Deconstructing the BPS sector of (2,0) Theories

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Motivation

Many years on still trying to understand the 6D $(2,0)_{\text{ADE}}$ theory!

See however, significant recent progress:

- Calculation of the (2,0) superconformal index via localisation/topological strings
  [Källén, Zabzine '12; Kim^2 '12; Lockhart, Vafa '12 ; ...]
- Connection to $\mathcal{W}_{\text{ADE}}$ algebras
  [Beem, Rastelli, van Rees '14]
- Superconformal bootstrap for (2,0) theories
  [Beem, Lemos, Rastelli, van Rees '15]

Interesting blend of old and new ideas and techniques

Today: We will use a diverse combination of ingredients

- The deconstruction of $(2,0)_{\mathsf{A}}$ theory on T^2
  [Arkani-Hamed, Cohen, Kaplan, Karch, Motl '01]
- The "$\frac{1}{2}$-BPS limit" of the (2,0) superconformal index
  [Bhattacharyya, Minwalla '09; Kim, Lee '12]
- The Higgs-branch Hilbert Series
  [Benvenuti, Feng, Hanany, He '06]
- SUSY localisation / the refined topological vertex
  [Pestun '07; Iqbal, Kozçaz, Vafa '07]

$\Rightarrow$ Obtain quantitative checks of the deconstruction proposal

Deconstructing the (2,0)$_{\mathsf{A}_{k-1}}$ theory

Begin with an N-noded 4D circular quiver theory with $SU(k)$ gauge groups.

$\Rightarrow$ This is an $\mathcal{N}=2\ SU(k)^N$ SCFT

Then take the Higgs-branch limit:

$$N \to \infty\ , \qquad G \to \infty\ , \qquad v \to \infty$$

while keeping

$$g_5^2 := \frac{G}{v} \to \text{fixed}\ , \qquad 2\pi R_5 := \frac{N}{Gv} \to \text{fixed}$$

For $E \ll 1/g_5^2$ reproduce 5D MSYM on a circle $S^1_{R_5}$

$\Rightarrow$ enhanced SUSY [Lambert, CP, Schmidt-Sommerfeld '12]

Gives rise to a spectrum of massive states

$$M_n^2 = \left(\frac{2\pi n}{R_5}\right)^2 , \qquad \widetilde{M}_n^2 = \left(\frac{4\pi^2 n}{g_5^2}\right)^2$$

$\Rightarrow$ M_n: KK modes on $S^1_{R_5}$

$\Rightarrow$ $\widetilde{M}_n$: Instanton solitons or KK modes on $S^1_{R_6}$, with $R_6 = \frac{g_5^2}{2\pi}$

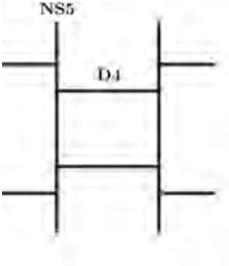
4D theory is UV complete and can consider circles of any size

$\Rightarrow$ Get (2,0)$_{\mathsf{A}_{k-1}}$ theory on $T^2 = S^1_{R_5} \times S^1_{R_6}$

[Arkani-Hamed, Cohen, Kaplan, Karch, Motl '01]

Brane Engineering

Can engineer the 4D $\mathcal{N} = 2$ SCFT using a Hanany–Witten construction



$\Rightarrow$ k D4s between N NS5s

$\Rightarrow$ periodically identify edges

$\Rightarrow$ At low energies, one gets the desired 4D $\mathcal{N} = 2$ SCFT

$\Rightarrow$ When D4s coincide across NS5s, the D4s reconnect

The k D4s can then be moved off the NS5s $\Rightarrow$ Higgsing

Matching $\frac{1}{2}$-BPS operators

This proposal is very natural but no quantitative checks

$\Rightarrow$ In the large-volume limit recover (2,0) SCFT in $\mathbb{R}^6$

$\Rightarrow$ Compare 4D/6D local operator spectrum

Hint: 4D $\mathcal{N}=2$ SCA embeds into the 6D (2,0) SCA

In particular, focus on and compare:

$\Rightarrow$ 6D supercharges annihilating primaries of $\frac{1}{2}$-BPS multiplets

$\Rightarrow$ Descend to 4D supercharges annihilating operators parametrising the Higgs-branch

$\exists$ various techniques for counting local operators in SCFTs

$\Rightarrow$ The 6D superconformal index

[Bhattacharya, Bhattacharyya, Minwalla, Raju '08]

$$I = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{-\beta\{Q,S\}} x^{\Delta+J_1} y_1^{h_1-h_2} y_2^{h_2+h_3} q^{h_1+h_2-h_3-3J_2}$$

The $\Delta,\ h_i,\ J_i$ are Cartans of $\mathfrak{u}(1) \oplus \mathfrak{so}(6) \oplus \mathfrak{sp}(2)_R \subset \mathfrak{osp}(8^*|4)$

Counts operators with signs, but in the limit $q \to 0$ only the $\frac{1}{2}$-BPS primaries contribute $\Rightarrow$ $\frac{1}{2}$-BPS index

For the A_{k-1} theory this gives [Bhattacharyya, Minwalla '09]

$$I_{\mathsf{A}_{k-1}} = \prod_{m=1}^{k} \frac{1}{1-x^m}$$

On the 4D side use the Higgs-branch Hilbert Series

[Benvenuti, Feng, Hanany, He '06]

$\Rightarrow$ Counts BPS operators parametrising the Higgs branch

For the N-noded circular quiver the result is given by the coefficient of the ν^k term in the expansion of

$$\mathrm{HS}_N(t,\nu) = \mathrm{PE}\left[\frac{(1-t^{2N})}{(1-t^2)(1-t^N)^2}\nu\right]$$

where the Plethystic Exponential is

$$\mathrm{PE}[g(t)] := \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}g(t^n)\right]$$

In the deconstruction limit, $N \to \infty$, this becomes, since $|t| < 1$,

$$\mathsf{HS}_{k,N}(t) = \prod_{m=1}^{k} \frac{1}{1 - t^{2m}} = I_{\mathsf{A}_{k-1}}(t^2)$$

$\Rightarrow$ We have an initial test of the deconstruction proposal

However:

- ◇ Difficult to extend HS calculation...
- ◇ Difficult to find a limit of the index that isolates Higgs-branch contributions...
- ◇ There are also selfual strings wrapping the torus...

$\Rightarrow$ Need a different tool to go further

Matching partition functions on $S^4_{\epsilon_1,\epsilon_2}$

Choose to compare partition functions for Ω-deformed theories

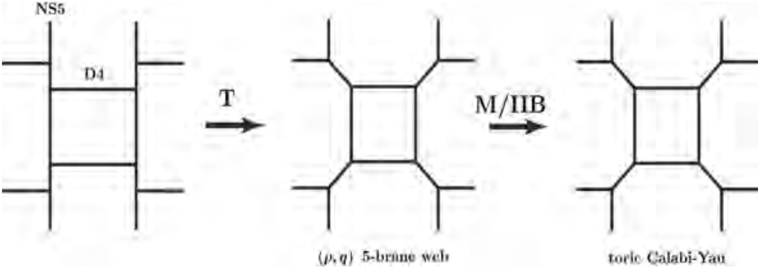
Start with the 4D circular-quiver result from localisation:

$$Z^{\rm 4D}_{S^4_{\epsilon_1,\epsilon_2}} = \int [da] Z^{\rm 4D}_{\rm IR} \overline{Z}^{\rm 4D}_{\rm IR}$$

No Higgs branch for 4D theory on S^4 but can be opened up by adding mass terms [Kim^2-Lee-Park '14]

$\Rightarrow$ Consider deconstruction limit directly on $Z^{\rm 4D}_{\rm IR}$

$Z_{\rm IR}^{\rm 4D}$ can also be calculated using topological strings [Iqbal, Kozçaz, Vafa '07]



$\Rightarrow$ Using brane intuition we provide a prescription for how to take the deconstruction limit on the parameters of $Z_{\rm IR}^{\rm 4D}$ (Coulomb, masses, couplings, ϵ_1, ϵ_2)

The result can be compared to the M5-brane IR partition function on $\mathbb{R}^4_{\epsilon_1,\epsilon_2} \times T^2$

[Haghighat, Iqbal, Kozçaz, Lockhart, Vafa '13]

⇒ They agree: $Z_{\mathrm{IR}}^{\mathrm{4D,Higgs}} = Z_{\mathrm{IR}}^{\mathrm{6D}}$

Extra dimensions deconstructed by combining infinite products of rational functions into trigonometric (c.f. q-deformation)

IR partition functions on $\mathbb{R}^4 \times T^2$ can be glued together to reproduce the full partition function on $S^4 \times T^2$ according to

$$Z^{\mathrm{6D}}_{S^4_{\epsilon_1,\epsilon_2}\times T^2} = \int [da] Z^{\mathrm{6D}}_{\mathrm{IR}} \overline{Z}^{\mathrm{6D}}_{\mathrm{IR}}$$

[Lockhart-Vafa '12]

Summary and Outlook

- Revisited deconstruction proposal for (2,0)$_{\mathsf{A}_{k-1}}$ theories
- Matched simple BPS operators using the 4D Hilbert Series and the 6D superconformal index
- Matched the full partition functions on S^4 by implementing the deconstruction limit on the IR partition function
- This provides a dictionary for exact calculations in 4D/6D
- Next: Can one use this to calculate observables in (2,0)?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