Siampos, D.C. Thompson and E. Sagkrioti

Motivation

- Exact beta-function and anomalous dimensions in quantum field theories.
  Traditionally these are computed perturbatively. It is a rare occasion to be able to compute them exactly.
- Systematic construction of new (integrable) deformations of (interacting) CFT's having explicit Lagrangian descriptions.
- Smoth RG flows (UV to IR) between CFTs.
- In conjunction with type-II supergravity embeddings use them in an AdS/CFT context.

Methods for constructing integrable theories from perturbing CFTs exist, i.e. [A.B. Zamolodchikov 89].
The emphasis is on the Lagrangian formulation, lacking in previous works.

Outline

- The theories of interest
- Construction of effective actions (self- and mutual-interacting)
- Exact beta-functions and anomalous dimensions.
  Some perturbative info plus symmetry and analyticity considerations lead to exact results.
- Concluding remarks

The theories of interest

Self-interacting theories

Let any 2-dim CFT with action S_k and a group G structure having holomorphic & anti-holomorphic currents $J^a(z)$ & $\bar{J}^a(\bar{z})$, obeying

$$J^a(z)J^b(w) = \frac{\delta_{ab}}{(z-w)^2} + \frac{f_{abc}}{\sqrt{k}}\frac{J^c(w)}{z-w} + \cdots$$

a nd similarly for the $\bar{J}^a(\bar{z})$'s.

We would be interested in:

- Study the theory away from the conformal point driven by a self-interaction current bilinears

$$S_{k,\lambda} = S_k - \frac{k}{\pi}\int d^2z\ \lambda_{ab}J^a\bar{J}^b\ .$$

In particular:

- To compute the RG flow eqs

$$\frac{1}{2}\mu\frac{d\lambda_{ab}}{d\mu} = \cdots \ .$$

- The currents' anomalous dims, as functions of λ_{ab} and k, as well as the anomalous dims of all operators.
- Search for new fixed points under the RG flow towards the IR.
- We would like to do that exactly in λ and in k, unlike traditional approaches.
- Construct effective, all loop in λ, actions.

Mutually-interacting theories

- Study two decoupled theories away from the conformal point driven by mutual-interactions of current bilinears

$$S_{k_1,k_2,\lambda_1,\lambda_2} = S_{k_1} + S_{k_2} - \frac{1}{\pi}\int d^2z \, \left(k_1\lambda_1^{ab} J_1^a \bar{J}_2^b + k_2\lambda_2^{ab} J_2^a \bar{J}_1^b\right) .$$

- Certain features different, i.e. when $k_1 \neq k_2$ a new fixed point in the IR. Identity the CFT.

Effective actions

Self-interacting theories [KS 13]

The starting point is the action

$$S(g,\tilde{g}) = S_{WZW,k}(g) + S_{\mathrm{PCM}}(\tilde{g}) \ .$$

- $S_{WZW,k}(g)$ is the WZW action for $g \in G$. This is a CFT; has a $G_{\mathrm{L,cur}} \times G_{\mathrm{R,cur}}$ current algebra symmetry.
- $S_{\mathrm{PCM}}(\tilde{g})$ is the PCM action for $(\tilde{g} \in G)$ with coupling κ^2

$$S_{\mathrm{PCM}}(\tilde{g}) = -\frac{\kappa^2}{\pi} \int \mathrm{Tr}(\tilde{g}^{-1}\partial_+\tilde{g}\tilde{g}^{-1}\partial_-\tilde{g}) \ .$$

  It is integrable with global $G_L \times G_R$ symmetry.
- We will gauge the group acting as

$$g \to \Lambda^{-1} g \Lambda \ , \qquad \tilde{g} \to \Lambda^{-1}\tilde{g} \ , \qquad \Lambda \in G \ .$$

Hence we consider the action

$$S_{k,\kappa^2}(g,\tilde{g}) = S_{\mathrm{g}WZW,k}(g,A_\pm) + S_{\mathrm{g}PCM}(\tilde{g},A_\pm) \ ,$$

- Choose the gauge fixing: $\tilde{g} = \mathbb{1}$.
- Integrating out the gauge fields we obtain the action

$$\boxed{S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k}{\pi}\int J_+^a(\lambda^{-1}\mathbb{I} - D^T)^{-1}_{ab} J_-^b} \ ,$$

where

$$J_+^a = -i\mathrm{Tr}(t^a\partial_+ gg^{-1}) \ , \quad J_-^a = -i\mathrm{Tr}(t^a g^{-1}\partial_- g) \ , \quad D_{ab} = \mathrm{Tr}(t_a g t_b g^{-1}) \ .$$

and

$$\lambda = \frac{k}{k+\kappa^2} \ .$$

is the deformation parameter.

- Generalization to $\lambda\delta_{ab} \to \lambda_{ab}$ straightforward.
- Remark: Non-Abelian T-duality is a zoom-in limit as $\lambda \to 1$.

- For small λ_{ab}

$$S_{k,\lambda}(g) = S_{\mathrm{WZW},k}(g) + \frac{k}{\pi} \int d^2\sigma \ \lambda_{ab} J_+^a J_-^b + \dots \ .$$

- This action has a duality-type symmetry [Itsios-KS-Siampos 14]

$$k \to -k \ , \qquad \lambda \to \lambda^{-1} \ , \qquad g \to g^{-1} \ .$$

- This symmetry was found using path integral arguments for the linear action [Kutasov 89]!
- It should be reflected as a symmetry in physical quantities.

Exact β-function and anomalous dims [Georgiou-Siampos-KS 15 & 16]

CFT and symmetry appoach

An integrable theory [KS 13, Hollowood-Miramontes-Schmidtt 14]

$$\lambda_{ab} = \lambda \delta_{ab} \ .$$

- We want to compute the 2-point functions

$$\langle J^a(x_1) J^b(x_2) \rangle_\lambda = \langle J^a(x_1) J^b(x_2) e^{-\frac{\lambda}{\pi}\int d^2 z J^a(z) \bar{J}^a(\bar{z})} \rangle \ ,$$
$$\langle J^a(x_1) \bar{J}^b(x_2) \rangle_\lambda = \langle J^a(x_1) \bar{J}^b(x_2) e^{-\frac{\lambda}{\pi}\int d^2 z J^a(z) \bar{J}^a(\bar{z})} \rangle \ ,$$

  perturbatively in λ by expanding the exponential.
- The basic correlators are

$$\langle J^a(x_1) J^b(x_2) \rangle = \frac{\delta_{ab}}{x_{12}^2} \ , \quad \langle J^a(x_1) J^b(x_2) J^c(x_3) \rangle = \frac{1}{\sqrt{k}} \frac{f_{abc}}{x_{12} x_{13} x_{23}}$$

  and similarly for the $\bar{J}^a$'s. Mixed $J\bar{J}$ correlators vanish.
- For higher correlators use Ward dentities

The perturbative β-function and anomalous dimensions

- The β-function is

$$\beta = \frac{1}{2}\mu\frac{d\lambda}{d\mu} = -\frac{c_G}{2k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) \ ,$$

  where c_G is the quadratic Casimir in the adjoint.

- The anomalous dimension of the currents is

$$\gamma^{(J)} = \mu\frac{d\ln Z^{1/2}}{d\mu} = \frac{c_G}{k}\left(\lambda^2 - 2\lambda^3 + \mathcal{O}(\lambda^4)\right) \ .$$

Task: Extend these exactly in λ?

Analyticity: λ-dependence of physical quantities

- Expand the action for $g = e^{ix^a t^a}$ around the identity

$$S_{k,\lambda} = \frac{k}{4\pi}\frac{1+\lambda}{1-\lambda}\int \partial_+ x^a \partial_- x^a + \cdots$$

- The β-function & anomalous dims may have poles at $\lambda = \pm 1$.
- The β-function & anomalous dims should be invariant under

$$k \to -k\ , \qquad \lambda \to \frac{1}{\lambda}\ ,$$

  for $k \gg 1$.
- Perturbative information to $\mathcal{O}(\lambda^2)$ and the above symmetry are enough to determine the β-function and the anomalous dimensions exactly in λ and to leading order in k.

The exact β-function and anomalous dimensions

The exact β-function and anomalous dimensions are of the form

$$\beta_\lambda = -\frac{c_G}{2k}\frac{f(\lambda)}{(1+\lambda)^2} , \qquad \gamma^{(J)} = \frac{c_G}{k}\frac{g(\lambda)}{(1-\lambda)(1+\lambda)^3} ,$$

where $f(\lambda)$ and $g(\lambda)$ are analytic in λ.

- They have a well defined non-Abelian and pseudodual limits.
- Due to the symmetry $(k,\lambda) \mapsto (-k,\lambda^{-1})$ we have that

$$\lambda^4 f(1/\lambda) = f(\lambda) , \qquad \lambda^4 g(1/\lambda) = g(\lambda) .$$

- $f(\lambda)$ and $g(\lambda)$ are polynomials of degree four, fixed by the above symmetry and the two-loop perturbative result.

- The final result for the beta-function is

$$\beta_\lambda = -\frac{c_G}{2k}\frac{\lambda^2}{(1+\lambda)^2} \leqslant 0$$

In agreement with [Kutasov 89] and [Gerganov-LeClair-Moriconi 01].

- For the anomalous dimension

$$\gamma^{(J)} = \frac{c_G}{k}\frac{\lambda^2}{(1-\lambda)(1+\lambda)^3} \geqslant 0 \ .$$

Agreement with perturbation theory to $\mathcal{O}(\lambda^3)$ and $\mathcal{O}(\lambda^4)$.

- Similarly for 3-point fucntions [Georgiou-KS-Siampos 16]

Gravitational approach

The one-loop β-functions are [Ecker-Honerkamp 71, Friedan 80, Braaten-Curtright-Zachos 85, Fridling-van de Ven 86]

$$\frac{\mathrm{d}G_{\mu\nu}}{\mathrm{d}t} + \frac{\mathrm{d}B_{\mu\nu}}{\mathrm{d}t} = R^{-}_{\mu\nu} + \nabla^{-}_{\nu}\xi_{\mu} \ , \quad t = \ln \mu^2 \ .$$

- The covariant derivatives and tensors include the torsion.
- If $G_{\mu\nu}$ and $B_{\mu\nu}$ retain their form, what flows with energy are the couplings λ_{ab}. The ξ^{μ}'s are diffs.
- Then the RG-flow eqs are [KS-Siampos 14]

$$\frac{d\lambda_{ab}}{dt} = \frac{1}{2k}\mathrm{Tr}\left(\mathcal{N}_a(\lambda)\mathcal{N}_b(\lambda^T)\right) \ .$$

were the matrices $\mathcal{N}_a(\lambda)$ have elements

$$(\mathcal{N}_a(\lambda))_b{}^c = (\lambda_{ae}\lambda_{bd}f_{edf} - f_{abe}\lambda_{ef})\, g^{fc} \ , \quad g_{ab} = (\mathbb{1} - \lambda^T\lambda)_{ab} \ .$$

- For $\lambda_{ab} = \lambda\delta_{ab}$ agreement with CFT approach.

Correlators of primary fields

The CFT contains affine primary fields $\Phi_{i,i'}(z,\bar{z})$ which are also Virasoro primaries [Knizhnik-Zamolodchikov 84]

$$\Delta_R = \frac{c_R}{2k+c_G}\ , \qquad \bar{\Delta}_{R'} = \frac{c_{R'}}{2k+c_G}\ ,$$

in the reps R and R' for left and right, with matrices t_a and $\tilde{t}_a$.

- We find the anomalous dimensions [Georgiou-KS-Siampos 16]

$$\gamma^{(I)}_{R,R'}(k,\lambda) = \frac{1}{2k(1-\lambda^2)}\left(c_R + \lambda^2 c_{R'} - 2\lambda N_I\right)\ .$$

  and N_I are the eigenvalues $t_a \times \tilde{t}_a^*$.

- Note that

$$\gamma^{(I)}_{R,R'}(-k,\lambda^{-1}) = \gamma^{(I)}_{R',R}(k,\lambda)\ .$$

Mutually-interacting theories [Georgiou-KS 16,17]

Modifying the gauging procedure two effective actions.
One with two and one with a single deformation matrice.
For $\lambda_{i,ab} = \lambda_i \delta_{ab}$, $i = 1, 2$ integrability.

Single matrix (easier to present). Then

$$S_{k_1,k_2,\lambda}(g_1, g_2) = S_{k_1}(g_1) + S_{k_2}(g_2) + \frac{\sqrt{k_1 k_2}}{\pi} \int d^2\sigma \lambda_{ab} J^a_{1+} J^b_{2-} .$$

- Beta-function for $\lambda_{ab} = \lambda \delta_{ab}$

$$\frac{d\lambda}{dt} = -\frac{c_G}{2\sqrt{k_1 k_2}} \frac{\lambda^2(\lambda - \lambda_0)(\lambda - \lambda_0^{-1})}{(1-\lambda^2)^2} .$$

  A new fixed point in the IR at $\lambda = \lambda_0 = \sqrt{\frac{k_1}{k_2}}$.

- Expressions for anomalous dimensions also exist.

Smooth RG flows

- At the IR fixed point at $\lambda = \lambda_0$ the action becomes

$$S_{\mathrm{IR}} = S_{k_1}(g_2 g_1) + S_{k_2-k_1}(g_2) \ .$$

- Therefore under the RG flow

$$G_{k_1} \times G_{k_2} \quad \overset{\mathrm{IR}}{\Longrightarrow} \quad G_{k_1} \times G_{k_2-k_1} \ ,$$

  In accordance with Zamolodchikov's c-theorem, since $c_{\mathrm{IR}} < c_{\mathrm{UV}}$.

Remark: For the case of two deformation matrices the RG flow is

$$G_{k_1} \times G_{k_2} \quad \overset{\mathrm{IR}}{\Longrightarrow} \quad \frac{G_{k_1} \times G_{k_2-k_1}}{G_{k_2}} \times G_{k_2-k_1} \ ,$$

For $G = SU(2)$ this argued to describe a theory of interacting fermions [Andrei-Douglas-Jerez 99]

Concluding remarks

- Computed exactly the beta-function and anomalous dimensions of operators in interacting current algebra theories.
- Based on leading order perturbative results and symmetries.
- New integrable σ-model theories, as all loop effective actions for current-current interactions of one, two or more exact CFT WZW models.
- Non-trivial smooth flows between exact CFTs.
- Future direction 1: For $k_1 \neq k_2$ embed to type-II supergravity. As in [KS-Thompson 14, Demulder-KS-Thompson 15, Borsato-Tseytlin 15]. Prototype example $AdS_3 \times S^3 \times S^3 \times S^1$.
- Future direction 2: New non-Abelian type T-dualities: The basic idea is to expand the group elements g_1 and/or g_2 around the identity and take the $k \to \infty$ limit.
- Future direction 3: Consider chains and/or webs of models (cyclic or infinite).

For Ioannis

- We first met in 1989 in Trieste
- Several common projects from 1995 to 2002. Most important:
  - T-duality and world sheet supersymmetry
    Ioannis Bakas (CERN), Konstadinos Sfetsos (Utrecht U.),
    Phys.Lett. B349 (1995) 448-457
  - States and curves of five-dimensional gauged supergravity
    Ioannis Bakas (Patras U.), Konstadinos Sfetsos (CERN),
    Nucl.Phys. B573 (2000) 768-810
  - PP waves and logarithmic conformal field theories
    Ioannis Bakas, Konstadinos Sfetsos (Patras U.),
    Nucl.Phys. B639 (2002) 223-240
- We were colleagues in the U. of Patras for several years.
- I remember him for his distinct approach to problem solving and humor.
- On the non-scientific life of Ioannis see talk by T. Vladikas in Xmas Theoretical Physics Workshop @Athens 2016: https://sites.google.com/site/xmasathens2016/home2