# Exotic RG Flows from Holography

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# Bibliography

Published work in:
E. Kiritsis, F. Nitti and LSP arxiv:1611.05493 [hep-th]

Based on earlier work:

F. Nitti, Wenliang Li and E. Kiritsis arxiv:1401.0888 [hep-th]

V. Niarchos and E. Kiritsis arxiv:1205.6205 [hep-th]

# Outline of the talk

1. Introduction to holographic RG flows.
2. 1st order formalism and "standard" RG flows
3. Introduce the exotic RG flows:
   a) **Irrelevant VEV flows**: flows between two minima of the bulk potential,
   b) **Skipping flows**: flow does not stop the nearest fixed point,
   c) **Bouncing flows**: multi-branched β-function.
4. Conclusions and Outlook

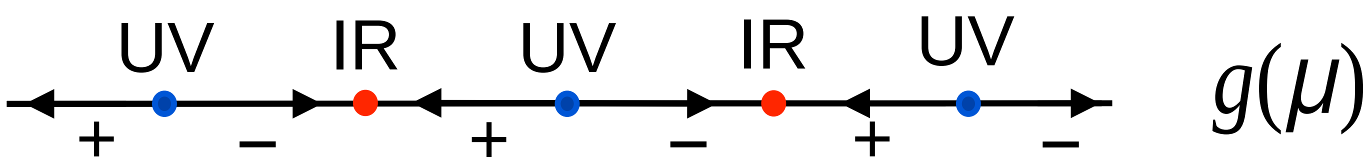
# 1 - Introduction

- (Perturbative) **QFT** RG flow: set of 1st order equations.
- For a single coupling g:

$$\frac{dg}{d\log\mu} = \beta(g)$$

- Flow direction $\longleftrightarrow$ sign of β.

- 1st order flows localized between consecutive zeroes of β; fixed points cannot be skipped.




- In AdS/CFT the RG flows become geometrical.

# The AdS/CFT correspondence

One-to-one correspondence between:

| $\mathbf{AdS_{d+1}}$<br>Bulk field | $\longleftrightarrow$ | $\mathbf{CFT_d}$<br>Gauge invariant, single-trace operators at d-dim Minkowski space |
|---|---|---|

In particular (invert):

Scalar operator $\mathcal{O}$ (x) $\longleftrightarrow$ Scalar field $\phi$ (x,u)

Stress-en. tensor $T_{\mu\nu}$(x) $\longleftrightarrow$ Metric $g_{\mu\nu}$(x,u)

# The AdS/CFT correspondence

- Consider a boundary scalar operator $\mathcal{O}(x)$ with conformal dimension $\Delta > d/2$.

- The asymptotic expansion of $\phi$ is:

$$\phi(x,u) = e^{u(\Delta - d)}\phi_0(x) + e^{u\Delta}\phi_+(x) + \cdots$$

where $\Delta(\Delta - d) = m^2$.

- The leading term contains the source of $\mathcal{O}(x)$:

$$\int d^d x\, \phi_0(x)\mathcal{O}(x).$$

- The sub-leading term is the VEV of $\mathcal{O}(x)$:

$$\langle \mathcal{O}(x)\rangle_{\phi_0} = (2\Delta - d)\phi_+.$$

# Holographic RG flows

| | | |
|---|---|---|
| Fields evolving along the holographic direction in asymptotically AdS space-times | $\longleftrightarrow$ | RG flows driven by the breaking of conformal symmetry in the boundary gauge theory |

Holographic dimension $\longleftrightarrow$ Energy scale in QFT

| | | |
|---|---|---|
| (Perturbative) QFT : $1^{st}$ order flow equations | $\longleftrightarrow$ | Holographic RG: $2^{nd}$ order flow equations |

Hamilton-Jacobi ($1^{st}$ order) formalism

# Holographic RG flows: the setup

$$S[g,\varphi] = \int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi)\right)$$

- Analytic and strictly negative $V(\phi)$.
- Solutions preserving Poincaré invariance:

$$ds^2 = du^2 + e^{2A(u)}\eta_{\mu\nu}dx^\mu dx^\nu$$

Coupling: $\phi = \phi(u)$

Energy scale: $\mu \equiv \mu_0 e^{A(u)}$

- Einstein equations:

$$2(d-1)\ddot{A}(u) + \dot{\phi}^2(u) = 0$$

$$d(d-1)\dot{A}(u)^2 - \frac{1}{2}\dot{\phi}^2(u) + V(\phi) = 0$$

# 2 - First order formalism and "standard" RG flows

# 1st order formalism: Superpotential

Define the function $W(\phi)$ via

$$W(\phi(u)) = -2(d-1)\dot{A}(u)$$

To satisfy the Einstein equation $2(d-1)\ddot{A}(u) + \dot{\phi}^2(u) = 0$

the scalar field must obey:

$$W'(\phi) = \dot{\phi}(u)\,.$$

The remaining Einstein equation takes the form:

$$\boxed{V(\phi) = \frac{1}{2}W'^2(\phi) - \frac{d}{4(d-1)}W^2(\phi)}$$

There are 3 integration constants.

# 1st order formalism: Superpotential

Coupling: $\phi = \phi(u)$

Energy scale: $\mu \equiv \mu_0 e^{A(u)}$

- Using the first order formalism,

$$\dot{\phi}(u) = W'(\phi) \qquad \dot{A}(u) = -\frac{W(\phi)}{2(d-1)}$$

the holographic β-function is:

$$\beta(\phi) \equiv \frac{d\phi}{dA} = -2(d-1)\frac{d\log W}{d\phi}$$

- Zeroes of the holographic β-function correspond to vanishing W'(ϕ): critical points.
- Each W(ϕ) determines a holographic RG flow.

# Properties of W(ϕ)

The non-trivial part is encoded in the superpotential and our goal is to solve the equation:

$$V(\phi) = \frac{1}{2}W'^2(\phi) - \frac{d}{4(d-1)}W^2(\phi)$$

for flows interpolating between fixed points.

We start with some general properties:

1) If W(ϕ) and V(ϕ) are finite the geometry is regular.
2) W(ϕ) is bounded from below by

$$B(\phi) := \sqrt{-\frac{4(d-1)}{d}V(\phi)}$$

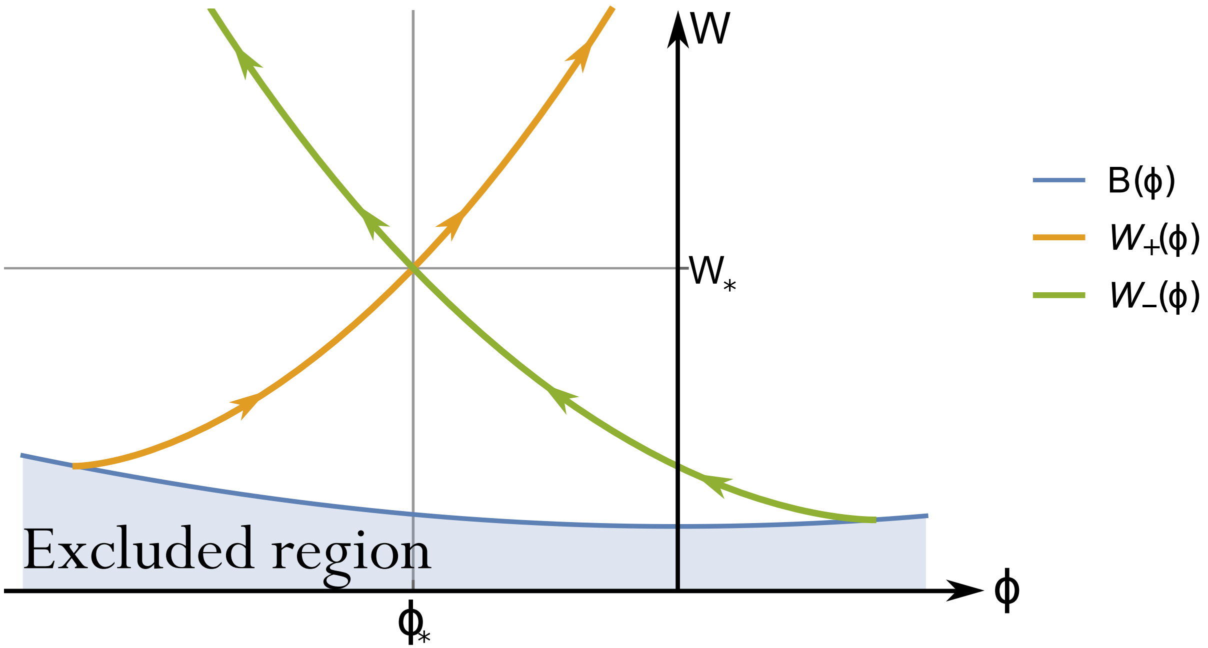
# Properties of W(ϕ)

3) At any point there are two solutions:

$$W'(\phi) \equiv \frac{dW}{d\phi} = \pm\sqrt{\frac{2d}{(d-1)}}(W^2 - B^2),$$

4) W(ϕ(u)) increases monotonically with u:

$$\frac{dW(u)}{du} = \frac{d\phi(u)}{du}\frac{dW(\phi)}{d\phi} = W'^2 \geqslant 0$$



# Critical points of W(ϕ)

Critical points of the superpotential equation are the extrema of $W(\phi)$ but not necessarily of $V(\phi)$.

$$\beta(\phi) \equiv \frac{d\phi}{dA} = -2(d-1)\frac{d\log W}{d\phi}$$

There are two kinds of critical points, those where:

a) $W''(\phi)$ is finite,

b) $W''(\phi)$ is infinite.

Through the superpotential equation they imply:

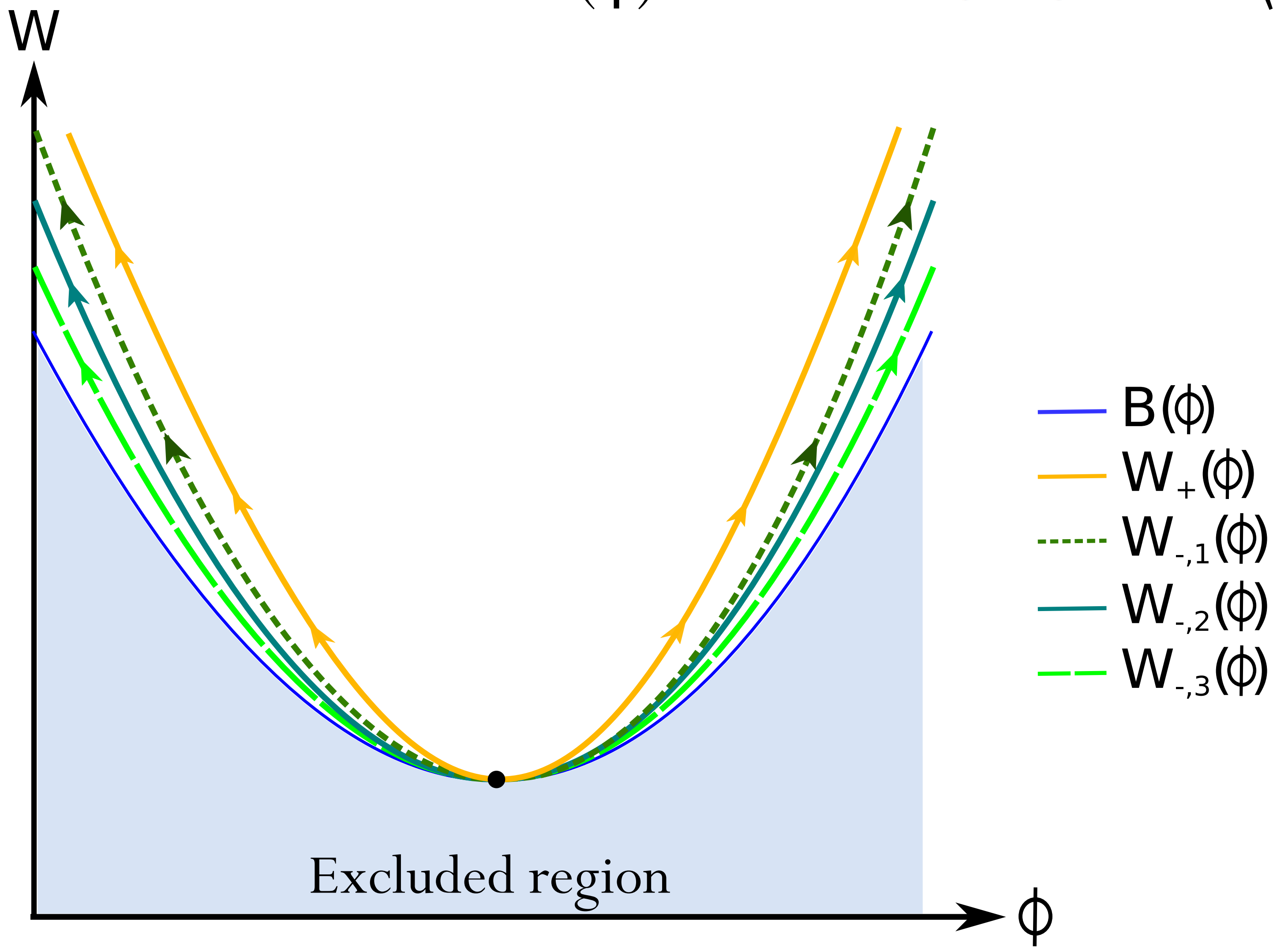
a) $V'(\phi)=0$, the scalar field stops (fixed points),

b) $V'(\phi)\neq 0$, the scalar does not stop (bounces).

# Critical points of $W(\phi)$ at extrema of $V(\phi)$: fixed points

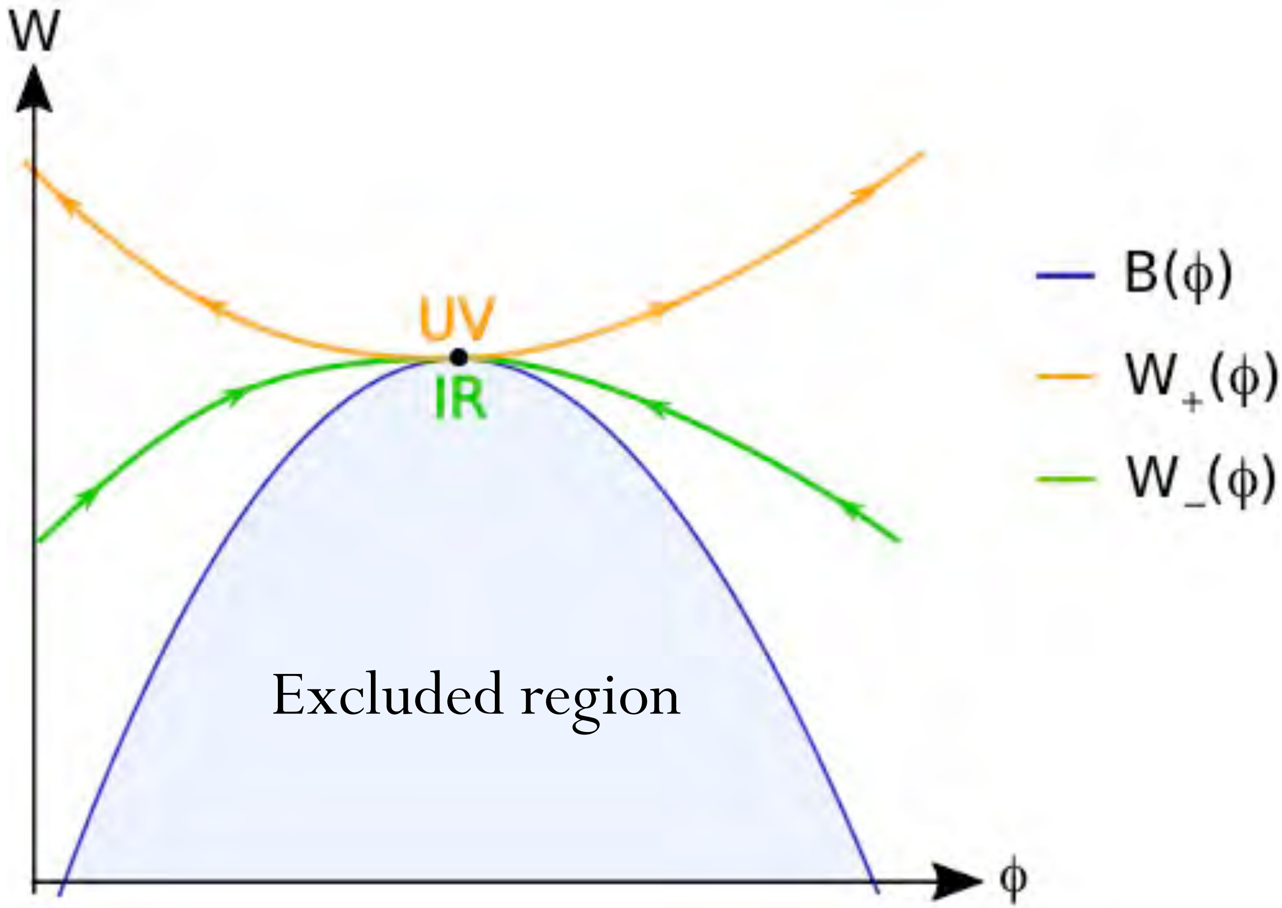
# Local maxima of V(ϕ) = UV fixed points

Integration constant for $W(\phi)$ at the UV $\longleftrightarrow$ VEV $\langle \mathcal{O}(x) \rangle_{\phi_0}$



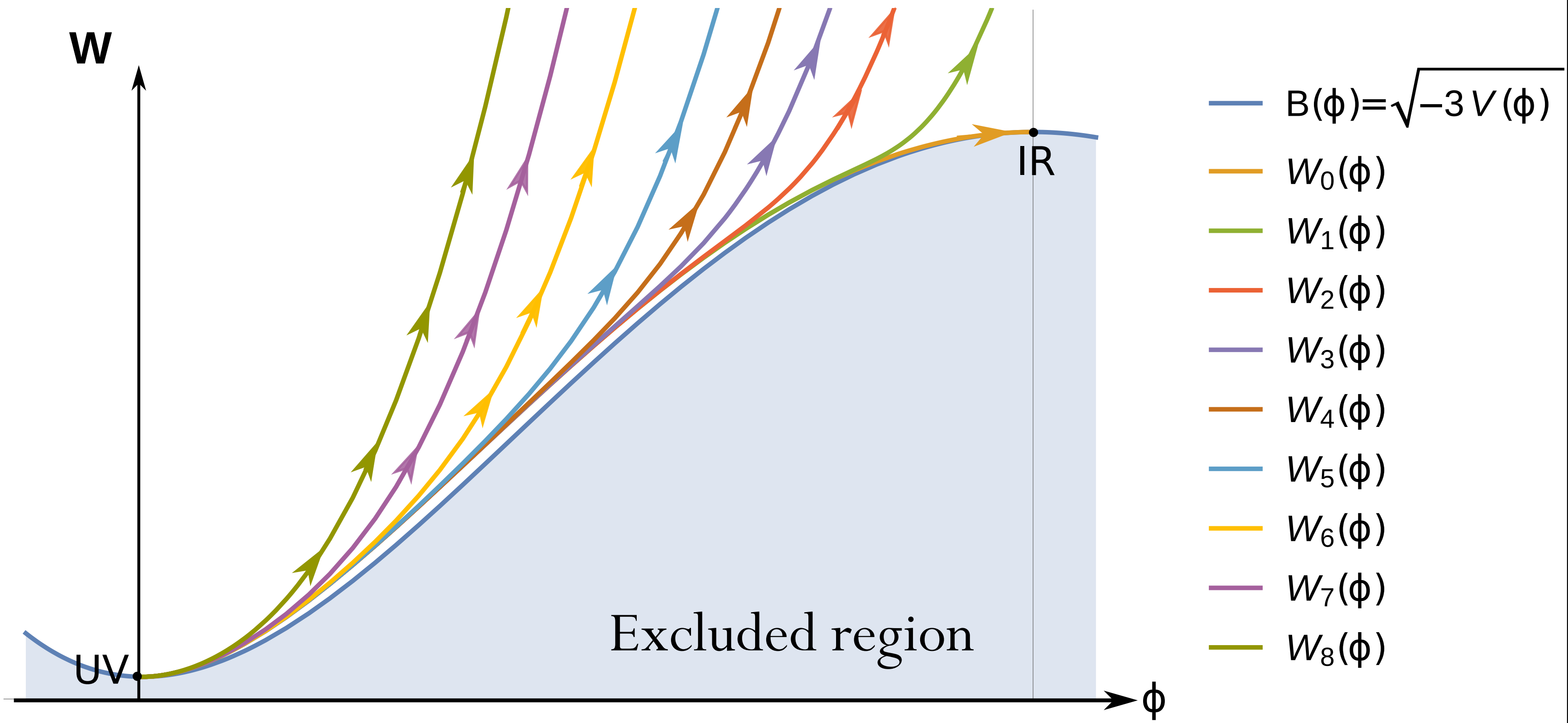
# Local minima of V = UV or IR fixed points

Only two solutions on each side of the minimum. One ends at an IR fixed point, the flows away from it with finite VEV and zero source.



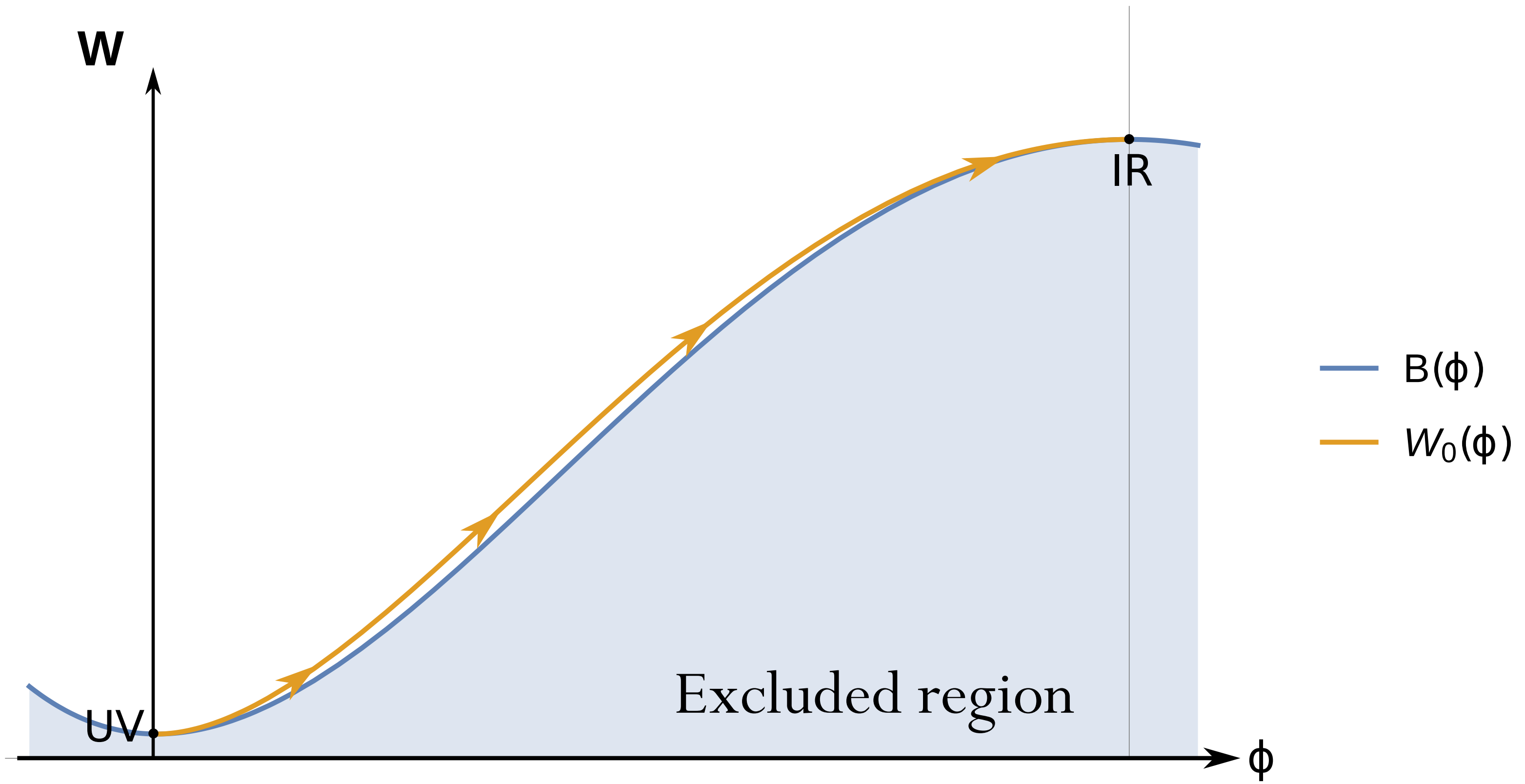
# Standard holographic RG flows

Most solutions W(ϕ) go to infinity. One is regular.



# Standard holographic RG flows

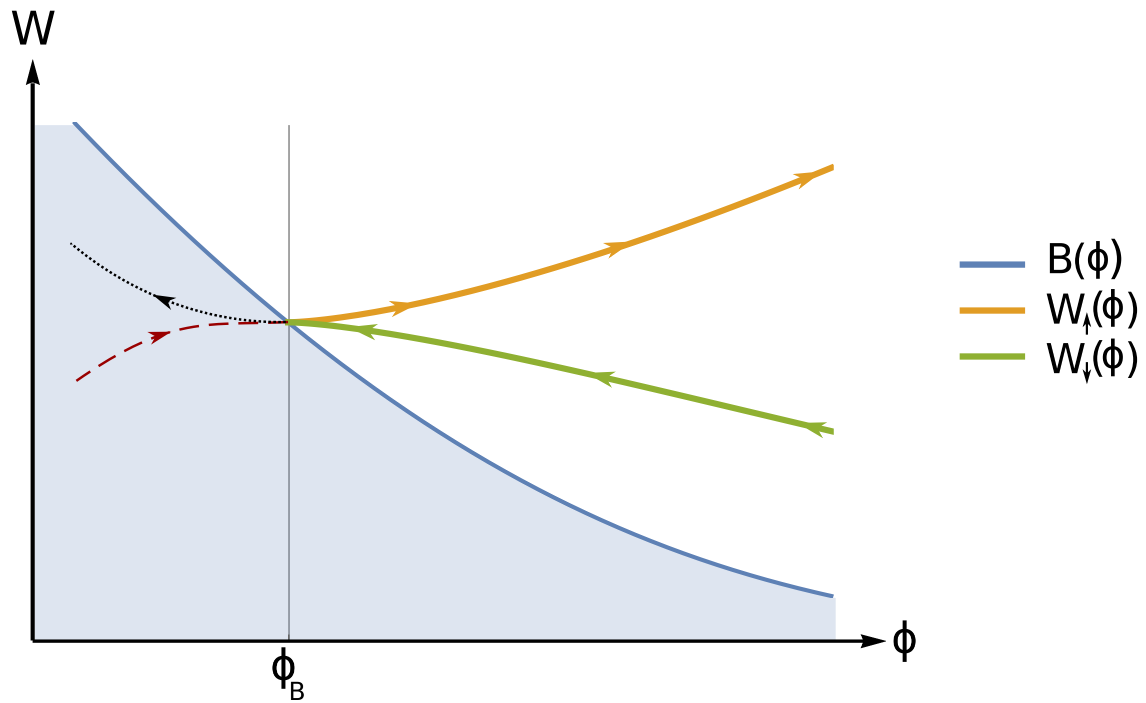
Most solutions $W(\phi)$ go to infinity. One is regular.



# Critical points of $W(\phi)$ with $V'(\phi)\neq 0$: bounces

# Bounces

The metric and scalar field are regular at the bounce.




The holographic β-function has the following form close to a bounce:

$$\beta(\phi) \propto \pm\sqrt{\phi - \phi_B} + \mathcal{O}(\phi - \phi_B)$$

# Bounces

Solving the flow equations we obtain the following regular scalar field profile:

$$\phi(u) = \phi_B + \frac{V'(\phi_B)}{2}(u - u_B)^2 + \mathcal{O}(u - u_B)^3$$

The scale factor has the expansion:

$$A(u) = A_B - \sqrt{\frac{V(\phi_B)}{d(d-1)}}(u - u_B) - \frac{(V'(\phi_B))^2}{4!(d-1)}(u - u_B)^4 + \mathcal{O}(u - u_B)^5$$

The two branches of the β-function, before and after the bounce, should be glued together, otherwise the solution is geodesically incomplete.

# Bounces

The holographic β-function has the following form close to a bounce:

$$\beta(\phi) \propto \pm\sqrt{\phi - \phi_B} + \mathcal{O}(\phi - \phi_B)$$

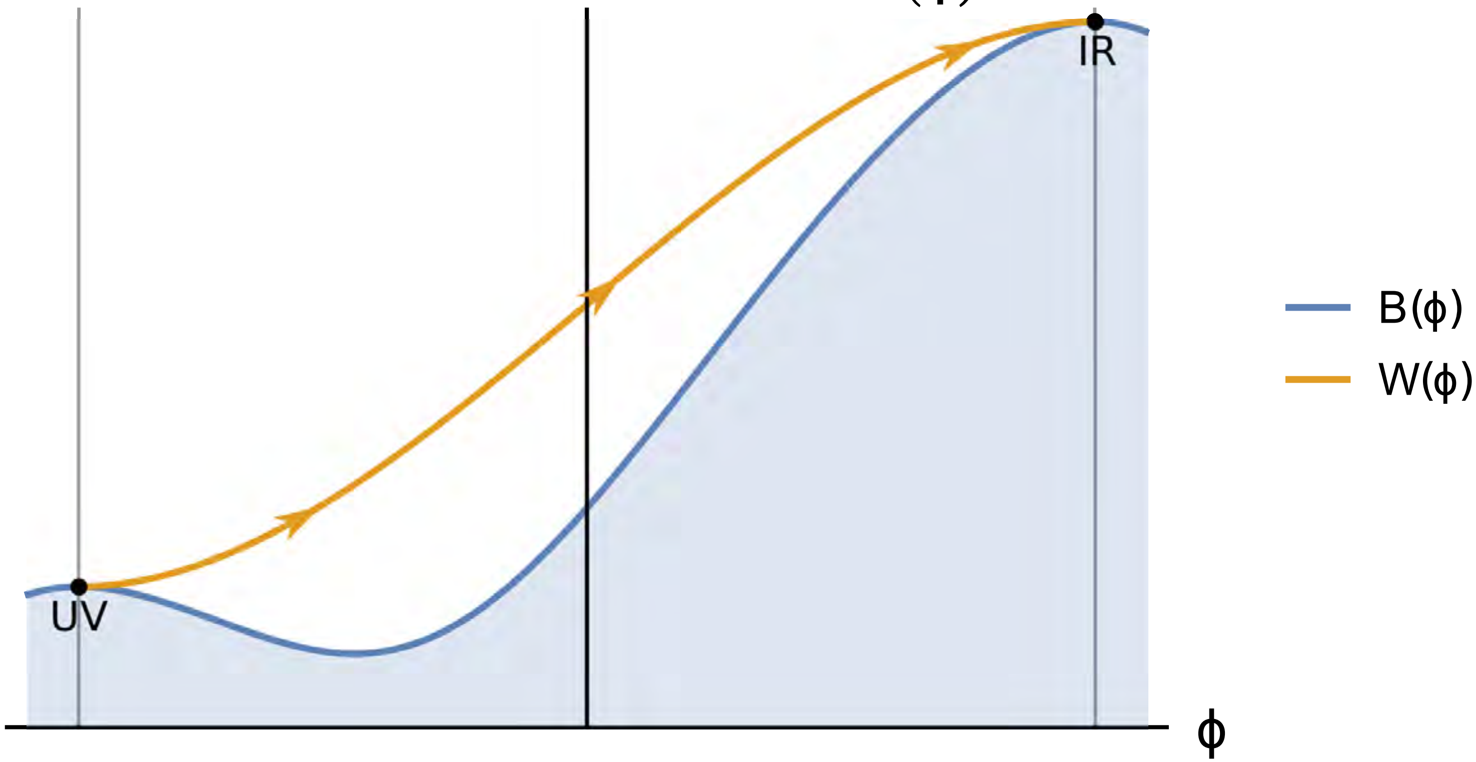
There is a branch change at $\phi = \phi_B$ and the β-function is patch-wise defined.

The β-function vanishes at the singularity but the flow does not stop, as the branches must be glued together.

# 3 - Exotic holographic RG flows

# Irrelevant VEV flows

Flows between two minima of $V(\phi)$.

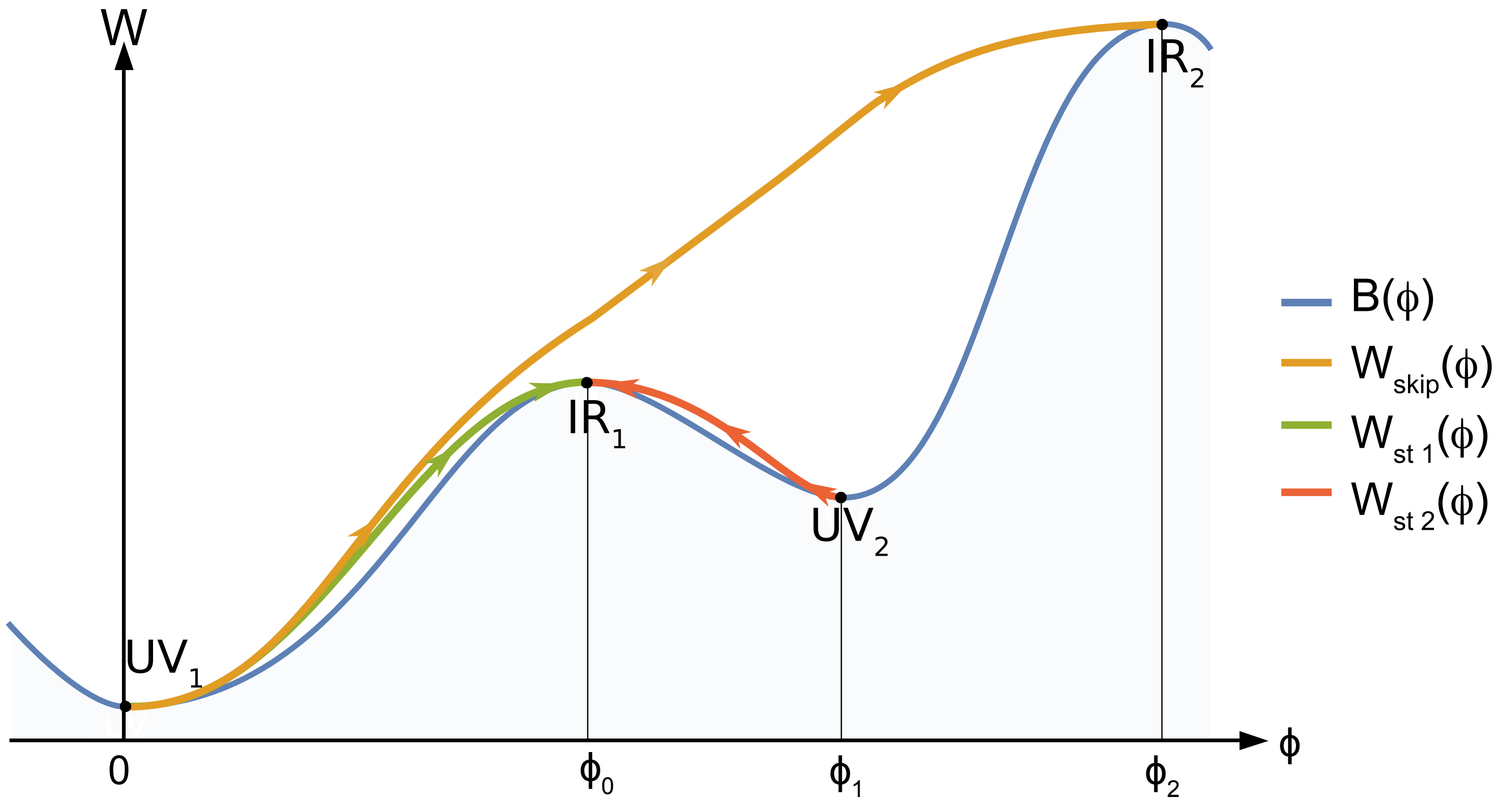



They exist in QFT. An example was discussed in a cosmological setting. Libanov+Rubakov+Sibiryakov

**RG flow driven by the VEV of an irrelevant operator.**

# "Skipping" flow

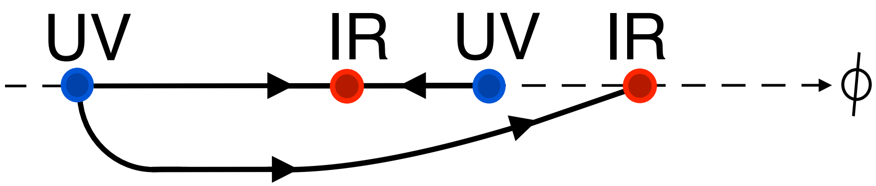
- Yellow flow does not stop at the nearest fixed point.




- No regular flow from $\phi_1$ with increasing $\phi$.
- Interpretation: analogous to $\lambda\phi^4$ theory where $\lambda<0$ yields an ill-defined theory.

# "Skipping" flow

The flow does not stop at the nearest IR fixed point.

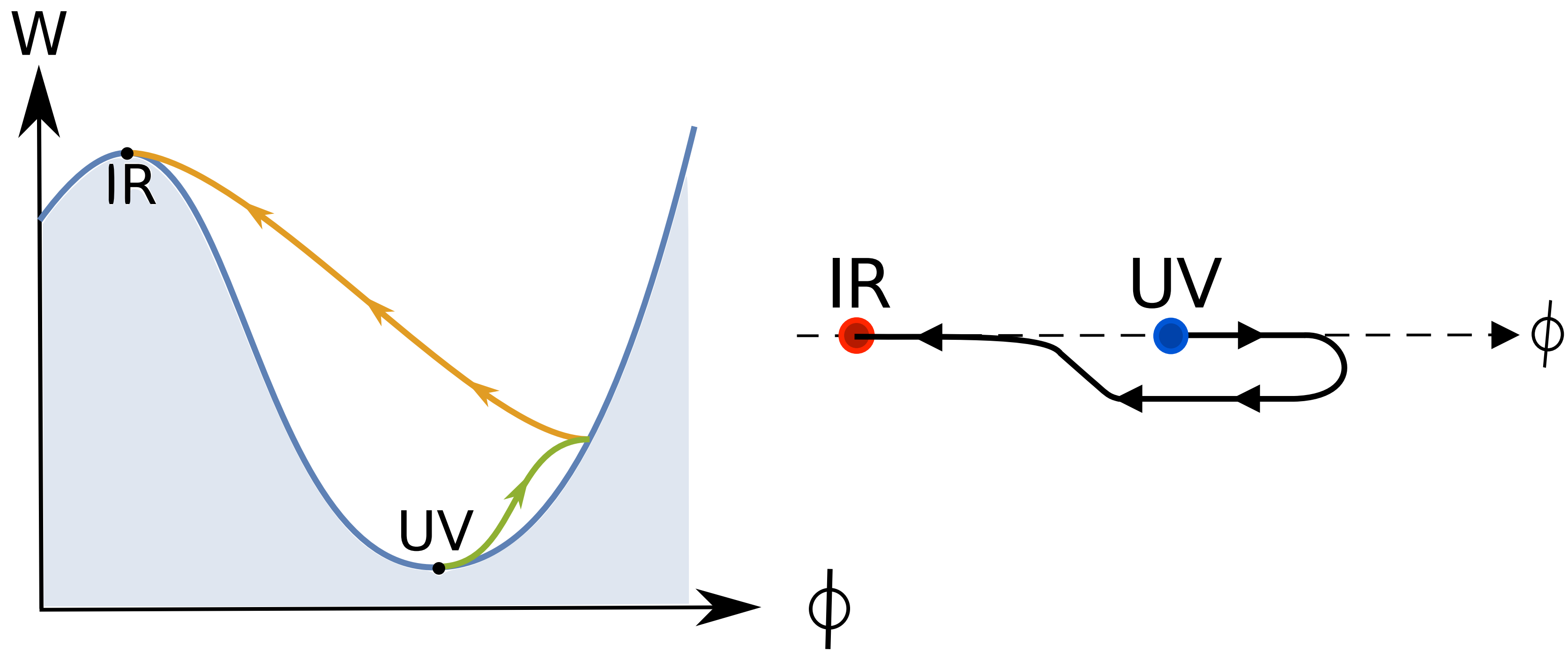



The "exotic" flow skipping the fixed point has lower free energy.

No (perturbative) QFT counterpart !

# Bouncing flows

A single running coupling changes direction along the RG flow.




From holography, these bouncing solutions should appear in RG flows in QFT.

This goes beyond perturbation theory.

# Bouncing flows

The local form of the holographic β-function near a bounce

$$\beta(\phi) \propto \pm\sqrt{\phi - \phi_B} + \mathcal{O}(\phi - \phi_B)$$

is the same as the one proposed as a way to obtain RG cycles without violating the *a*-theorem. Curtright, Jin & Zachos, '11

However, there are **no cycles** in our setup.

Cycles would required infinitely multi-branched **potentials** and we consider a single-valued bulk potential.

# Conclusions

Holographic RG flows are richer than perturbative QFT flows for a single renormalized coupling, where:

- A single renormalized coupling can skip fixed points;
- A multi-branched $\beta$-function allows the couplings to reverse direction along the flow and continue;

This may be teaching us something new about the strong coupling regime of QFT.

# Perspectives

For the single-field case:

- Verify if the exotic single-field flows exist for potentials coming from String Theory and SUGRA,
- Finite temperature case in superpotential language,
- Look for QFT realizations of the exotic flows.

(Ongoing) generalization to the multi-field case.

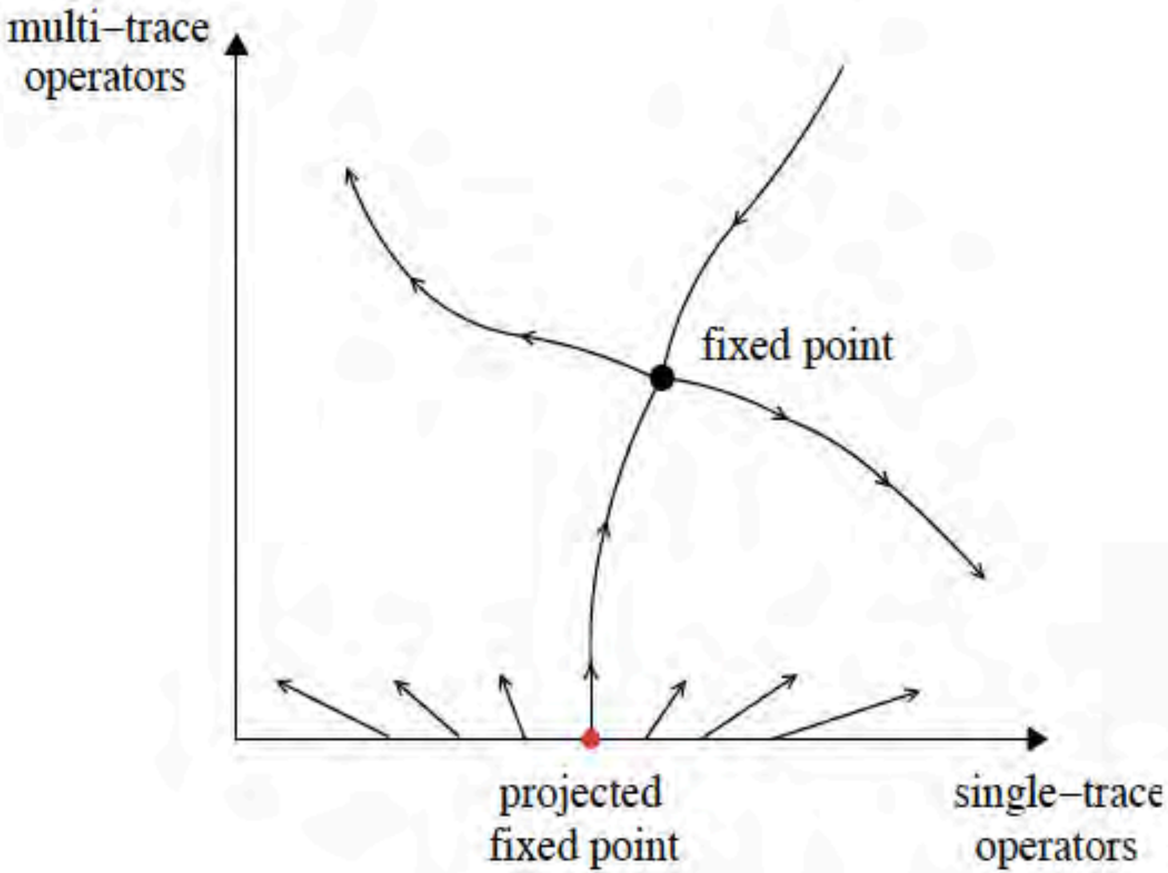
Thank you !

# Extra slides

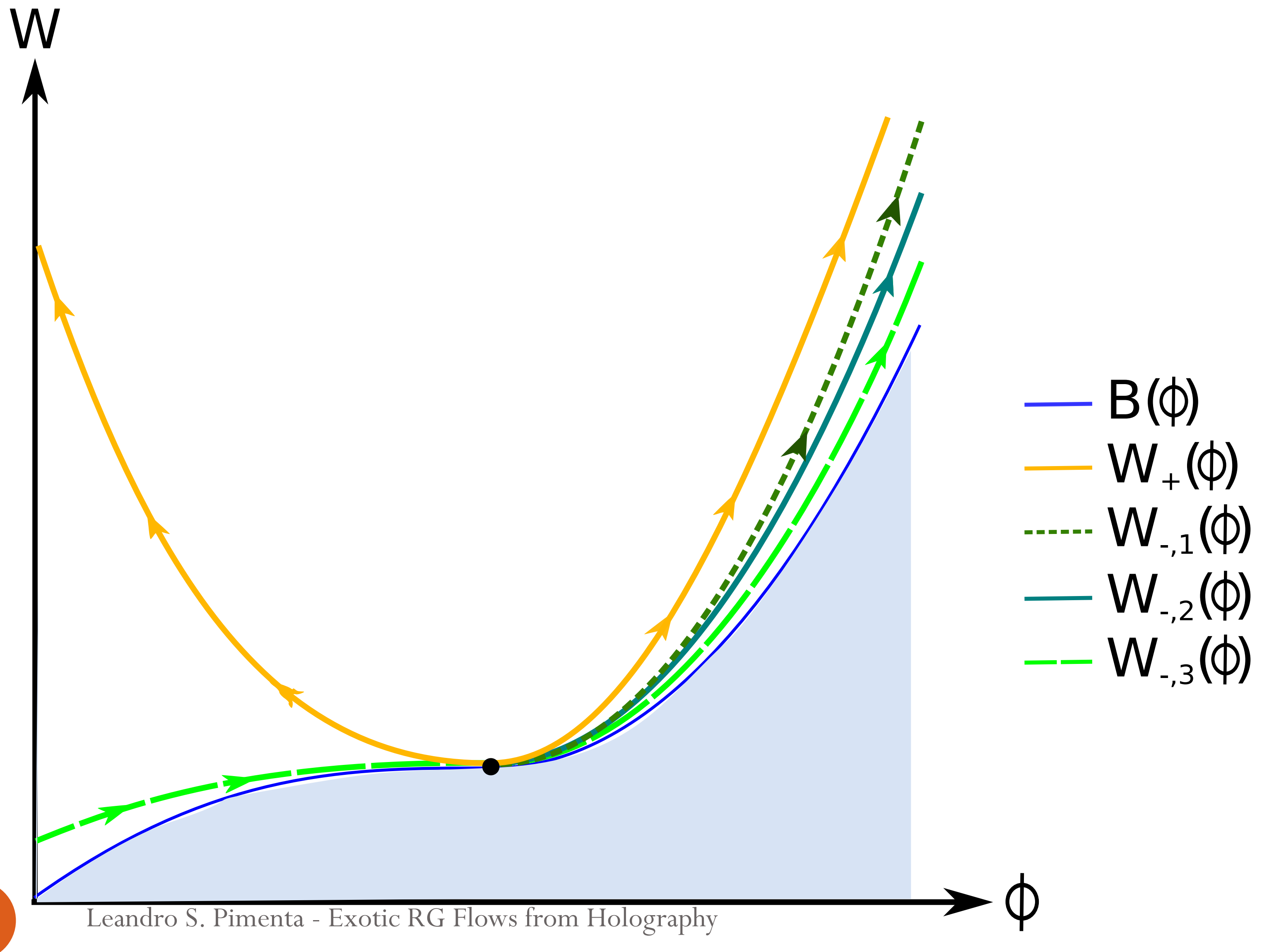
# Quantum RG flows

It has also been shown that there is a description of RG flows which leads to 2nd order equations. These are the so called Quantum RG flows.

S. S. Lee - 1305.3908 [hep-th]



# Inflection points = UV or IR fixed points



# Choosing a vacuum

If the IR is regular or computable, the on-shell action is given by the UV contribution alone:

$$S^{ren}_{on-shell} = (C - C_{ct}) \int d^d x \, \phi_-^{d/(d-\Delta)}$$

where $C$ is the integration constant in

$$W_-(\phi) = \frac{1}{\ell}\left[2(d-1) + \frac{\Delta_-}{2}\phi^2 + \mathcal{O}(\phi^3)\right] + C|\phi|^{d/\Delta_-}\left[1 + \mathcal{O}(\phi)\right] + \mathcal{O}(C^2)$$

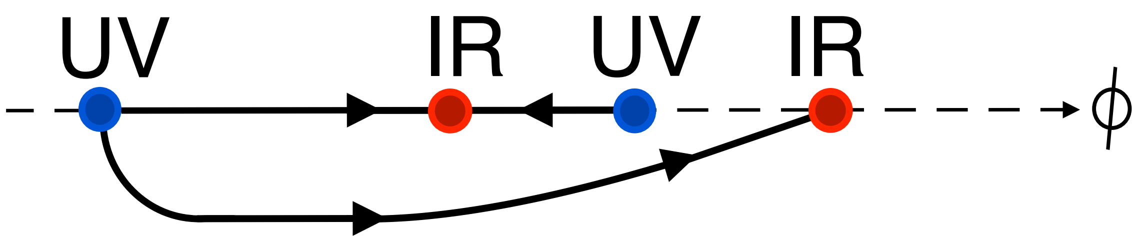
which selects a solution among the continuous family. $C_{ct}$ is a constant from the counter-term.

# Choosing a vacuum

The free-energy equals the Euclidean on-shell action:

$$\mathcal{F}_i = -(C_i - C_{ct}) \int d^4x \, \phi_-^{d/(d-\Delta)}$$

Therefore, the regular solutions with larger VEV are favorable.




The exotic flow is preferred here. The standard flow is non-perturbatively unstable by buble nucleation.