The bulk phase shift, the Regge Limit and Einstein Gravity

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Introduction

Holography relates quantum filed theories to gravitational theories. The most well-studied and tractable examples, involve conformal field theories (CFTs).

The principle of holography raises several questions:

- *Which CFTs have a (local) gravitational description?*
- *Can we see gravity emerge from the CFT dynamics?*
- *Which local theories of gravity correspond to consistent CFT theories?*

Introduction

Holographic CFTs: Large number of degrees of freedom and large gap in the spectrum of "single trace" operators.

- Holographic CFTs have a local bulk dual.

  Crossing symmetry leads to:

$$\# \; of \; \frac{1}{N^2} \; corrections = \# \; of \; local \; bulk \; couplings$$

[Heemskerk, Penedones, Polchinski, Sully]

$*$ **Are all local bulk couplings consistent?** $*$

Introduction

Example: Lanczos-Gauss-Bonnet gravity:

$$S \sim \int d^5x \sqrt{-g} \left(R - 2\Lambda + \alpha_{GB} \left(\mathcal{R}^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2\right)\right)$$

Despite the higher derivative term, this theory leads to second-order equations of motion. Can it be the holographic dual of a CFT?

Note that: $\alpha_{GB} \propto a - c$.

String theory intuition and recent gravitational analysis suggests that the answer is NO.

Introduction

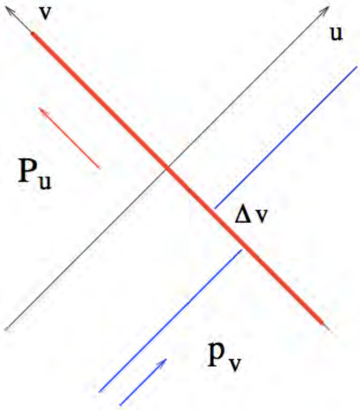
[Camanho, Edelstein, Maldacena, Zhiboedov] showed that:

$$S = \frac{1}{\ell_p^{D-2}} \int d^D x \sqrt{g} \left[R - 2\Lambda + \alpha_{GB} \tilde{\ell}^2 (R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) \right]$$

when $\alpha_{GB} \, \tilde{\ell}^2 >> \ell_p^2$, the theory violates *causality*.
Certain higher curvature corrections to Einstein gravity must be accompanied by new massive higher spin particles at the same scale.

Method:
Consider a shock-wave geometry created by a highly energetic particle. When a probe spinning-particle traverses the geometry, it experiences a time-delay. The time-delay may become a time advance when $\alpha_{GB} \neq 0$.

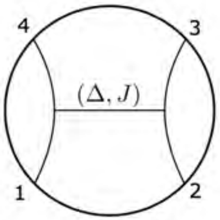
Introduction



A particle creates a shockwave localized at $u = 0$. A proble particle propagates on the geometry and experiences a time delay Δv. The two particles are separated along the transverse directions.

Introduction

- The shock wave computation corresponds to 2 $\rightarrow$ 2 particle scattering.



Relevant Witten diagram for the exchange of a spin-J field.

- The scattering phase shift is directly proportional to the time delay:

$$\delta(S, L) = S^{J-1}\Pi(L) \propto \Delta v$$

[Dray,'t Hooft][Kabat, Ortiz][Cornalba, Costa, Penedones, Schiappa]

Introduction

Objective: Study the bulk phase shift in CFT language.

- Consider a holographic CFT and evaluate the phase shift in the Regge limit (high energy, fixed impact parameter).
- Impose physical consistency conditions, unitarity/causality, to restrict the possible interaction terms of the graviton.

Summary of results

Combining the CFT techniques with these assumptions, we will show that:

- $j_0 = 2$. When $\Delta_{gap} = \infty$, the large-N theory does not contain single-trace primaries of spin $j_0 > 2$.

  This is like a gravity theory. [Maldacena, Shenker, Stanford]

- The OPE coefficients of the stress-tensor T satisfy a special constraint.

  For supersymmetric theories in $d = 4$ dimensions, the constraint reads:

  $$a - c = 0$$

  This is Einstein-Hilbert gravity.

[M.K., A. Parnachev and A. Zhiboedov]

[Li, Meltzer and Poland]

Outline

- CFT: general aspects.
- Holographic CFTs.
- The correlation function.
- The Regge limit and conformal regge theory.
- A first result:
  The gap in the spectrum of holographic CFTs should appear at spin $J = 2$, i.e., the stress-enrgy tensor.

CFT: general aspects

The operators of a CFT are classified by their spin j and conformal dimension Δ. The basic building blocks are *primary* operators $\mathcal{O}_{\Delta}^{j}$, anihilated by the generator of special conformal transformations .

Conformal symmetry determines the form of the two- and three-point correlation functions up to a few independent parameters.
Example: (scalar operators)

$$\left\langle \mathcal{O}^i(x_1)\mathcal{O}^k(x_2)\right\rangle = \frac{\delta^{ik}}{x_{12}^{2\Delta}}, \qquad x_{ik} = x_i - x_k$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{\alpha}{x_{12}^{\Delta_{12}} x_{23}^{\Delta_{23}} x_{13}^{\Delta_{13}}}, \qquad \Delta_{ik} = \Delta_i - \Delta_k$$

CFT: general aspects

A special example is the stress-energy tensor $T_{\mu\nu}(x)$.

2-point function:

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = c\, \frac{\mathcal{I}_{\mu\nu,\rho\sigma}(x)}{x^{2d}}$$

3-point function:

$$\langle T^{\mu\nu}(x_3) T^{\rho\sigma}(x_2) T^{\tau\kappa}(x_1)\rangle = \frac{a f_1^{\mu\nu\rho\sigma\tau\kappa}(x) + c f_2^{\mu\nu\rho\sigma\tau\kappa}(x) + b f_3^{\mu\nu\rho\sigma\tau\kappa}(x)}{|x_{12}|^d |x_{13}|^d |x_{23}|^d}$$

CFT: general aspects

A conformal field theory is characterized by:

- Its spectrum. A set of primary operators $\mathcal{O}_j^{\Delta}$ with conformal dimensions Δ and spin j.

- The coefficients of the Operator Product Expansion (OPE):

  Example:

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{j,\,\Delta} \frac{\lambda_{12\mathcal{O}}}{|x|^{\Delta_1+\Delta_2-\Delta_3+j}} \mathcal{O}^{\Delta}_{\mu_1\cdots\mu_j}\, x^{\mu_1}\cdots x^{\mu_j}$$

Clearly, the undetermined parameters in the three-point functions and the OPE coefficients represent the same set of data.

CFT: general aspects

The four point function is fixed by conformal invariance to be of the form:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \frac{\mathcal{A}(u,v)}{x_{12}^{2\Delta_1}x_{34}^{2\Delta_2}}$$

with (u,v) *the conformal cross ratios*:

$$u \equiv \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad v \equiv \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

and $\mathcal{A}(u,v)$ an undetermined function.

CFT: general aspects

Using the OPE the four-point function can also be written as:

$$\mathcal{A}(u,v) = \sum_{\mathcal{O}^j_\Delta} \lambda_{11\mathcal{O}}\lambda_{22\mathcal{O}} g_{\mathcal{O}}(u,v)$$

with $g_{\mathcal{O}}(u,v)$ known as *the conformal block*.

Due to conformal symmetry, the conformal block satisfies a 2nd order differential equation, *the Casimir differential equation*. Solutions are explicitly known in any even d and as integral representations or power series in odd d.

Holographic CFTs

Holographic CFTs:

The CFT has a stress-tensor operator T and two large paramaters:

1. Large number of degrees of freedom N.

   At $N = \infty$ the CFT correlations functions factorize:

   $$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_2 \rangle = \langle \mathcal{O}_1 \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \mathcal{O}_2 \rangle + \frac{1}{N^2}(\cdots)$$

2. A characteristic scale Δ_{gap}.

   When $\Delta_{gap} = \infty$ the CFT contains only a finite number of primary single-trace operators with spin $j \leq j_0$.

Holographic CFTs

- "single-trace" primaries: $\mathcal{O}_1, \mathcal{O}_2, \cdots, J^\mu, T^{\mu\nu}, \cdots, J^{\mu_1\mu_2\cdots\mu_{j_0}}$
- "double-trace" primaries:

$$M_2 : \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_\ell} (\partial^2)^n \mathcal{O}_2, \quad \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_\ell} (\partial^2)^n J^\mu, \cdots$$

- "multi-trace" primaries (not relevant for this talk-subleading):

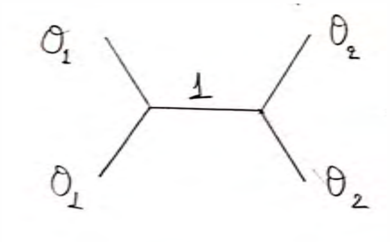
$$M_{n>2} : \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_a} (\partial^2)^n \mathcal{O}_2 \partial_{\mu_1 \ldots} \partial_{\mu_b} (\partial^2)^m \mathcal{O}_1 \partial_{\mu_1 \ldots} \partial_{\mu_c} (\partial^2)^k J^\mu, \cdots$$

$$\langle \mathcal{O}_1 \mathcal{O}_1 \rangle \sim 1 + \cdots, \qquad \langle M_2 M_2 \rangle \sim 1 + \cdots$$

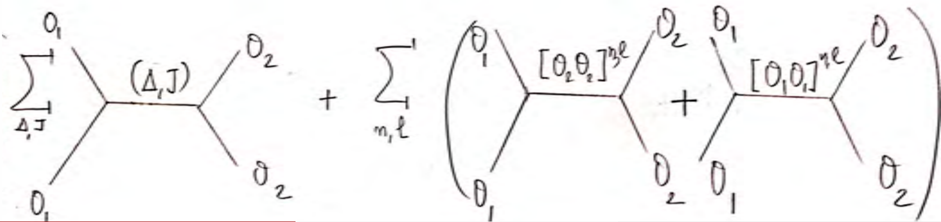
$$\langle \mathcal{O}_1 \mathcal{O}_1 \, T \rangle \sim \frac{1}{N} + \cdots, \quad \langle \mathcal{O}_1 \mathcal{O}_1 M_2^{22} \rangle \sim \frac{1}{N^2} + \cdots, \quad \langle \mathcal{O}_1 \mathcal{O}_1 M_2^{11} \rangle \sim 1 + \cdots$$

Holographic CFTs

$\mathcal{O}(1)$:



$\mathcal{O}(\frac{1}{N^2})$:



Holographic CFTs

Strategy:

- Study a 4-point correlation function in the large-N limit.
- Simplify the correlator by focusing on a special region of the conformal cross ratios u, v: Regge limit.
- Impose consistency conditions, such as unitarity and causality, which will restrict the OPE coefficients and undetermined parameters.

Holographic CFTs

The correlation function :

$$A(x,\bar{x}) \equiv \left\langle \mathcal{O}_1\left(-\frac{x}{2}\right)\mathcal{O}_2\left(-\frac{\bar{x}}{2}\right)\mathcal{O}_2\left(\frac{\bar{x}}{2}\right)\mathcal{O}_1\left(\frac{x}{2}\right)\right\rangle = \frac{\mathcal{A}(u,v)}{(-x^2)^{\Delta_1}(-\bar{x}^2)^{\Delta_2}}$$

with $\mathcal{O}_1, \mathcal{O}_2$ operators of conformal dimensions Δ_1, Δ_2.

$\mathcal{O}_{1,2}$ are scalar operators
(later, $\mathcal{O}_1$ will be replaced by an operator with spin).

Focus: on the first, non-trivial $\frac{1}{N^2}$ correction.

Holographic CFTs

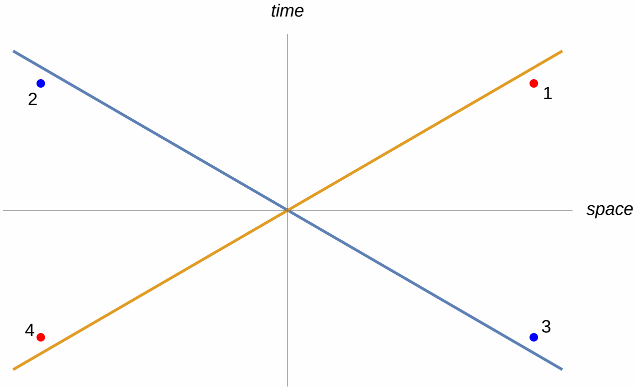
$$A(x,\bar{x}) = \left(\frac{x^+}{2}\right)^{-2\Delta_1} \left(\frac{\bar{x}^+}{2}\right)^{-2\Delta_2} \langle \mathcal{O}_1(y_1)\mathcal{O}_2(y_3)\mathcal{O}_2(y_4)\mathcal{O}_1(y_2)\rangle$$

where:

$$y_1^+ = -y_2^+ = \frac{2}{x^+}, \quad y_3^- = -y_4^- = \frac{2}{\bar{x}^+}\ ,$$
$$y_1^- = -y_2^- = \frac{x^2}{2x^+}, \quad y_3^+ = -y_4^+ = \frac{\bar{x}^2}{2\bar{x}^+}\ ,$$
$$\vec{y}_1 = \vec{y}_2 = \frac{\vec{x}}{x^+}, \quad \vec{y}_3 = \vec{y}_4 = \frac{\vec{\bar{x}}}{\bar{x}^+}.$$

and $(x^\mu, \bar{x}^\mu)$ future-directed timelike vectors.

Holographic CFTs



Ananlytic Continuation: $x^2 \to x^2 - i\epsilon x^0,$ and $\bar{x}^2 \to \bar{x}^2 - i\epsilon \bar{x}^0.$

Holographic CFTs

Simplify $\mathcal{A}(u,v)$ by using the *partial wave decomposition* to write the sum over conformal dimensions as an integral:

$$\mathcal{A}(u,v) = \sum \lambda_{11\mathcal{O}}\lambda_{22\mathcal{O}} g_{\mathcal{O}}(u,v) \sim \sum_J \int_{-\infty}^{\infty} d\nu \, b_J(\nu^2) \, F_{\nu,J}(u,v)$$

For a generic CFT, $b_J(\nu^2)$ has poles at

$$\nu^2 + \left(\Delta - \frac{d}{2}\right)^2 = 0$$

for all the operators $\mathcal{O}_\Delta^J$ which appear in the conformal block decomposition of $\mathcal{A}(u,v)$.

For a large N CFT, $b_J(\nu^2)$ has poles only on single- and double-trace primaries.

Holographic CFTs

Focus on the *Regge-limit*:

$$u = \sigma^2, \qquad v = 1 - 2\sigma \cosh\rho + \sigma^2$$
$$\sigma \to 0, \qquad \rho = \textit{fixed}.$$

The scalar blocks in this limit are known in any d:

$$g_{\Delta, J}(\sigma, \rho) \simeq \frac{\Pi_{\Delta-1}(\rho)}{\sigma^{j-1}}$$

$\Pi_{\Delta-1}(\rho)$ is a known function: it is a solution to the scalar propagator equation in hyperbolic space $\mathcal{H}_{d-1}$

Holographic CFTs

Determine the behaviour of $\mathcal{A}(u, v)$ in the Regge limit with conformal Regge theory. [Cornalba][Costa,Concalves,Penedones]

By analytically continuing the spin-J, we turn the sum over the spins of the intermediate operators to an integral in the complex plane.

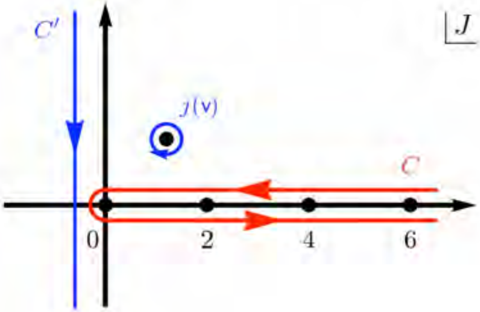
$$\sum_j \quad \Rightarrow \quad \int dj$$

Deforming the contour computes the integral by picking up only the contribution from the *leading Regge trajectory*: the operator of the highest spin-j for a given conformal dimension.

The analytic continuation of the $b_J(\nu^2)$ exists [S. Caron-Huot].

Holographic CFTs

Contour deformation. We deform the contour and pick up the contribution from the maximal $Re[J]$, which we assume to be a pole.



Large-N: Only single-trace primaries lie on the leading Regge trajectory.

Holographic CFTs

The result is:

$$\mathcal{A}(\sigma,\rho) \simeq 1 + \frac{1}{N^2}\, 2\pi i \int_{-\infty}^{+\infty} d\nu\, \alpha(\nu)\, \sigma^{1-j(\nu)}\, \Pi_{\Delta(\nu)-1}(\rho)$$

- Unity denotes the disconnected piece of the four-point function.
- $\Pi_{\Delta(\nu)}(\rho)$ is the scalar block in the Regge limit.
- $j(\nu)$ is the highest spin of the "single-trace" operators in the theory.

Holographic CFTs

For $\Delta_{gap} \gg 1$:

$$j(\nu) = j_0 + \cdots$$
$$\alpha(\nu) = c_0 \gamma(\nu)\gamma(-\nu) \frac{1}{\nu^2 + (\Delta_{j_0}^{min} - d/2)^2} + \cdots, \quad c_0 > 0$$

We close the contour below the axis and pick up the contribution from:

- The Regge pole at $\nu = -i\left(\Delta_{j_0}^{min} - d/2\right)$
- The poles hidden in $\gamma(\nu)\gamma(-\nu)$, which correspond to "double trace" operators.

Holographic CFTs

The result is

$$\mathcal{A} \simeq 1 - i\frac{1}{N^2}\,\frac{f(\rho)}{\sigma^{j_0-1}}\,,$$

with $f(\rho)$ a known function.

When the four-point function respects unitarity [Hartman, Jain, Kundu] :

$$|\mathcal{A}| \leq 1, \quad \mathrm{Im}(\sigma) > 0 \quad \Longrightarrow \quad j_0 \leq 2\,.$$

For a theory with a stress-tensor $j_0 = 2$.

[Maldacena,Shenker,Stanford]

Holographic CFTs

To obtain new constraints we need:

- To consider a correlation function of external operators with spin. The simplest example: vector operators J^μ.

$$\left\langle J^m\left(-\frac{x}{2}\right) J^n\left(\frac{x}{2}\right)\mathcal{O}\left(\frac{\bar{x}}{2}\right)\mathcal{O}\left(-\frac{\bar{x}}{2}\right)\right\rangle = \frac{\mathcal{A}^{mn}(x,\bar{x})}{x^{2\Delta_J}\bar{x}^{2\Delta}}$$

Need to compute the conformal block for an intermediate operator of any spin-j!

Impact Parameter Representation

- To move to the impact parameter representation (Fourier transform).

$$\frac{\mathcal{A}^{mn}(x,\bar{x})}{x^{2\Delta_1}\bar{x}^{2\Delta_2}} = \int dp\, d\bar{p}\ e^{ixp}\ e^{i\bar{x}\bar{p}}\ \frac{\mathcal{B}^{mn}(p,\bar{p})}{(-p^2)^{d/2-\Delta_1}(-\bar{p}^2)^{d/2-\Delta_2}}$$

where:

$$\mathcal{B}^{mn} \simeq 1 + i\delta^{mn}(p,\bar{p})$$

and the Regge limit is:

$$s^2 = p^2\bar{p}^2 \to \infty, \quad \cosh L = -\frac{p\cdot\bar{p}}{\sqrt{p^2\bar{p}^2}} = fixed$$

A little miracle occurs....but more in the next talk!

Summary-Results

- We computed a special correlation function in the Regge limit, which is the position-space analog of the high energy scattering in gravity.
- We used conformal Regge theory to constrain the spin of the Regge pole in a Holographic CFTs.

Natural generalizations:

- Go beyond the infinite gap limit.
- Free theories/ theories with flavors.
- Consider theories with broken conformal symmetry.:
  RG flows, finite temperature,...