Logarithmic corrections to black hole entropy from Kerr/CFT

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Outline

- Logarithmic corrections to $S_{BH} = \frac{A_H}{4G}$
- Kerr/CFT correspondence $\Rightarrow S_{BH} = S_{Cardy}$
- Match of log corrections to S_{BH} with those to S_{Cardy}

Logarithmic corrections to Bekenstein-Hawking

[Sen (2014), and references therein]

$$S = \ln d(q)$$

- Bekenstein-Hawking formula valid only for classical, two-derivative gravity
  - Wald formula includes classical higher-derivative "stringy" corrections to the action
- Full quantum+stringy corrections would require doing path integral near the BH horizon over infinite number of fields in string theory.
- Approximation: Compute $\ln d(q)$ for large q

Logarithmic corrections to Bekenstein-Hawking

- Typically, the leading entropy is a homogeneous function of the *q*'s:
$$S_{BH}(aq) = a^{D-2} S_{BH}(q)$$
- In the limit $a \to \infty$ the leading corrections to S_{BH} are logarithmic:
$$\Delta S \propto \ln a$$
- Log corrections arise from 1-loop corrections to the leading saddle point result for $Z = \int d\Psi e^{-\text{Action}}$ from loops of *massless fields only*

  $\Rightarrow$ Log corrections from low energy data only: spectrum of massless fields and their coupling to BH background

Logarithmic corrections to Bekenstein-Hawking

- For D-dim *non-extremal* BH with angular momenta $\vec{J}$ and charges $\vec{Q}$ define the microcanonical entropy as:

  $e^{S_{mc}(M,\vec{J},\vec{Q})}\delta M$ = # of microstates with angular momenta $\vec{J}$ and charges $\vec{Q}$ in the mass range δM.

- Sen's main result: [Sen (2013)]

$$S_{mc}(M,\vec{J},\vec{Q}) = S_{BH}(M,\vec{J},\vec{Q}) + \\ + \left(C_{\text{local}} - \frac{D-4}{2} - \frac{D-2}{2}N_C - \frac{D-4}{2}n_V \right) \ln a$$

  where:

  - a is the size of the BH such that $A \sim a^{D-2}$
  - $N_C = \left[\frac{D-1}{2}\right]$ is the # of Cartan generators of $SO(D-1)$
  - n_v is the # of $U(1)$ gauge fields in the theory.
  - C_{local} is related to the trace anomaly due to massless fields in the BH background. Note that: $C_{\text{local}} = 0$ when $D = \text{odd}$.

The Kerr/CFT correspondence

[Guica, Hartman, Song, Strominger (2009)]

‘weak’ Kerr/CFT

the *fact* that gravitational dynamics in the near horizon region of a near-extreme Kerr are constrained by an infinite-dim conformal symmetry

- Suffices for interesting questions in observational astronomy.
    1. Gravitational waves from extreme-mass-ratio-inspirals
    2. Optical appearance of electromagnetic sources

‘strong’ Kerr/CFT

the *conjecture* that quantum gravity in the near horizon region of a near-extreme Kerr is dual to a (warped/dipole/???) 2D CFT

- Relevant for quantum black hole puzzles (e.g. $S_{BH} = S_{Cardy}$)
- Bottom-up: a few dictionary entries are known and some string theoretic constructions are partially understood

The Kerr/CFT correspondence

[Bredberg,Keeler, Lysov, Strominger (2011); Compere (2012)]

- Near-horizon metric of rotating near-extreme BHs:

$$ds^2 = \Omega\left[-r(r+2\kappa)dt^2 + \frac{dr^2}{r(r+2\kappa)} + \Lambda\,(d\psi + (r+\kappa)dt + \ldots)^2 + \ldots\right]$$

- Isometry group contains: $SL(2,\mathbb{R})_R \times U(1)_L$.
  ∂_t in $SL(2,\mathbb{R})_R$, ∂_ψ is $U(1)_L$ [BH rotates in ψ direction]
- Asymptotic symmetry group (ASG): Virasoro (L and/or R), Virasoro-Kac-Moody, ... [depends on boundary conditions]
- Central charge: $c \propto J$, where $J =$ BH angular momentum.
- Frolov-Thorne temperatures (T_L, T_R) defined by:

$$e^{-\frac{\omega - m\Omega_H}{T_H}} = e^{-\frac{n_L}{T_L} - \frac{n_R}{T_R}}$$

- To linear order in T_H we have:

$$S_{BH} = S_{\text{Cardy}}\,, \quad \text{i.e.} \quad \frac{A_H}{4G} = \frac{\pi^2}{3} c\, T_L + \frac{\pi^2}{3} c\, T_R\,.$$

Match of logarithmic corrections to S_{BH} from Kerr/CFT

A stringy embedding of Kerr/CFT

[Guica, Strominger (2011); Song, Strominger (2012)]

- Supersymmetric M/string compactifications to 5D contain also non-supersymmetric Kerr-Newman BHs
- Action (bosonic sector of minimal 5D SUGRA):

$$S_5 = \frac{1}{4\pi^2}\int d^5x \left(\sqrt{-g}\left(R - \frac{3}{4}F^2\right) + \frac{1}{4}\epsilon^{abcde}A_a F_{bc} F_{de}\right)$$

- BH solution:

$$\begin{aligned} ds_5^2 &= -\frac{(a^2+\hat{r}^2)(a^2+\hat{r}^2-M_0)}{\Sigma^2}\, d\hat{t}^2 + \Sigma\left(\frac{\hat{r}^2 d\hat{r}^2}{f^2 - M_0\hat{r}^2} + \frac{d\theta^2}{4}\right) \\ &\quad -\frac{M_0 F}{\Sigma^2}\,(d\hat{\psi} + \cos\theta\, d\hat{\phi})\, d\hat{t} + \frac{\Sigma}{4}(d\hat{\psi}^2 + d\hat{\phi}^2 + 2\cos\theta\, d\hat{\psi}\, d\hat{\phi}) \\ &\quad + \frac{a^2 M_0 B}{4\Sigma^2}\,(d\hat{\psi} + \cos\theta\, d\hat{\phi})^2\,, \\ A &= \frac{M_0 \sinh 2\delta}{2\Sigma}\left(d\hat{t} - \frac{1}{2}a e^{\delta}(d\hat{\psi} + \cos\theta\, d\hat{\phi})\right) \end{aligned}$$

where:

$$\Sigma = \hat{r}^2 + a^2 + M_0 s^2\,, \quad f = \hat{r}^2 + a^2\,, \quad s = \sinh\delta\,, c = \cosh\delta\,,$$

$$F = a(\hat{r}^2 + a^2)(c^3 + s^3) - aM_0 s^3\,, \quad B = a^2 + \hat{r}^2 - 2M_0 s^3 c^3 - M_0 s^4(2s^2 + 3)$$

Match of logarithmic corrections to S_{BH} from Kerr/CFT

The macroscopic entropy from the bulk

- Three-parameter (a, M_0, δ) BH family. Physical quantities:

$$M = \frac{3M_0}{2}\cosh 2\delta\,, \quad J_L = aM_0\,(c^3 + s^3)\,, \quad Q = M_0 sc$$

- $SU(2)_R$ charge set to $J_R = 0$. $SU(2)_L$ angle $\hat{\psi} \sim \hat{\psi} + 4\pi$
- Near extremality,

$$\kappa \equiv \sqrt{M_0 - 4a^2}/a \ll 1$$

the Bekenstein-Hawking entropy is given by:

$$S_{BH\,\mathrm{near\,ext}} = 8\pi a^3(c^3 - s^3) + 4\pi a^3(c^3 + s^3)\,\kappa + \mathcal{O}(\kappa^2)$$

- In the $a \to \infty$ limit the logarithmic correction to S_{BH} is ($D = 5 \Rightarrow C_{\mathrm{local}} = 0\,, N_C = 2$ and $n_V = 1$):

$$\Delta S_{\mathrm{bulk}} = -4\ln a$$

Match of logarithmic corrections to S_{BH} from Kerr/CFT

The near-horizon solution

- Making the coordinate+gauge transformation

$$t = \frac{1}{2}\Omega_{L\,\text{ext}}\,\hat{t}\,, \quad r = \frac{\hat{r}^2 - r_+^2}{r_+^2}\,, \quad \psi = \hat{\psi} - \Omega_{L\,\text{ext}}\hat{t}\,, \quad A \to A - \Phi_{\text{ext}} d\hat{t}$$

the near-horizon solution becomes:

$$\begin{aligned} ds_5^2 &= \frac{M_{\text{ext}}}{12}\Big[-r(r+2\kappa)dt^2 + \frac{dr^2}{r(r+2\kappa)} + d\theta^2 + \sin^2\theta d\phi^2 \\ &\qquad + \frac{27 J_{L\,\text{ext}}^2}{M_{\text{ext}}^3}\big(\pi T_L(d\psi + \cos\theta\, d\phi) + (r+\kappa)dt\big)^2\Big] \\ A &= -\frac{1}{2} a\, e^{\delta}\tanh 2\delta\,\big(d\psi + \cos\theta d\phi + e^{-2\delta}(r+\kappa)dt\big) \end{aligned}$$

- Here (T_L, T_R) are the Frolov-Thorne temperatures conjugate to $(t_L, t_R) = (\psi, t)$:

$$T_L = \frac{1}{\pi}\frac{c^3 - s^3}{c^3 + s^3}\,, \qquad T_R = \frac{\kappa}{2\pi}$$

Match of logarithmic corrections to S_{BH} from Kerr/CFT

The microscopic entropy from the CFT

- Consider a CFT with the global symmetries enhanced as

$$SL(2,R)_R \times U(1)_L \to Vir_R \times Vir_L$$

- ASG analysis $\Rightarrow c_L = 6J_{L\,\mathrm{ext}}$. Expect: $c_R = c_L$.
- Cardy formula reproduces Bekenstein-Hawking entropy:

$$\begin{aligned} S_{Cardy} &= \frac{\pi^2}{3} c_L T_L + \frac{\pi^2}{3} c_R T_R \\ &= 8\pi a^3 (c^3 - s^3) + 4\pi a^3 (c^3 + s^3)\,\kappa = S_{BH} \end{aligned}$$

- Cardy formula follows from modular invariance of

$$Z(\tau, \bar{\tau}, \vec{\mu}) = \mathrm{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} \bar{L}_0 + 2\pi i \mu_i P^i}$$

  where $4\pi\tau = \beta_L - \beta_R + i(\beta_L + \beta_R)$ and $(\mu_1, \mu_2) = (\mu_R, \mu_Q)$ are chemical potentials corresponding to the additional global symmetries $SU(2)_R \times U(1)_Q$ of our Kerr-Newman.

Match of logarithmic corrections to S_{BH} from Kerr/CFT

The microscopic entropy from the CFT

- The modular transformation rule for this partition function is

$$Z(\tau, \bar{\tau}, \vec{\mu}) = e^{-\frac{2\pi i \mu^2}{\tau}} Z\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}, \frac{\vec{\mu}}{\tau}\right)$$

  where $\mu^2 \equiv \mu_i \mu_j k^{ij}$ with k^{ij} the matrix of Kac-Moody levels of the associated currents. We expect that $k_J \propto c$ and we compute: $k_Q = 12\,(2\pi T_L)^2 a e^{\delta} \tanh 2\delta$

- For small τ, this implies that we project onto the vacuum:

$$Z(\tau, \bar{\tau}, \vec{\mu}) \approx e^{-\frac{2\pi i \mu^2}{\tau}} e^{-\frac{2\pi i E_L^v}{\tau} + \frac{2\pi i E_R^v}{\bar{\tau}} + \frac{2\pi i \mu_i p_v^i}{\tau}}$$

- The density of states at high temperatures then becomes

$$\rho(E_L, E_R, \vec{p}) \simeq \int d\tau d\bar{\tau}\, d^2\mu\, e^{2\pi i\left(-\frac{\mu^2}{\tau} - \frac{E_L^v}{\tau} + \frac{E_R^v}{\bar{\tau}} - E_L \tau + E_R \bar{\tau} - \mu_i p^i\right)}$$

  where we assumed an electrically neutral vacuum, $p_v^i = 0$. This integral may be evaluated by saddle point methods

Match of logarithmic corrections to S_{BH} from Kerr/CFT

The microscopic entropy from the CFT

- The leading contribution at the saddle gives:

$$S = \log \rho_0 = 2\pi\sqrt{-E_L^v\left(4E_L - \mathcal{P}^2\right)} + 2\pi\sqrt{-E_R^v\left(4E_R\right)}$$

  Here $\mathcal{P}^2 \equiv p^i p^j k_{ij}$ and the matrix k_{ij} is the inverse of k^{ij}.

  This is the Cardy formula in microcanonical form; putting $E_L^v = E_R^v = -c/24\,,\, E_L - \mathcal{P}^2/4 = (\pi^2/6)c\,T_L^2\,,\, E_R = (\pi^2/6)c\,T_R^2\,,$ yields the canonical version $S = \frac{\pi^2}{3}c\,T_L + \frac{\pi^2}{3}c\,T_R$.

- The logarithmic correction to the above is generated by Gaussian fluctuations about the saddle:

$$\Delta S = -\frac{1}{2}\log\frac{1}{16}\left(-E_L^v\right)^{-\frac{3}{2}}\left(4E_L - \mathcal{P}^2\right)^{\frac{5}{2}}\left(-E_R^v\right)^{-\frac{1}{2}}\left(4E_R\right)^{\frac{3}{2}}\det k^{ij}$$

  We have the following scalings,

$$E_L^v\,,\, E_R^v\,,\, E_L - \mathcal{P}^2/4\,,\, E_R \sim a^3\,,\quad k_Q \sim a\,,\quad k_J \sim a^3$$

  so we obtain:

$$\Delta S = -5\ln a$$

Match of logarithmic corrections to S_{BH} from Kerr/CFT

The microscopic entropy from the CFT

- Recall the bulk answer:

$$\Delta S_{\text{bulk}} = -4 \ln a \tag{1}$$

- From the CFT we got:

$$\Delta S = -5 \ln a \tag{2}$$

- Q: What went wrong? A: For a sensible comparison, one must ensure both results are given in the same ensemble.
- Eq (1) assumes the entropy is a function of the energy $Q[\partial_{\hat{t}}]$ conjugate to the asymptotic time. Eq (2) is a function of the energy $Q[\partial_t]$ conjugate to the near-horizon time
- Adjusting by the appropriate Jacobian we get

$$\rho_{\text{bulk}} = \frac{\delta Q[\partial_t]}{\delta Q[\partial_{\hat{t}}]}\,\rho \quad \Rightarrow \quad \Delta S_{\text{bulk}} = \Delta S + \ln a$$

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Thank you