CFT Regge limit and Einstein gravity

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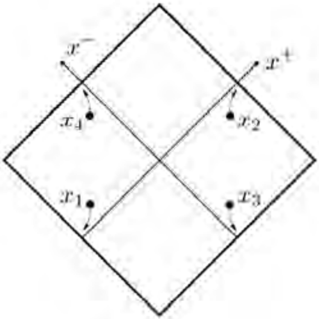
Introduction and summary

Goal: analyze the emergence of gravity using CFT methods. Will focus on the Regge limit of CFT four-point function $\langle J_\mu J_\nu \psi\psi\rangle$. Assume large gap in the spectrum of operators with spin> 2; large central charge.

- In the Regge limit correlator is dominated by operators of spin 2: the stress tensor and the double trace operators
- To decouple double trace operators, will perform the Fourier transform
- The resulting object (the phase shift) must be positive
- Its positivity constrains $\langle J_\mu J_\nu T_{\alpha\beta}\rangle$ correlators
- Choosing J to be the $U(1)_R$ current in a SCFT this leads to $a = c$

Outline

Regge limit



$$\langle \phi\left(-\frac{x}{2}\right)\psi\left(-\frac{\bar{x}}{2}\right)\psi\left(\frac{\bar{x}}{2}\right)\phi\left(\frac{x}{2}\right)\rangle = \frac{A(u,v)}{(-x^2)^{\Delta_\phi}(-\bar{x}^2)^{\Delta_\psi}}$$

Regge Limit

$$u = (1-z)(1-\bar{z}) = (1-\sigma e^{-\rho})(1-\sigma e^{\rho}), \qquad v = z\bar{z} = \sigma^2 \approx x^2\bar{x}^2$$

Regge limit is $\sigma \to 0$, ρ fixed. The result is

$$A \simeq 1 - 2\pi i \int_{-\infty}^{\infty} d\nu\, \alpha(\nu) \sigma^{1-j(\nu)} \Omega_{i\nu}(\rho) + \ldots$$

where

$$\Omega_{i\nu}(\rho) = \frac{i\nu}{2\pi} \left(\Pi_{i\nu+\frac{d}{2}-1}(\rho) - \Pi_{-i\nu+\frac{d}{2}-1}(\rho) \right)$$

Regge Limit

For example, in $d = 4$

$$\Omega_{i\nu}(\rho) \simeq \frac{\nu \sin \nu\rho}{\sinh \rho}$$

Note that $\alpha(\nu)$ has poles at $\nu = \pm id/2$ (stress-tensor) and at values of ν which correspond to appearance of double trace operators such as $\phi\partial^{2n}\partial_\mu\partial_\nu\phi$ and $\psi\partial^{2n}\partial_\mu\partial_\nu\psi$.

We can exactly reproduce the graviton exchange Witten diagram by setting $j(\nu) = 2$.

Phase shift

Fourier transform

$$B(p,\bar{p}) = \int d^d x d^d \bar{x} e^{ip.x} e^{i\bar{p}.\bar{x}} \frac{A(x,\bar{x})}{(-x^2)^{\Delta_\phi}(-\bar{x}^2)^{\Delta_\psi}}$$

One can write this as

$$B(p,\bar{p}) = B_0\left(1+\delta(S,L)\right)$$

where $\delta(S,L)$ is the phase shift we are after, and S, L are energy and impact parameter

$$\cosh L = -\frac{p.\bar{p}}{\sqrt{-p^2}\sqrt{-\bar{p}^2}}, \quad S = \sqrt{p^2\bar{p}^2}$$

Phase shift

Substituting the correlator in the Regge limit we get

$$\delta(S,L) \simeq \int_{-\infty}^{\infty} d\nu\, \beta(\nu)\, S^{j(\nu)-1}\, \Omega_{i\nu}(L)$$

where $\beta(\nu) \sim \alpha(\nu)$ but with the poles due to double trace operators removed.

Phase shift

This quantity has an interesting interpretation in the dual channel. It is equal to (minus) the anomalous dimension of the double-trace operator $\phi\partial^{2n}\partial_{\mu_1}...\partial_{\mu_s}\psi$ The map is via

$$h^2\bar{h}^2 = p^2\bar{p}^2, \qquad \frac{h}{\bar{h}} + \frac{\bar{h}}{h} = -2\frac{p.\bar{p}}{|p||\bar{p}|}$$

where the spin is $s = \bar{h} - h$ and the conformal dimension is $\Delta = \bar{h} + h$.

Phase shift

There are a couple of ways to argue positivity of the phase shift for theories with gravitational duals.

The anomalous dimension of a double trace operator corresponds to gravitational binding energy between the two states. Gravity is attractive.

Using eikonal approximation in scattering, phase shift is the same as time delay. Causality implies positivity.

Would be nice to have chaos-like bound

$$|1+\delta(S,L)|<1$$

External operators with spin

This can be repeated for external operators with spin. Consider conserved spin-1 current J^m.

$$\langle J^m(y_4)\,\psi(y_3)\,\psi(y_2)\,J^n(y_1)\rangle = \frac{A^{mn}}{(y_{23}^2)^{\Delta_\psi}(y_{14}^2)^d}$$

we can write

$$A^{mn} = \sum_{\Delta,J} \lambda_{\psi\psi\mathcal{O}_{\Delta,J}} \sum_{k=1}^{5} Q^{(k)mn} f_k(u,v)$$

where $Q^{(k)mn} = \{-2y_{14}^m y_{14}^n + y_{14}^2\eta^{mn}, \ldots\}$ and

$$f_1 = \left[-\frac{1}{3}(n_s + 12n_f) + \frac{1}{6}n_s u\partial_u\right] g_{\Delta,J}(u,v)$$

External operators with spin

Substituting our kinematics and taking the Regge limit, we can match the result to the following expression

$$A^{mn}(x,\bar{x}) \simeq \int_{-\infty}^{\infty} d\nu \sum_{k=1}^{4} \alpha_k(\nu) \frac{\left(x^2 \mathcal{D}_k^{mn}\right) \Omega_{i\nu}^{j}(\rho)}{(-x^2)^d(-\bar{x}^2)^{\Delta_\psi}(-\sqrt{x^2\bar{x}^2})^{j(\nu)-1}}$$

where

$$\alpha_k(\nu) = a_k \alpha(\nu)$$

and $\mathcal{D}_k^{mn}$ are diff operators, e.g.

$$x^2\mathcal{D}_4^{mn} = (x^2)^2\partial^m\partial^n + x^2(x^m\partial^n + x^n\partial^m) - \frac{1}{d-1}(x^2\eta^{mn} - x^m x^n)x^2\partial^2$$

External operators with spin

Performing the Fourier transform we obtain

$$\delta^{mn}(S,L) \simeq \sum_{k=1}^{4} \int_{-\infty}^{\infty} d\nu \beta_k(\nu) S^{j(\nu)-1} \hat{\mathcal{D}}_k^{mn} \Omega_{i\nu}(L)$$

where

$$\hat{\mathcal{D}}_1^{mn} = \eta^{mn} - \frac{p^m p^n}{p^2}, \quad \hat{\mathcal{D}}_2^{mn} = \frac{p^m p^n}{p^2}, \qquad \hat{\mathcal{D}}_3^{mn} = p^m \partial^n + p^n \partial^m$$

and

$$\hat{\mathcal{D}}_4^{mn} = p^2 \partial^m \partial^n + (p^m \partial^n + p^n \partial^m) - \frac{1}{d-1} \left(\eta^{mn} - \frac{p^m p^n}{p^2} \right) p^2 \partial^2 \,,$$

External operators with spin

The results are

$$\beta_2 = \beta_3 = 0$$

$$\beta_1 \simeq \frac{2d^2}{(d-1)}(n_s + 4(d-2)n_f)\frac{1}{\nu^2 + d^2/4}$$

and

$$\beta_4 \simeq -\frac{2d}{d-1}(n_s - 4n_f)\frac{1}{\nu^2 + d^2/4}$$

External operators with spin

The contribution of the graviton pole to the integral gives rise to $\Pi_{i\nu+\frac{d}{2}-1}(L) \sim \exp(-dL)/L$. Term with β_4 comes with 2 derivatives, so is enhanced by a factor of $1/L^2$ for small L.

For the phase shift to be always positive, it must be that $|\beta_4| < \beta_1 L^2$. In the infinite gap limit we can take $L \to 0$, which implies $\beta_4 = 0$ or $n_s = 4n_f$.

This implies that $\langle J_\mu J_\nu T_{\alpha\beta}\rangle$ has only one structure (holographically dual to $F_{\mu\nu}F^{\mu\nu}$). Adding supersymmetry to this implies that holographic theory cannot have generic higher derivative corrections. In $d = 4$ SCFT this implies $a = c$.

Comments

To estimate the value of $\beta_4 \sim L^2_{min}$ consider the leading Regge trajectory

$$j(\nu) = 2 - 2\frac{\nu^2 + \frac{d^2}{4}}{\Delta^2_{gap}} + \ldots$$

Saddle point is at

$$\nu_* = -\frac{iL\Delta^2_{gap}}{4\log S}.$$

For graviton contribution to be much larger than that of the saddle it must be that

$$L^2_{min} \sim \frac{\log S_*}{\Delta^2_{gap}} \sim \frac{\log \Delta_{gap}}{\Delta^2_{gap}}$$

So $|\beta_4| \sim |a-c|/c < \log \Delta_{gap}/\Delta^2_{gap}$

Summary

- Witten diagram with graviton exchange decomposes into conformal blocks due to the exchange of the stress tensor and of double traces. Fourier transform disentangles stress tensor contribution from double traces
- We can see precisely how this picture emerges in a CFT using Conformal Regge Theory
- In a holographic CFT (with a gap for higher spin operators) positivity of the result leads to $a = c$. (Dual theory is Einstein gravity)

To do list

- Repeat for external stress tensor (straightforward)
- Understand the miracle of the Fourier transform
- What happens outside the Regge limit?
- Non-conformal examples?
- Theories with controlled higher spin sector (CFT "semi-holography").

THE END

Thank you!