## Kolymbari, 13 July, 2017



# Elias Kiritsis











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# I. Bakas and higher spin algebras

• One of Ioannis' most important contributions to physics is the introduction and study of higher spin algrebras.

• They were introduced in the context of two dimensional theories

 $\bullet$  This came a bit after Zamolodchikov introduced the  $W_N$  algebra in 2d CFTs.

## THE LARGE-N LIMIT OF EXTENDED CONFORMAL SYMMETRIES

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We study the large-N limit of the operator algebra  $W_N$  generated by primary conformal fields with integer spin 1, 2, ..., N. It is shown that  $W_\infty$  provides a representation of a certain infinite dimensional (sub)algebra of the area-preserving diffeomorphisms of the 2-plane (compactified or not). We also discuss certain applications of this result to quantum field theory.

• Ioannis started from the Drinfeld-Sokolov reduction  $SL(N) \to W_N$  and suggested to take the limit  $N \to \infty$ .

 $\bullet$  Showed that classical  $W_\infty$  algebra obtained is related to area-preserving diffeos

• In a highly prescient conclusion page he suggested that:

1.  $W_{\infty}$  is related to quantum theories with area preserving symmetry like membranes, hydrodynamics and fermi surfaces.

2. It is related to an "enveloping" algebra of all  $W_N$  symmetries of Zamolodchikov.

3. Suggested that the construction of  $W_N$  and  $W_\infty$  strings is possible and interesting.

4. Suggested that the  $N \to \infty$  of  $Z_N$  parafermions, is a  $W_\infty$  theory that is related to area preserving diffeos.

5. Suggested that they are similar to the higher-spin theories of Fradkin and Vasiliev.

# The Structure of the $W_{\infty}$ Algebra

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Abstract. We prove rigorously that the structure constants of the leading (highest spin) linear terms in the commutation relations of the conformal chiral operator algebra  $W_{\infty}$  are identical to those of the Diff<sub>0</sub><sup>+</sup>  $\mathbb{R}^2$  algebra generated by area preserving diffeomorphisms of the plane. Moreover, all quadratic terms of the  $W_N$  algebra are found to be absent in the limit  $N \to \infty$ . In particular we show that  $W_{\infty}$  is a central extension of Diff<sub>0</sub><sup>+</sup>  $\mathbb{R}^2$  with non-trivial cocycles appearing only in the commutation relations of its Virasoro subalgebra. We also propose a representation of  $W_{\infty}$  in terms of a single scalar field in 2 + 1 dimensions and discuss its significance in the context of quantum field theory.

• Fast forward to Berkeley in 1990:

Ioannis visited me, and while he came another important paper came out:



Woo and the Racah-Wigner algebra

C.N. Pope,\* L.J. Romans<sup>†</sup> and X. Shen\*

#### ABSTRACT

We examine the structure of a recently-constructed  $W_{\infty}$  algebra, an extension of the Virasoro algebra that describes an infinite number of fields with all conformal spins 2, 3, ..., with central terms for all spins. By examining its underlying SL(2, R) structure, we are able to exhibit its relation to the algebras of SL(2, R) tensor operators. Based upon this relationship, we generalise  $W_{\infty}$  to a one-parameter family of inequivalent Lie algebras  $W_{\infty}(\mu)$ , which for general  $\mu$  requires the introduction of formally negative spins. Furthermore, we display a realisation of the  $W_{\infty}(\mu)$  commutation relations in terms of an underlying associative product, which we denote with a lone star. This product structure shares many formal features with the Racah-Wigner algebra in angular momentum theory. We also discuss the relation between  $W_{\infty}$  and the symplectic algebra on a cone, which can be viewed as a co-adjoint orbit of SL(2, R).

## BOSONIC REALIZATION OF A UNIVERSAL W-ALGEBRA AND Z<sub>∞</sub> PARAFERMIONS\*

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Received 30 March 1990

We construct a field theoretic representation of the universal W-algebra proposed by Pope, Romans and Shen, using a free complex boson in two dimensions. The resulting symmetry algebra is generated by conformal fields with spin 2, 3, 4, ... and has central charge c = 2. Highest-weight representations are also given in terms of vertex operators. Furthermore, we discuss the relation of this representation to the theory of  $Z_{\infty}$  parafermions.

- Our work realized the symmetry algebra as written by PRS, with a FREE complex massless boson and introduced ''colored'' generalizations of  $W_\infty$
- $\bullet$  Later, Pope Romans and Shen realized  $W_{1+\infty}$  in terms of a FREE Dirac Fermion.
- The way was open for realizations of  $W_{\infty}$  in many concrete systems.
- Almost 25 years later, Maldacena and Zhibhoedov proved the reverse: only free theories have  $W_{\infty}$  symmetries.

### BOSONIZATION OF NON-RELATIVISTIC FERMIONS AND W-INFINITY ALGEBRA

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#### Received 18 November 1991

We discuss the bosonization of non-relativistic fermions in one-space dimension in terms of bilocal operators which are naturally related to the generators of W-infinity algebra. The resulting system is analogous to the problem of a spin in a magnetic field for the group W-infinity. The new dynamical variables turn out to be W-infinity group elements valued in the coset W-infinity/H where H is a Cartan subalgebra. A classical action with an H gauge invariance is presented. This action is three-dimensional. It turns out to be similar to the action that describes the color degrees of freedom of a Yang-Mills particle in a fixed external field. We also discuss the relation of this action with the one recently arrived at in the Euclidean continuation of the theory using different coordinates.

# Fermions in the lowest Landau level. Bosonization, $W_{\infty}$ algebra, droplets, chiral bosons

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We present field theoretical descriptions of massless (2+1) dimensional nonrelativistic fermions in an external magnetic field, in terms of a fermionic and bosonic second quantized language. An infinite dimensional algebra,  $W_{\infty}$ , appears as the algebra of unitary transformations which preserve the lowest Landau level condition and the particle number. In the droplet approximation it reduces to the algebra of area-preserving diffeomorphisms, which is responsible for the existence of a universal chiral boson lagrangian independent of the electrostatic potential. We argue that the bosonic droplet approximation is the strong magnetic field limit of the fermionic theory. The relation to the c=1 string model is discussed.

- Boris Khesin (student of Arnold), then professor at Berkeley Math department had introduced the log of derivative cocycle to introduce central terms in algebras of pseudo-differential operators.
- We teamed up.....

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# The Logarithm of the Derivative Operator and Higher Spin Algebras of $W_{\infty}$ Type\*

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# • An important reason that made $W_{\infty}$ popular was due to the expectation that the W<sub> $\infty$ </sub> charges would be important in the resolution of the black hole information paradox of the two dimensional black hole.

#### BEYOND THE LARGE N LIMIT: NON-LINEAR $W_{\infty}$ AS SYMMETRY OF THE SL(2,R)/U(1) COSET MODEL\*

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#### ABSTRACT

We show that the symmetry algebra of the  $SL(2,R)_k/U(1)$  coset model is a non-linear deformation of  $W_{\infty}$ , characterized by k. This is a universal W-algebra which linearizes in the large k limit and truncates to  $W_N$  for k = -N. Using the theory of non-compact parafermions we construct a free field realization of the non-linear  $W_{\infty}$  in terms of two bosons with background charge. The W-characters of all unitary SL(2,R)/U(1) representations are computed. Applications to the physics of 2-d black hole backgrounds are also discussed and connections with the KP approach to c=1 string theory are outlined.

- $\bullet$  Since then  $W_\infty$  activity quieted down
- Vasiliev went ahead to develop to develop his higher spin theories in AdS completely unnoticed (in the 1990's)
- Things were revived in 2010.....

# PHYSICAL REVIEW D 83, 066007 (2011) An AdS<sub>3</sub> dual for minimal model CFTs

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We propose a duality between the 2d  $W_N$  minimal models in the large N 't Hooft limit, and a family of higher spin theories on AdS<sub>3</sub>. The 2d conformal field theories (CFTs) can be described as Wess-Zumino-Witten coset models, and include, for N = 2, the usual Virasoro unitary series. The dual bulk theory contains, in addition to the massless higher spin fields, two complex scalars (of equal mass). The mass is directly related to the 't Hooft coupling constant of the dual CFT. We give convincing evidence that the spectra of the two theories match precisely for all values of the 't Hooft coupling. We also show that the renormalization group flows in the 2d CFT agree exactly with the usual AdS/CFT prediction of the gravity theory. Our proposal is in many ways analogous to the Klebanov-Polyakov conjecture for an AdS<sub>4</sub> dual for the singlet sector of large N vector models.



- Ioannis had also several other important contributions in physics.
- Beyond physics, he was a remarkable personality: he was warm, honest, with high standards, both on moral issues and on quality.
- He will live on in our minds, as long as we live.
- But his scientific legacy will outlive all of us.

Some photos











I. Bakas and  $W_\infty$  symmetries,

Elias Kiritsis