Five dimensional gauge theories and Higgs branch at infinite coupling

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Five dimensional gauge theory

- Existence of SCFTs in 4 dimensions and higher
- Brane systems
- D4 - D8
- Webs of five branes
- M theory of CY singularity
- F theory with 8 supercharges

Gauge theory in 5d
SQCD

- Today focus on gauge theories with a single gauge group
- Relatively few theories which have a UV fixed point in 5d
- Number of flavors should be small enough
- Conditions similar to the condition for asymptotic freedom in 4d, but not the same

CS coupling

- The CS level k can be turned on, but fewer number of flavors are needed to have a 5d UV SCFT

SU(n) gauge theory

- Further restrict to SU(n) with N flavors and CS level k
- The level k is 1/2 + integer for N odd
- Integer for N even

Coulomb branch parameters

- Inverse gauge coupling $1/g^2$
- Masses for flavors
- Both have dimension 1
- Contribute to the central charge formula for BPS states

SU(n) with 2n+4 flavors and 0 CS level

- This theory has a 6d UV fixed point
- Trade 1 flavor by 1/2 CS level increase by integrating out a massive flavor
- Get a condition for the existence of a 5d UV fixed point
- $N + 2k <= 2n + 4$
- Excluding the case $N=2n+4$, $k=0$ which has 6d UV SCFT

Classical flavor symmetry

- U(N) x U(1)
- Two U(1) global symmetries
- Baryon number and the topological U(1) symmetry which counts instanton number
- For n=2 this changes to SO(2N) x U(1)

Gauge instantons in 5d

- This is a particle with a contribution to the mass that scales like $|I|/g^2$, with instanton charge I
- At infinite coupling, at the origin of the coulomb branch, becomes massless
- Increases the global symmetry
- Many new flat directions along the Higgs branch

Higgs branch at infinite coupling

- Given a theory with n, N, k set all parameters to 0 and ask what is the Higgs branch at infinite coupling

Higgs branch
hyper Kähler cone

- Recall all parameters are set to zero - have a cone
- SU(2) R symmetry
- Chiral ring
- Operators characterized by representation under SU(2) R and representations under the global symmetry

HyperKähler cones
Global symmetry

- Theorem
- All operators with spin 1 under SU(2) R transform in the adjoint representation of the global symmetry

Instanton operators

- Assume there exists chiral operators in the chiral ring which transform under the U(1) instanton, SU(2) R, and global symmetries
- Non perturbative operators which do not show in the classical theory and become crucially relevant to the UV SCFT
- Analogs of ‘t Hooft monopole operators in 3d

Instantons and global symmetry

- If the instanton carries spin 1 under SU(2) R the global symmetry is enhanced

- All such instantons are highly restricted as they must complete the existing classical global symmetry C to form the adjoint representation of a bigger global symmetry F which contains the classical symmetry as a subgroup

- In all cases we know, C is a Levy subgroup of F - the ranks are equal

Global symmetry at infinite coupling

- $N = 2n + 3$, $k = 1/2$: $SO(4n + 8)$
- $N = 2n + 2$, $k = 1$: $SO(4n + 4) \times SU(2)$
- $N = 2n + 2$, $k = 0$: $SU(2n + 4)$
- $N = 2n + 1$, $k = 3/2$: $SO(4n + 2) \times U(1)$
- $N = 2n + 1$, $k = 1/2$: $SO(2n + 2) \times SU(2)$
- $N = 2n$, $k = 2$: $SO(4n) \times U(1)$

More global symmetry

- $N = 2n$, $k = 1$: $SU(2n + 1) \times U(1)$
- $N = 2n$, $k = 0$: $SU(2n) \times SU(2) \times SU(2)$
- $N = 2n - 1$, $k = 5/2$: $SO(4n - 2) \times U(1)$
- $N = 2n - 1$, $k = 3/2$: $SU(2n) \times U(1)$
- $N = 2n - 1$, $k = 1/2$: $SU(2n - 1) \times SU(2) \times U(1)$

Instantons and Higgs branch

- If the instanton has a spin under SU(2) R which is higher than 1, the Higgs branch becomes larger, by a significant amount
- Roughly double the dimension

Instantons transform under SU(2) R

- To compute this quantity, we need to evaluate the energy of a vacuum state in the topological sector of 1 instanton
- For a gauge group G this is h/2 where h is the dual Coxeter number of G
- Closest analog to the 4d beta function for a SYM theory

Instanton zero modes

- Each flavor of quarks contributes a fermionic zero mode
- Easy to see in the D4-D8 system by looking at D0-D8
- Or D1-D7 for a five brane web
- As a result, the instanton transforms under the global symmetry as
- spinor representation if it is a rotation group
- Antisymmetric representation if it is a unitary group

Higgs branch at infinite coupling

- Given this data, how to proceed?
- Two ways of constructing hyperKahler cones
- HyperKahler quotient (Higgs branch) F & D terms
- 3D N=4 Coulomb branch

Construction using 3d techniques

- Assume
- The answer is given by a Coulomb branch of 3d N=4 theory
- Use the global symmetry as an input
- The instantons transform in spin n/2 of SU(2) R and spinor or maximally antisymmetric representation of the global symmetry

3d N=4 Coulomb branch Global symmetry

- An imbalance of a node is the number of flavors minus twice the number of colors
- A node is balanced if the imbalance is 0
- The subset of balanced nodes forms the Dynkin diagram of the global symmetry
- If the imbalance of a node is n-2 then there is an operator in the chiral ring with spin n/2 under SU(2) R and representation under the global symmetry given by the node is attached to

E8 sequence

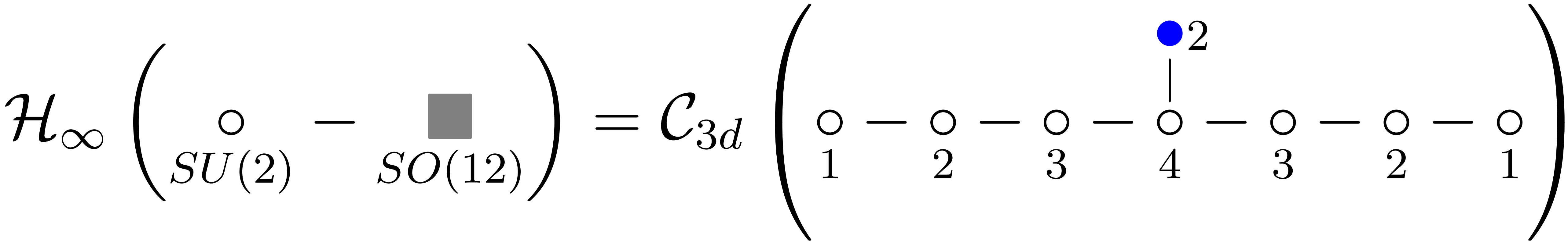
$$\mathcal{H}_\infty\left(\underset{SU(2)}{\circ} \;-\; \underset{SO(14)}{\blacksquare}\right) = \mathcal{C}_{3d}\left(\begin{array}{ccccccccccccccc} & & & & & & & & & & \circ\, 3 & & & & \\ & & & & & & & & & & | & & & & \\ \underset{1}{\circ} & - & \underset{2}{\circ} & - & \underset{3}{\circ} & - & \underset{4}{\circ} & - & \underset{5}{\circ} & - & \underset{6}{\circ} & - & \underset{4}{\circ} & - & \underset{2}{\circ} \end{array}\right)$$

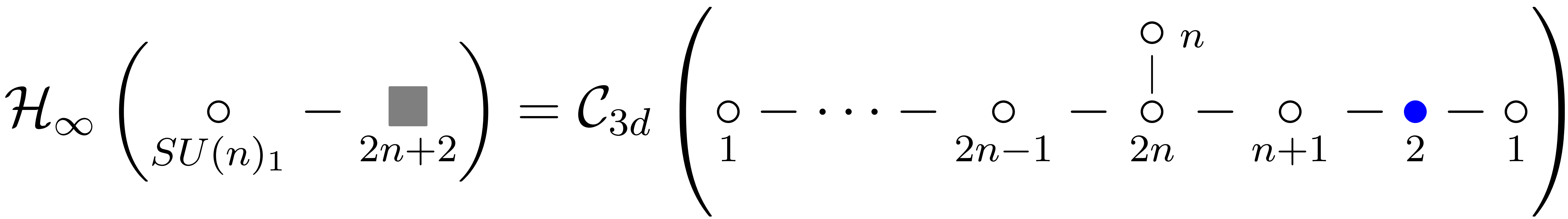
$$\mathcal{H}_\infty\left(\underset{SU(n)_{\frac{1}{2}}}{\circ} \;-\; \underset{2n+3}{\blacksquare}\right) = \mathcal{C}_{3d}\left(\begin{array}{ccccccccccc} & & & & & & \circ\, n+1 & & & & \\ & & & & & & | & & & & \\ \underset{1}{\circ} & - & \cdots & - & \underset{2n+1}{\circ} & - & \underset{2n+2}{\circ} & - & \underset{n+2}{\circ} & - & \underset{2}{\bullet} \end{array}\right)$$

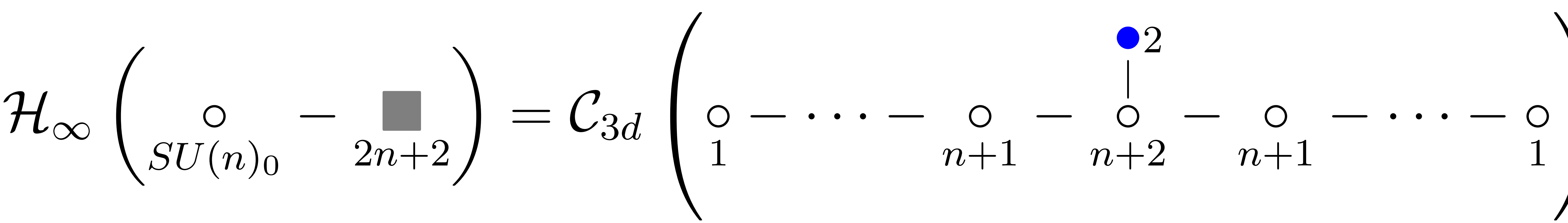
Generators of the chiral ring

- The global symmetry of this Coulomb branch SO(4n + 8)
- An adjoint valued operator at spin 1 of SU(2) R
- A spinor valued operator at spin n/2 of SU(2) R
- Mesons, baryons, gaugino bilinear, instantons
- satisfying relations that are dictated by the quiver and the symmetries

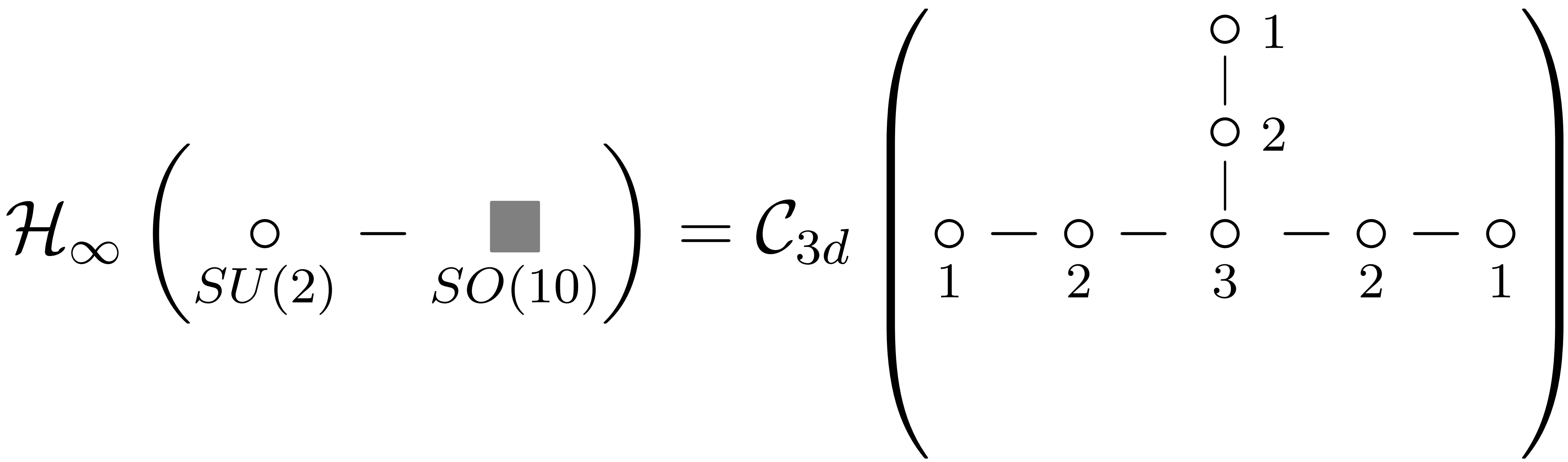
E7 sequences

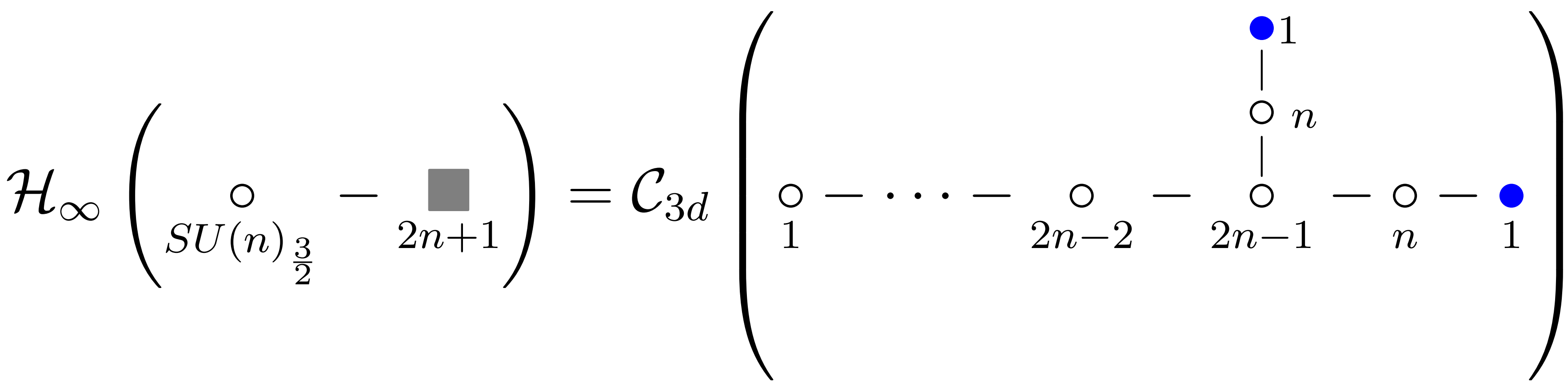


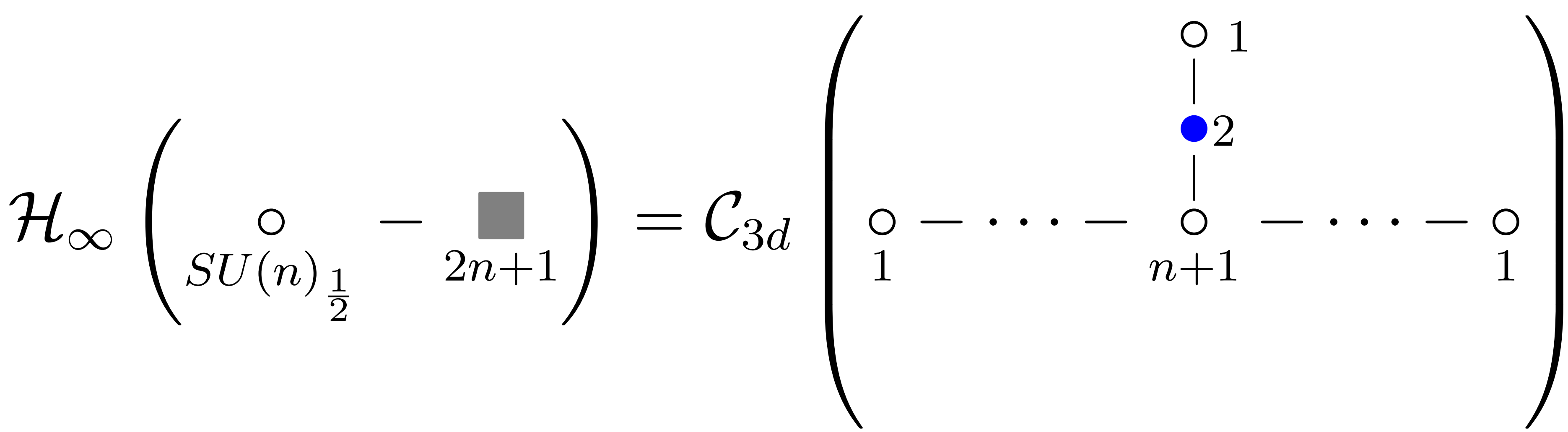




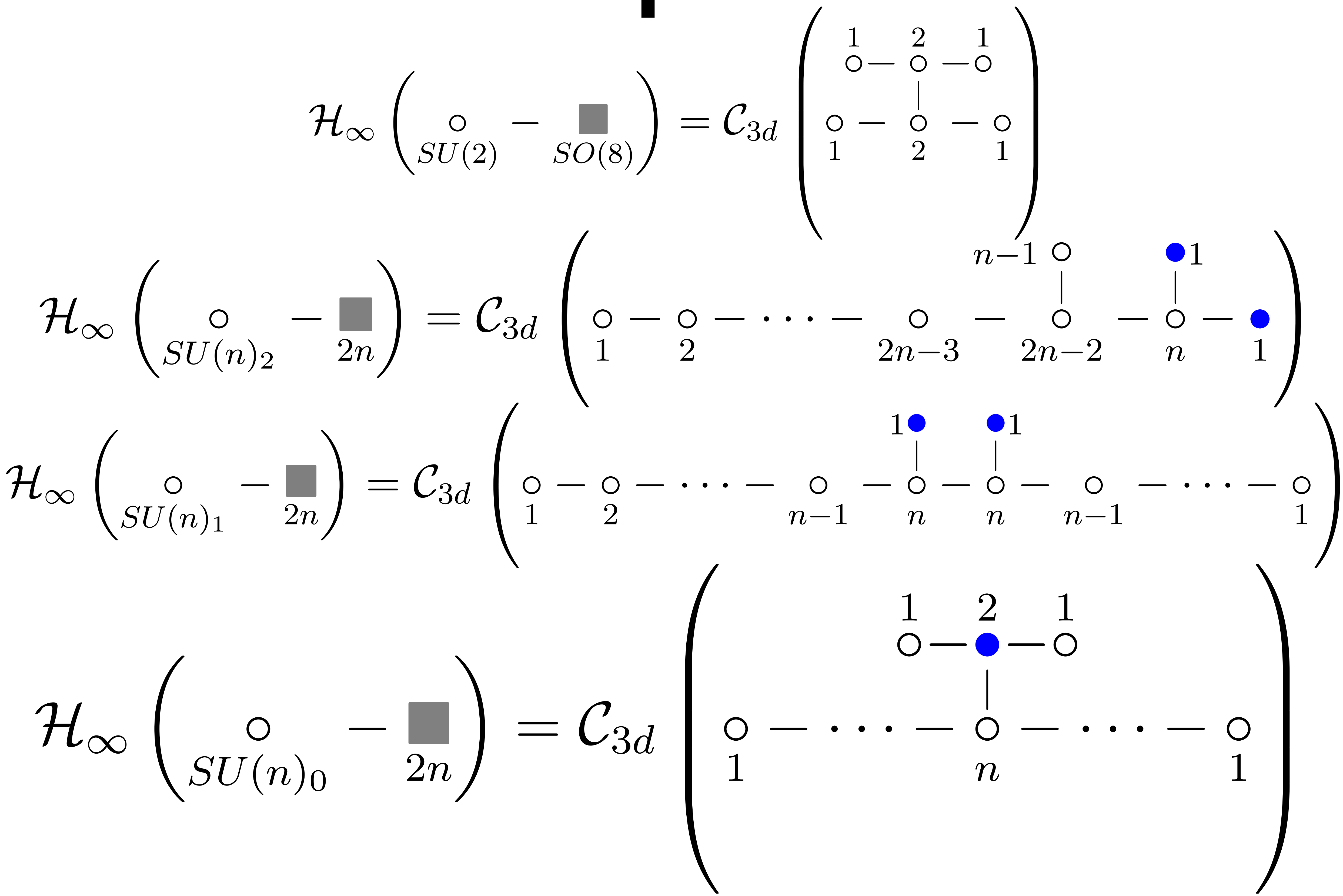
E6 sequences



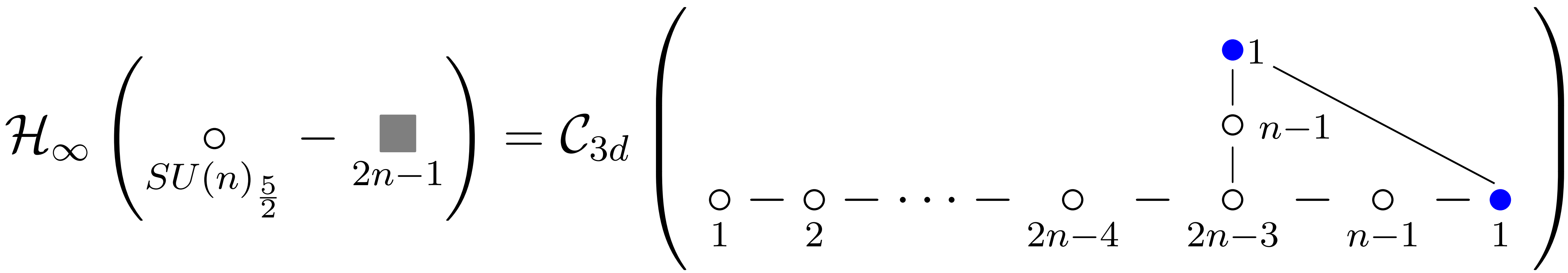


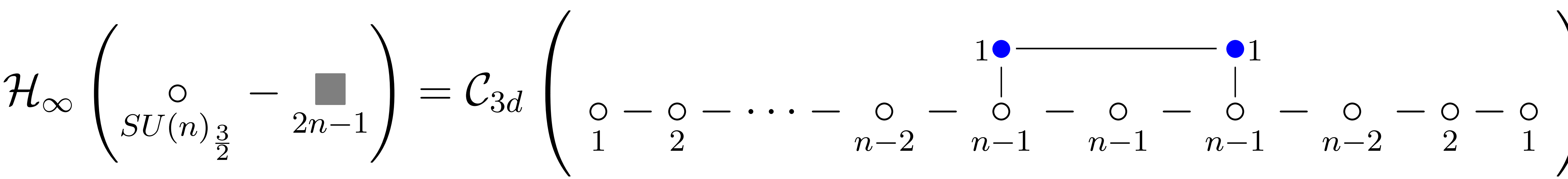


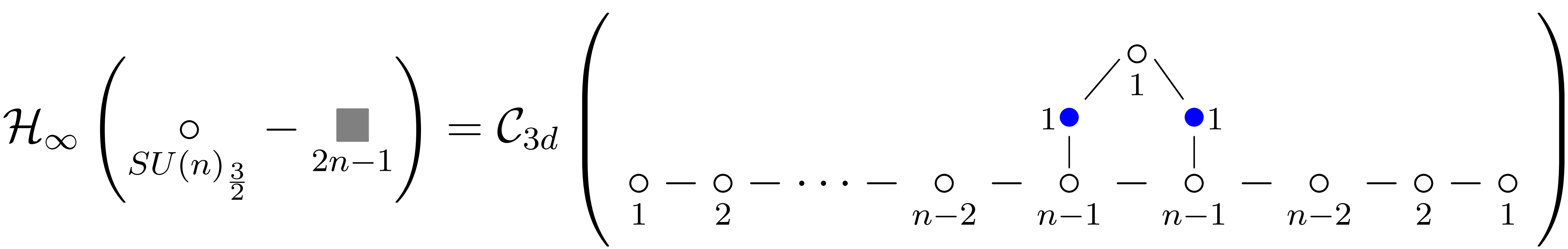
E5 sequences



E4 sequences







Minimally unbalanced quivers

- Quivers with very few unbalanced nodes
- Global symmetry
- 1 unbalanced node gives either a single non Abelian factor or 2 non Abelian factors
- If there are p unbalanced nodes, expect p-1 U(1) factors
- Gradually complicated moduli spaces

Summary

- The Higgs branch at infinite coupling of a 5d theory is conveniently encoded by a coulomb branch of a 3d N=4 theory

- The chiral ring can be derived from this description

- The generators of the chiral ring are simple and describe the solution to this problem

- Get a window to non perturbative effects made by instantons and the precise way they correct classical relations in the chiral ring

Thank you !