Holographic self-tuning of the cosmological constant

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work with Elias Kiritsis and Christos Charmousis, 1704.05075

Introduction: the CC and QFT

The Cosmological Constant (CC) problem arises as a clash between classical GR and QFT (in the modern effective FT sense).

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QFT: the source of semiclassical gravity becomes $\langle T_{\mu\nu} \rangle$:

$$\langle T_{\mu\nu} \rangle = \mathcal{E}_{vac} \, g_{\mu\nu} \qquad \text{in the vacuum}$$

$$G_{\mu\nu} = \Lambda_{eff} \, g_{\mu\nu}, \quad \Lambda_{eff} = \Lambda_0 + 8\pi G_N \mathcal{E}_{vac}.$$

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The problem: In a low energy EFT it is possible to compute the generic contributions to $\mathcal{E}_{vac}$ by (almost) flat space physics. The result is grossly incompatible with the observed small curvature of space-time.

Possible way out

Disconnect vacuum energy from curvature: allow large $\mathcal{E}_{vac}$ but make it so it does not gravitate.

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- braneworld in extra dimension:
  - Λ_{eff} curves the bulk, but not the brane.
  - *self-tuning:* For a generic value of $\mathcal{E}_{vac}$ flat 4d space is a solution to the field equations. Early attempts: Arkani-Hamed *et al.* '00; Kachru,Schulz,Silverstein '00.

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- Tighlty connected to holography: fundamental theory is a (UV complete) strongly coupled QFT in flat space and 4d gravity is an emergent low-energy phenomenon.

  idea explored by E. Kiritsis and myself in '06

Outline

- Setup
- Flat vacua: self-tuning
- Tensor perturbations: emergent braneworld gravity
- Scalar perturbations: stability
- Conclusions and perspectives

Setup and motivation

Consider a model with a UV conformal fixed point, made out of:

1. A strongly coupled large-N CFT, deformed by a relevant operator;
2. The weakly coupled Standard Model fields;
3. Some heavy messangers with mass scale Λ, coupling the first two.

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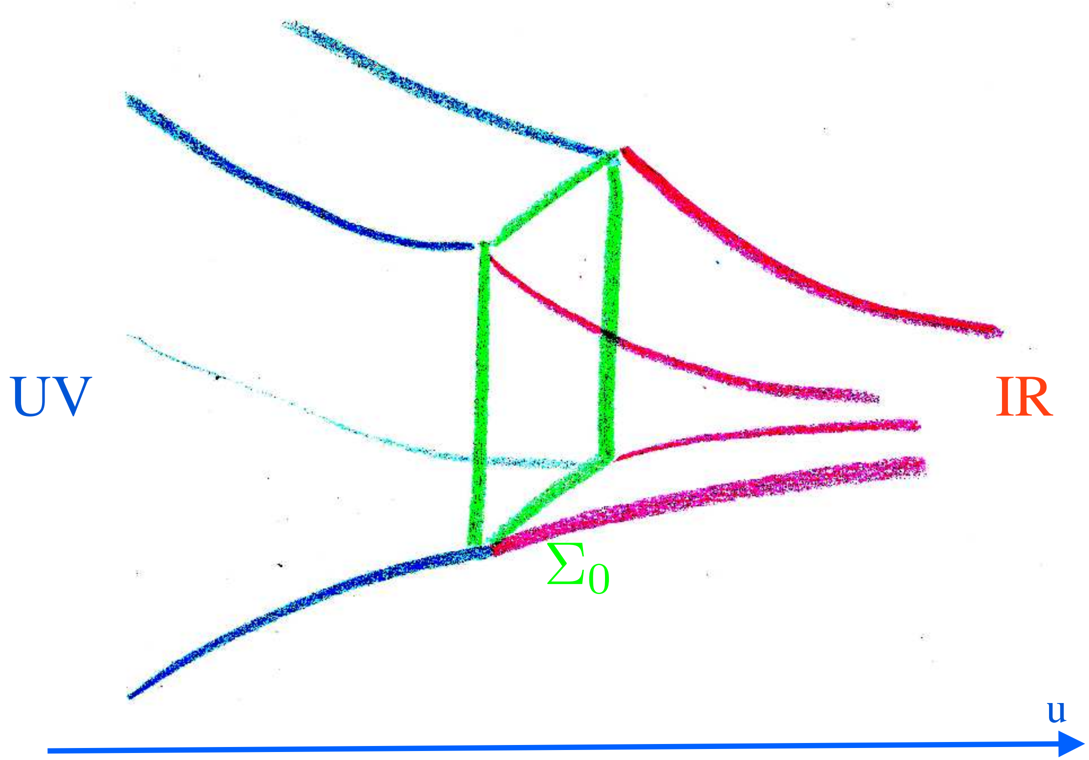
semi-holographic description:

- Describe the strongly coupled large-N theory by a 5d gravity dual with the metric g_{ab} and some bulk scalar fields φ_i, dual to the operators that drive the CFT to the IR.
- The weakly coupled SM fields have a standard field-theoretical description, and they sit on a 4d defect in th 5d dual geometry.

Semi-holographic setup

$$S = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$ ← 5d Gravity dual of 4d CFT

$$+ \int_{\Sigma_0} d^4\sigma \sqrt{-\gamma} \mathcal{L}(\psi_i, H, W^a, \ldots, \varphi, \gamma_{\mu\nu}).$$ ← Weakly coupled 4d QFT



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- Action is the most general up to two derivates preserving 4d diffeos.

Field equations and matching conditions

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Geometry is determined by the bulk Einstein equations:

$$G_{ab} = \frac{1}{2} \partial_a \varphi \partial_b \varphi - \frac{1}{2} g_{ab} \left(\frac{1}{2} g^{cd} \partial_c \varphi \partial_d \varphi + V(\varphi) \right),$$

plus Israel junction conditons at the defect ($[\ ] \equiv$ jump across Σ_0):

$$\left[\gamma_{\mu\nu} \right] = \left[\varphi \right] = 0; \quad \left[K_{\mu\nu} - \gamma_{\mu\nu} K \right] = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\Sigma_0}}{\delta \gamma^{\mu\nu}}; \quad \left[n^a \partial_a \varphi \right] = -\frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\Sigma_0}}{\delta \varphi}$$

Self-tuning vacua

- Are there solutions with *flat space* as the 4d geometry, for *generic* potentials W_B of order (UV cut-off)^4 ?
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Explored before (2000-2002) but:

- IR geometry not fully understood (in particular meaning of the singularity)
- 4d gravity regime seemed incompatible with self-tuning (partly because models were not general enough)

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- Holographic picture clarifies how to organize the space of solutions: what integrations constants are fixed, which are dinamically determied, and what IR geometries are acceptable.
- Framework is general enough to allow for emergent 4d gravity.

Bulk equations

$$S_5 = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

Vacuum (Poincaré invariant) solutions:

$$ds^2 = du^2 + e^{2A(u)} \eta^{\mu\nu} dx_\mu dx_\nu, \qquad \varphi = \varphi(u)$$

$$6\ddot{A} + \dot{\varphi}^2 = 0, \qquad 12\dot{A}^2 - \frac{1}{2}\dot{\varphi}^2 + V(\varphi) = 0.$$

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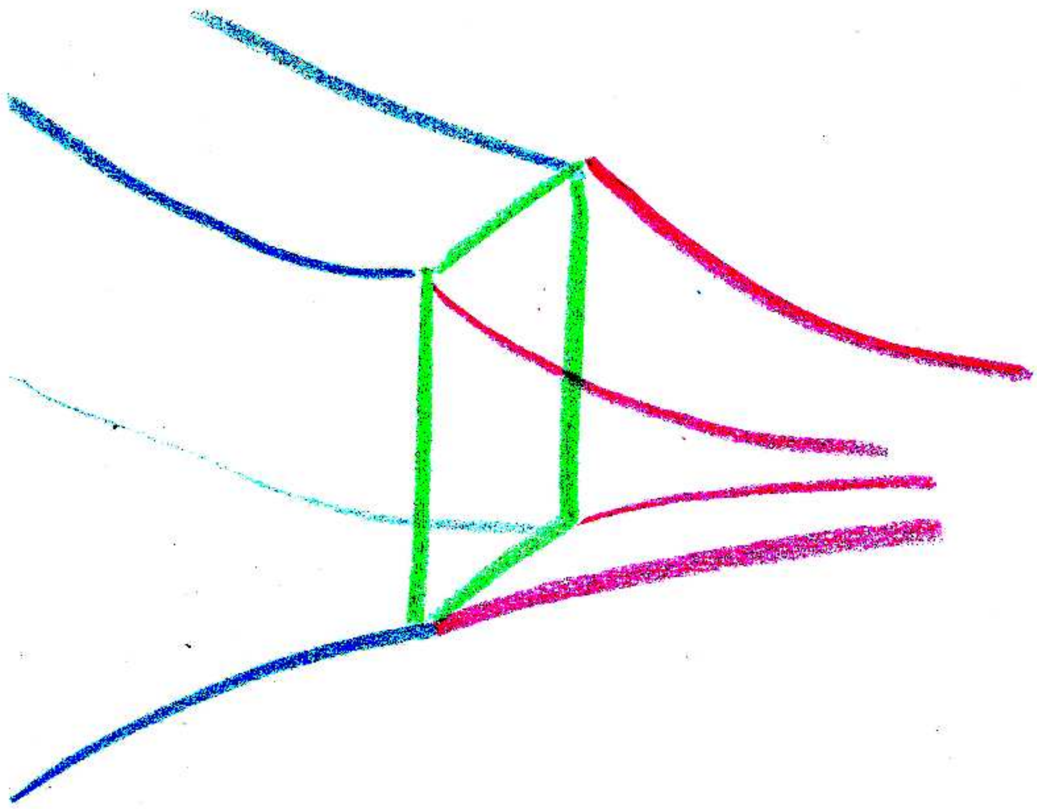
One has to *solve independently on each side* of the defect (at $u = u_0$), and glue the solutions using Israel junction conditions:

$$\left[A\right] = \left[\varphi\right] = 0; \quad \left[\dot{A}\right] = -\frac{1}{6} W_B(\varphi(u_0)); \quad \left[\dot{\varphi}\right] = \frac{dW_B}{d\varphi}(\varphi(u_0))$$

Vacuum Geometry



$A_{UV}(u), \varphi_{UV}(u)$ $A_{IR}(u), \varphi_{IR}(u)$

$e^{A_{UV}} \to +\infty,\ \varphi_{UV} \to 0$

AdS boundary as $u \to -\infty$

$ds^2 \simeq du^2 + e^{-2u/\ell} \eta_{\mu\nu} dx^\mu dx^\nu$

$\varphi \simeq \varphi_- e^{(4-\Delta)u/\ell} + \ldots$

$V \simeq -\frac{12}{\ell^2} + \frac{m^2}{2}\phi^2 + \ldots\ \Delta(\Delta - 4) = m^2.$

$e^{A_{IR}} \to 0,\ \varphi_{IR} \to \varphi_*$

Regular AdS interior or naked singularity as $u \to +\infty$.

Superpotential

Write Einstein's equations as first order flow equations, with an auxiliary scalar function $W(\Phi)$ $\quad (' = d/d\Phi)$:

$$\dot{A} = -\frac{1}{6}W(\Phi) \qquad \dot{\Phi} = W'(\Phi),$$

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- UV side: C_{UV} controls the vev of the dual operator and it is *not* fixed by UV boundary conditions.

$$\varphi(u) = \varphi_- e^{(4-\Delta)u/\ell}\Big(1+\ldots\Big) + \varphi_-{}^{\Delta/(d-\Delta)} C_{UV} e^{\Delta u/\ell}\Big(1+\ldots\Big)$$

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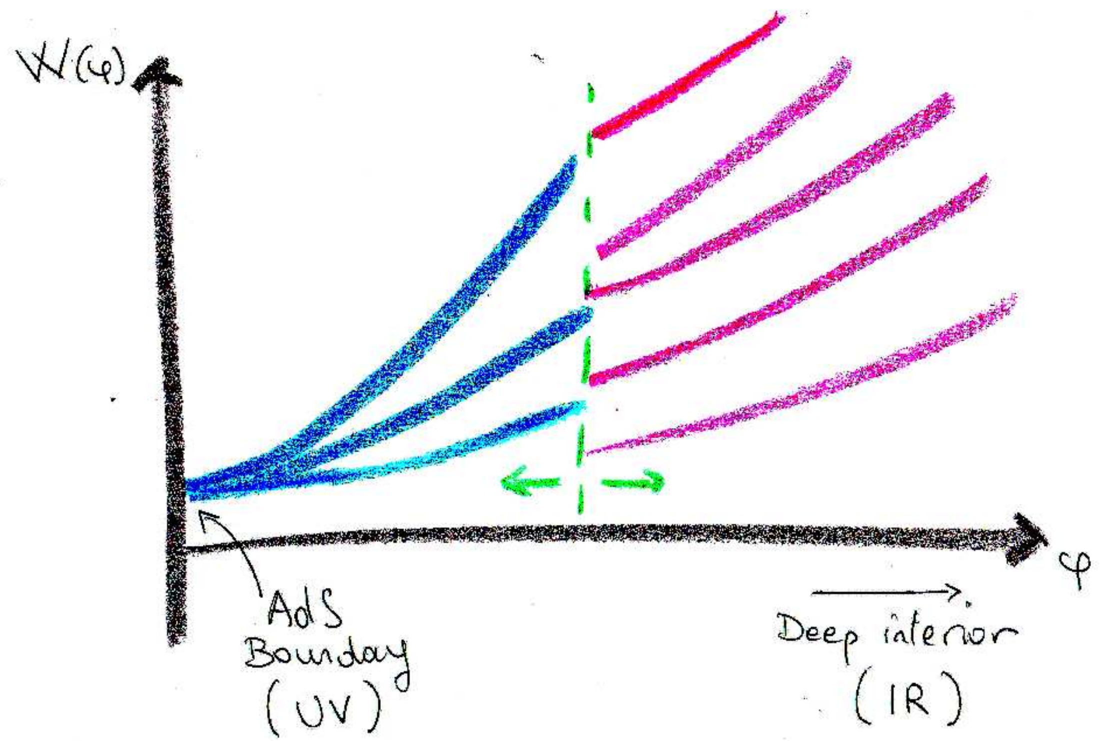
- IR side: C_{IR} fixed by regularity of the IR geometry.

Junction conditions for the superpotential



Junction conditions take a simple form:

$$W^{IR}(\varphi_0) - W^{UV}(\varphi_0) = W_B(\varphi_0), \quad \frac{dW^{UV}}{d\varphi}(\varphi_0) - \frac{dW^{IR}}{d\varphi}(\varphi_0) = \frac{dW_B}{d\varphi}(\varphi_0)$$

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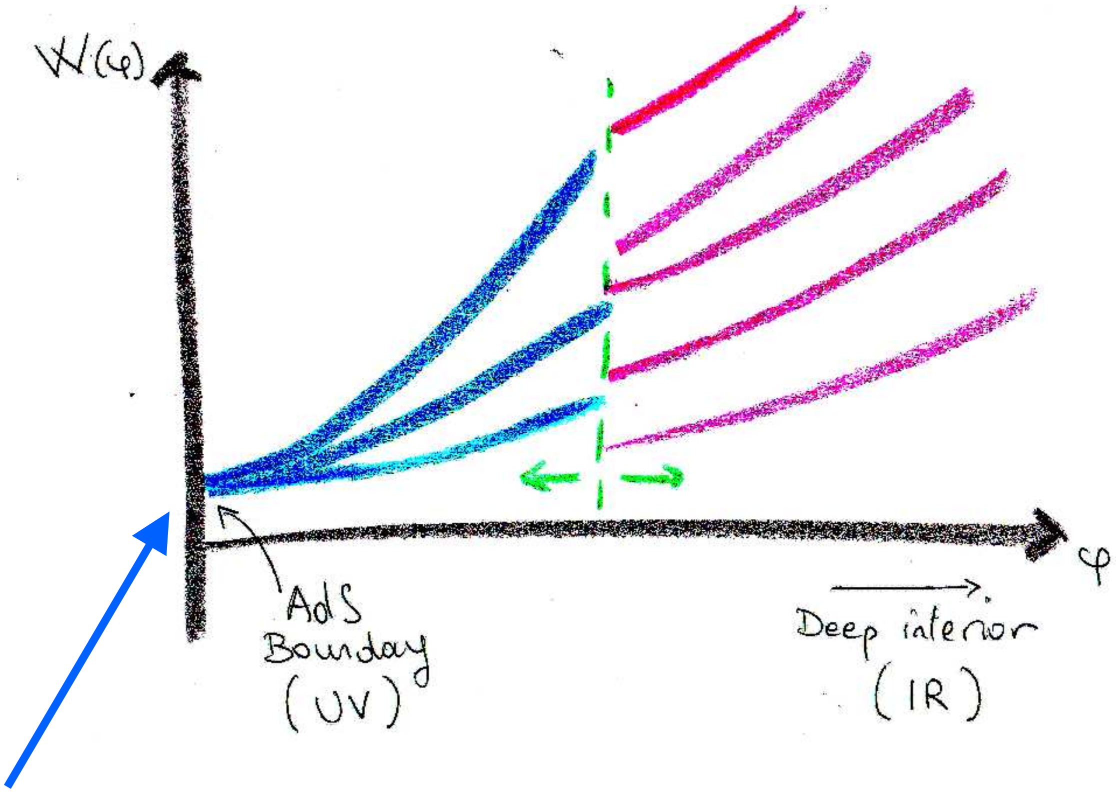

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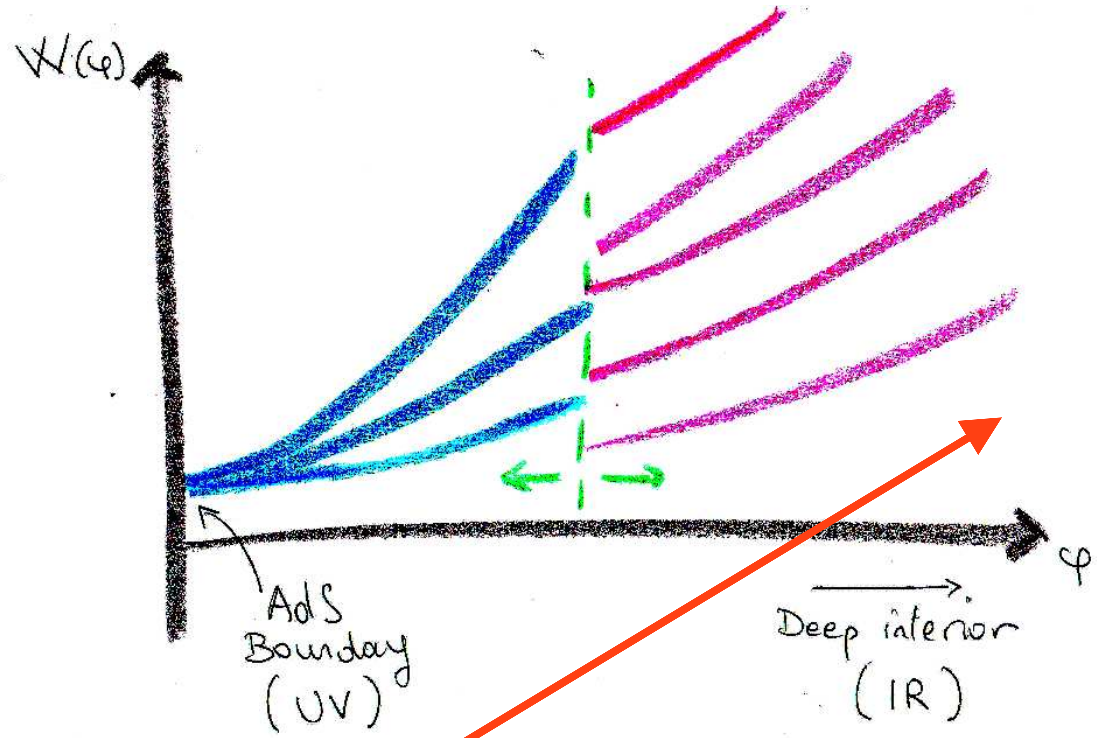

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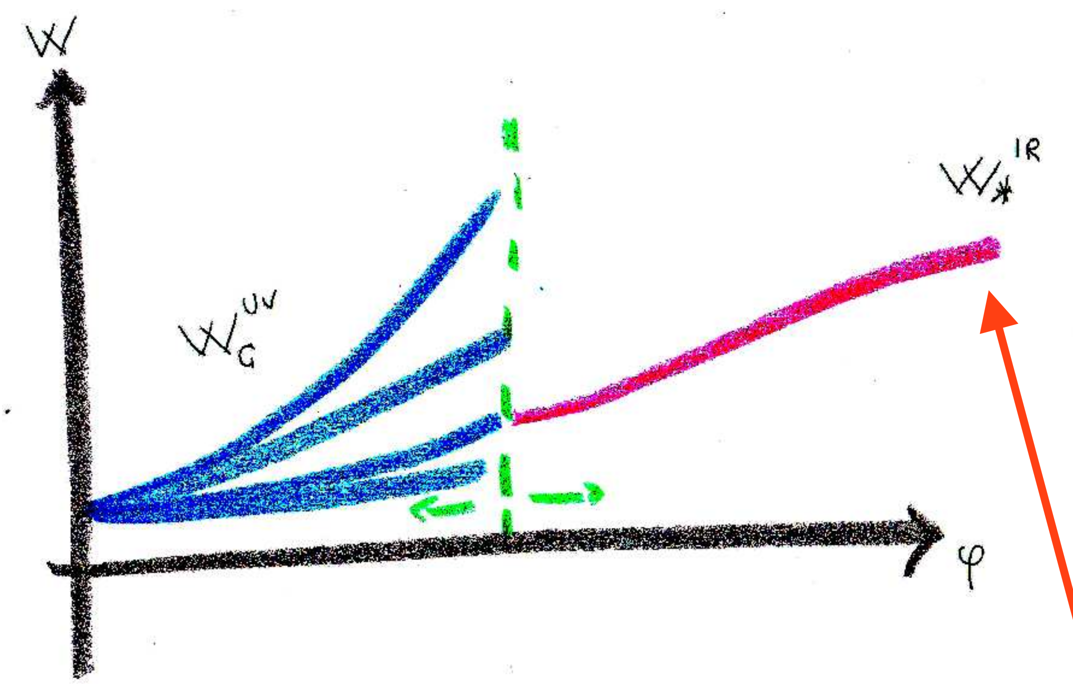
IR side: Only certain IRs are acceptable (IR AdS fixed point, “good” singularities). This picks out a single solution W^{IR} and fixes $C_{IR} = C_*$ (or at most a finite number)

IR Selection



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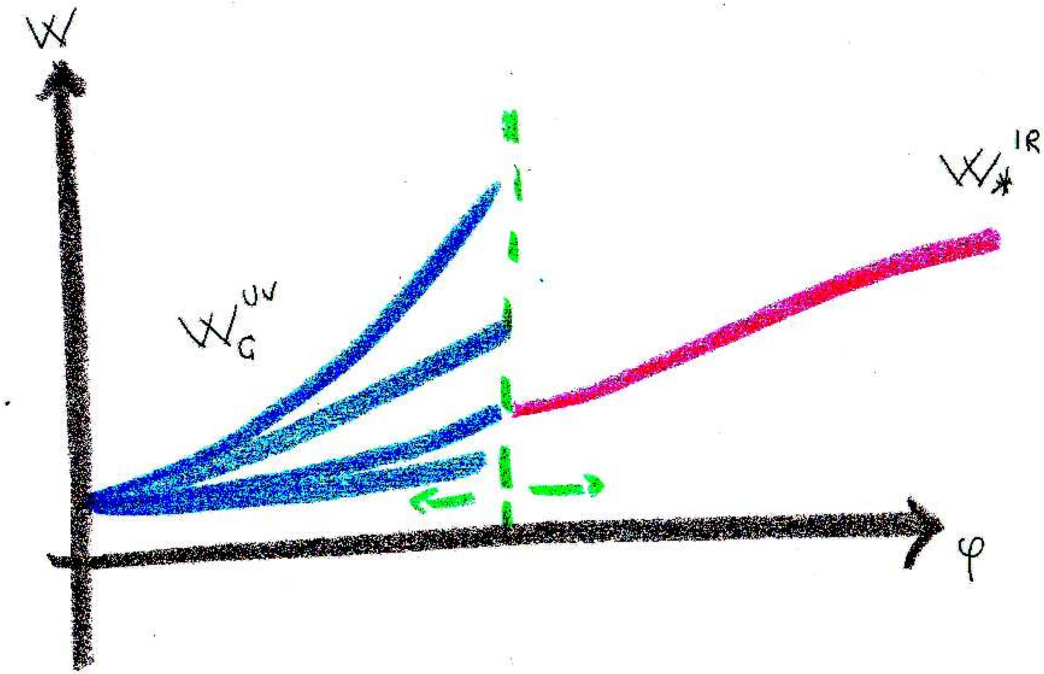
Equilibrium solution



$$W^{UV}(\varphi_0) = W_*^{IR}(\varphi_0) - W_B(\varphi_0), \qquad \frac{dW^{UV}}{d\varphi}(\varphi_0) = \frac{dW_*^{IR}}{d\varphi}(\varphi_0) - \frac{dW_B}{d\varphi}(\varphi_0)$$

Two equations for two unknowns C_{UV}, φ_0. Generically there exist a unique (or a discrete set of) solutions with C_{UV}, φ_0 determined.

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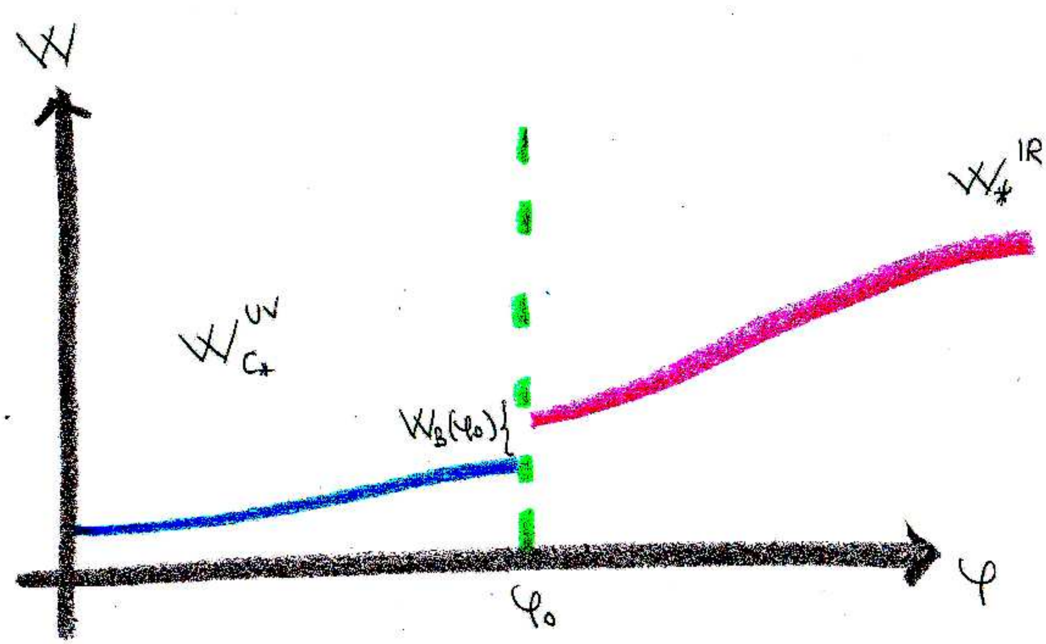

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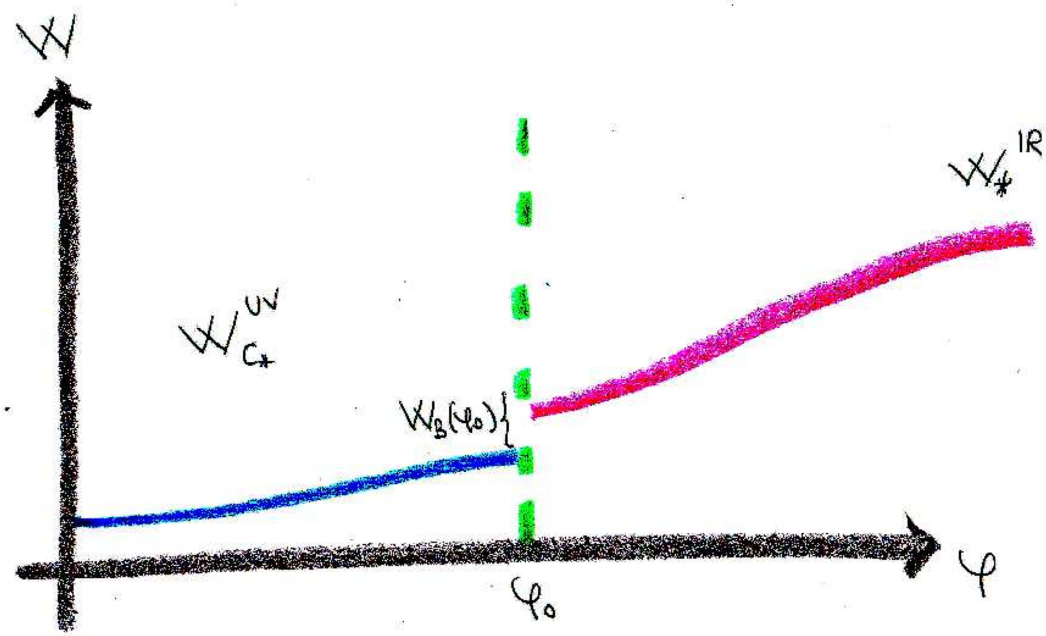

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For generic brane vacuum energy $\sim \Lambda^4$, geometry (VEVs and brane position) adjusts so that the brane is flat and the UV glues to the regular IR through the junction (*self-tuning*).

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- The induced Einstein term on the defect allows for the existence of a 4d-like graviton resonance (Dvali,Gabadadze,Porrati, '00)

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- Bulk curvature $\Rightarrow$ 4d massive graviton at *very* large distances.

Scales of braneworld gravity

Two competing scales:

1. "DGP" transition length: $r_c \approx U(\varphi_0)$
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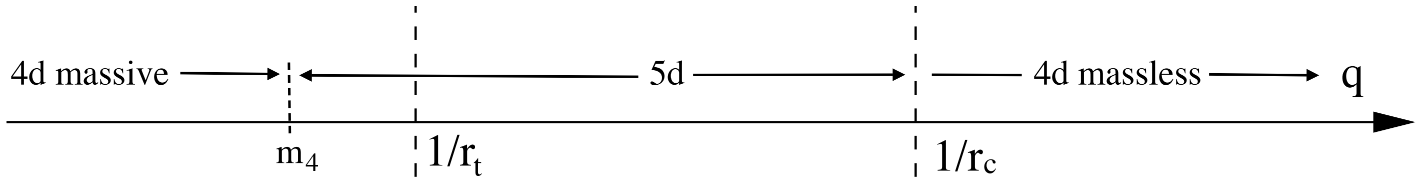

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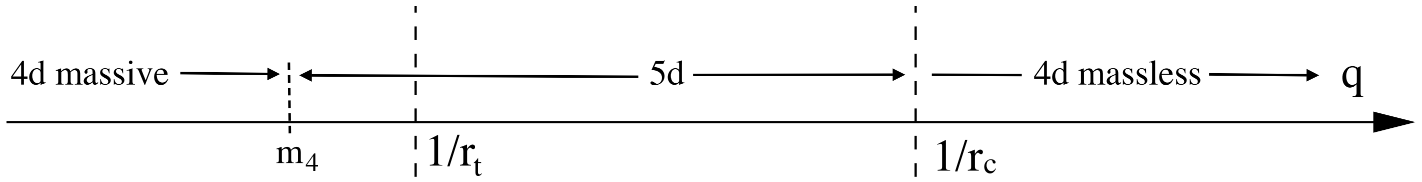

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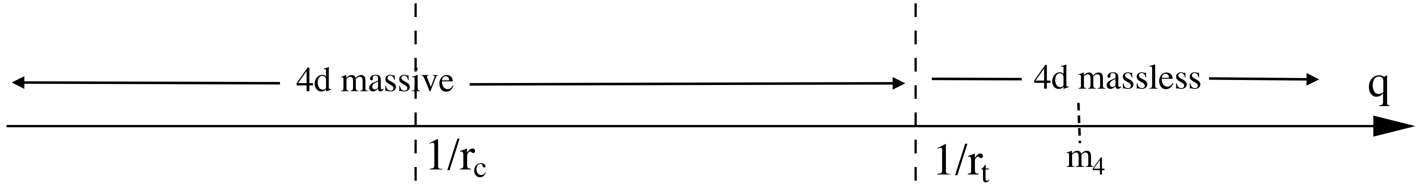

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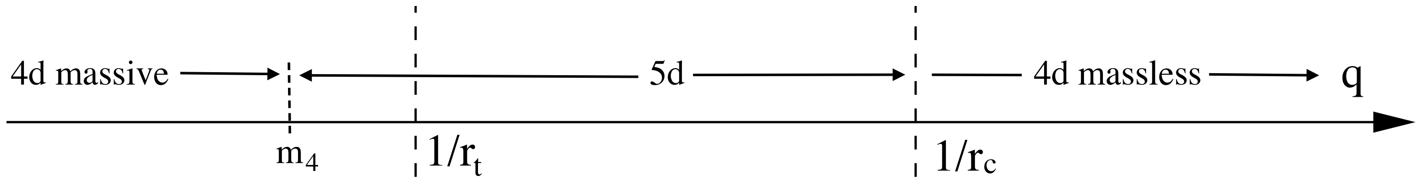

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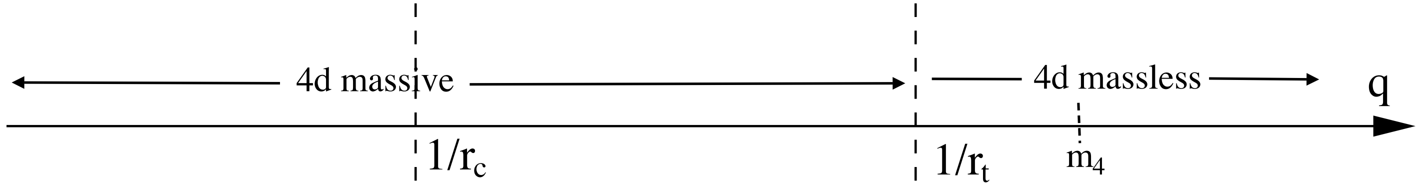

$$M_p^2 \approx M^3 U_0, \qquad m_g^2 \approx \frac{\mathcal{R}_0}{U_0}$$

Scalar perturbations

- Determine whether vacuum solution (flat brane at $r = r_0$) is stable.
- Possible light scalar mediated interactions (fifth force, violations of equivalence principle) $\Rightarrow$ pheno constraints.

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1.

$$\tau_0 > 0, \quad Z_0 > 0, \quad Z_0\tau_0 > 36 \left(\frac{dU_B}{d\varphi}\bigg|_{\varphi_0} \right)^2$$

$$\tau_0 \equiv 6 \left(6 \frac{W_B}{W_{IR} W_{UV}} - U \right)_{\varphi_0}, \quad Z_0 \equiv Z(\varphi_0)$$

$\Rightarrow$ No ghost instabilities

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2.

$$\tilde{\mathcal{M}}^2 \equiv \left(\frac{d^2 W_B}{d\varphi^2}(\varphi_0) - \left[\frac{d^2 W}{d\varphi^2} \right]_{UV}^{IR} \right) \geq 0$$

$\Rightarrow$ No tachyonic instabilities.

Conclusion and outlook

We constructed a framework where Self-tuning of the CC is generically realized. Challenge now is model-building: construct phenomenologically viable model.

- Acceptable values of M_p, r_c, m_g given large UV cutoff;
- Compliance with stability requirements;
- Deal with vDVZ discontinuity (Role of non-linearities, Veinshtein mechanism);
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Conclusion and outlook

We constructed a framework where Self-tuning of the CC is generically realized. Challenge now is model-building: construct phenomenologically viable model.

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- Avoidance of fifth force constraints;

If this all goes through, one can do more phenomenology:

- Add SM and Higgs field (see Lukas Witkowski's talk)
- Study the space of solutions: non-flat brane, time-dependent solutions (cosmology) (ongoing work with Lukas Witkowski and Jewek Ghosh)
- The framework can potentially addess EW hierarchy problem (via stabilized warped extra dimensions) and late-time acceleration (cosmology close to the equilibrium position).

Example

$$V(\varphi) = -12 - \left(\frac{\Delta(4-\Delta)}{2} - \frac{b^2}{4}\right)\varphi^2 - V_1 \sinh^2 \frac{b\varphi}{2},$$

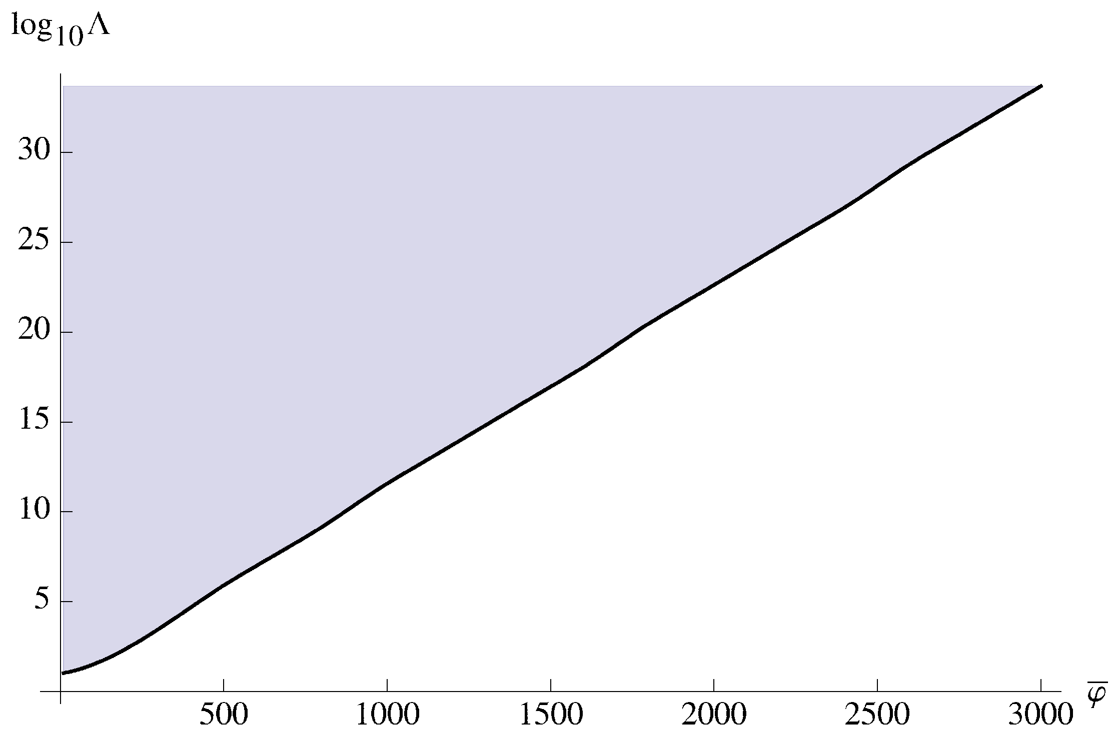
- supports an AdS fixed point at $\varphi = 0$ ($\ell_{UV} = 1$)
- good IR solution:

$$W_{IR}(\varphi) \sim \sqrt{\frac{2}{(32/3) - b^2}} \exp \frac{b\varphi}{2}, \qquad \varphi \to +\infty.$$

How large can Λ be?

$$W_B(\varphi) = \Lambda^4 \left[-1 - \frac{\varphi}{s} + \left(\frac{\varphi}{s} \right)^2 \right]$$

$$b = \frac{1}{\sqrt{6}}, \ \Delta = 3, \ V_1 = 1$$



$$\varphi_0 \simeq \bar{\varphi} \approx 1.6\, s$$

Effective 4d Green's function

Introuce tensor perturbations:

$$\delta g_{\mu\nu} = e^{2A(r)} h_{\mu\nu}(r, x^{\alpha}), \qquad h^{\mu}_{\mu} = \partial^{\mu} h_{\mu\nu} = 0$$

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Solve classical linearized equation for tensor fluctuations with localized source:

$$h_{\mu\nu}(x, r) = \int d^4x G_{\mu\nu}{}^{\rho\sigma}(x - x'; r, r_0) T_{\rho\sigma}(x', r_0),$$

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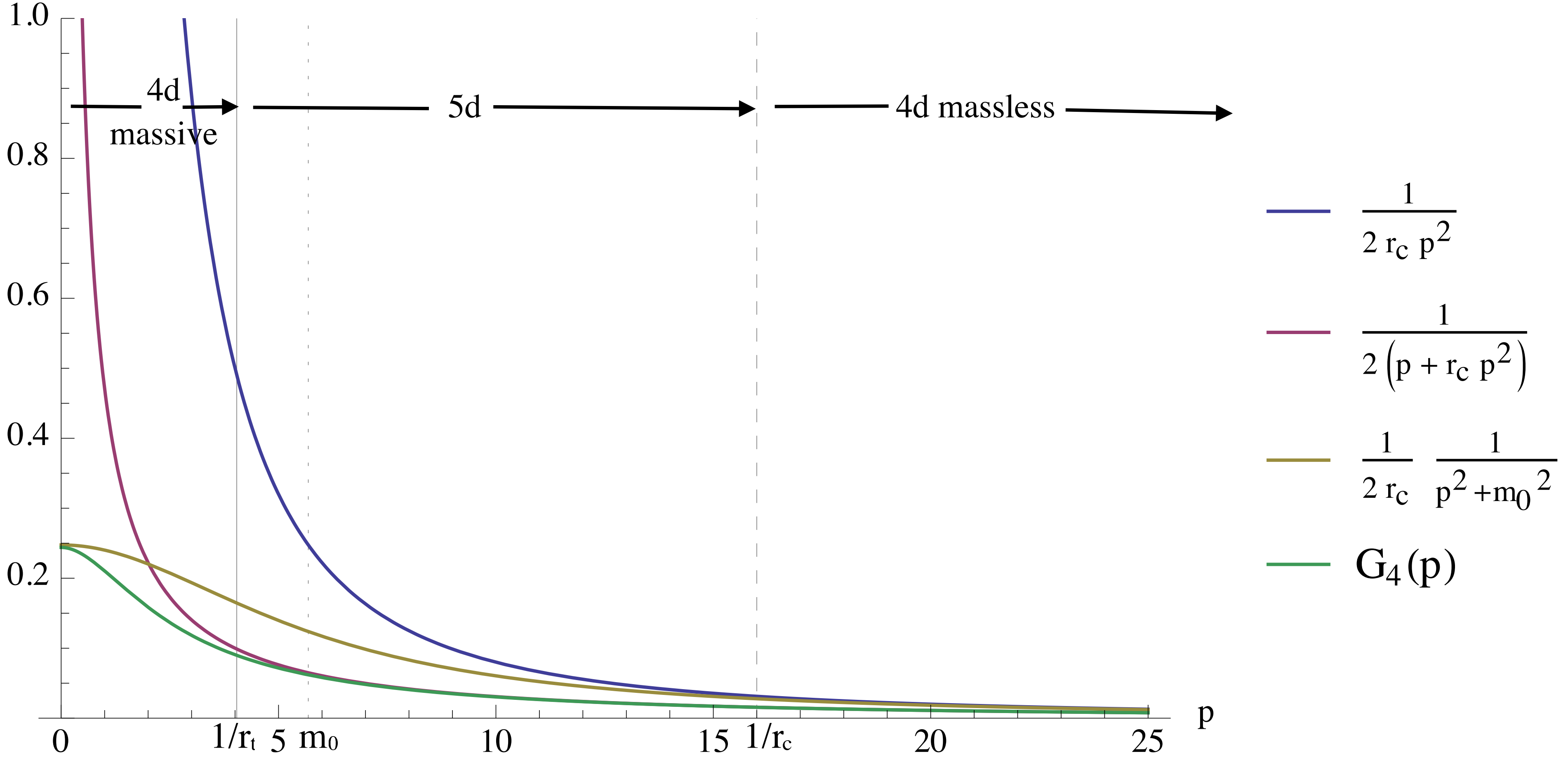
Tree-level interaction described in purely 4d terms by an effective Green's function:

$$S_{int}(T) = \int \frac{d^4p}{(2\pi)^4} \tilde{G}_4(p) \left[T_{\mu\nu}(p) T^{\mu\nu}(-p) - \frac{1}{3} T(p) T(-p) \right]$$

$$G_4(x) \equiv G(x, r_0, r_0).$$

4d-5d transition

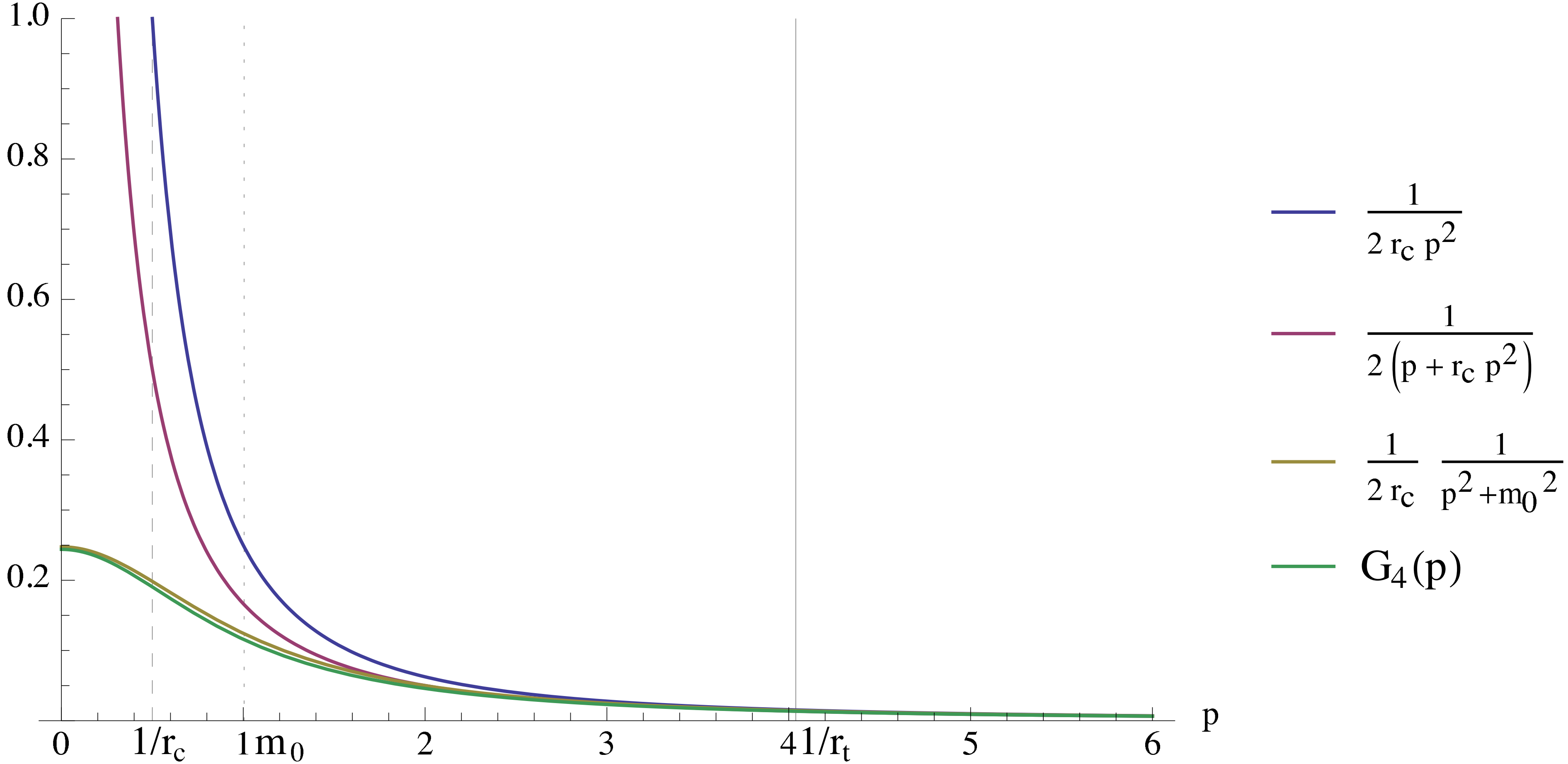
$r_c < r_t$: DGP-like transition, at intermediate distances.



$$r_c = U_0, \; r_t = \frac{e^{-A_0}}{\mathcal{R}_0}, \quad M_p^2 \approx M^3 U_0, \quad m_0^2 \approx \frac{\mathcal{R}_0}{U_0},$$

Massless/Massive gravity transition

$r_c > r_t$ massive graviton propagator all the way.



$$r_c = U_0,\ r_t = \frac{e^{-A_0}}{\mathcal{R}_0}, \quad M_p^2 \approx M^3 U_0, \quad m_0^2 \approx \frac{\mathcal{R}_0}{U_0},$$

Looking for solutions

Junction conditions can be rewritten as a non-linear equation for φ_0:

$$-\frac{Q^2}{2}\Big(W^{IR}(\varphi_0) - W^B(\varphi_0)\Big)^2 + \frac{1}{2}\left(\frac{dW^{IR}}{d\varphi} - \frac{dW^B}{d\varphi}\right)^2_{\varphi_0} = V(\varphi_0),$$

$$Q \equiv \sqrt{\frac{d}{2(d-1)}}$$

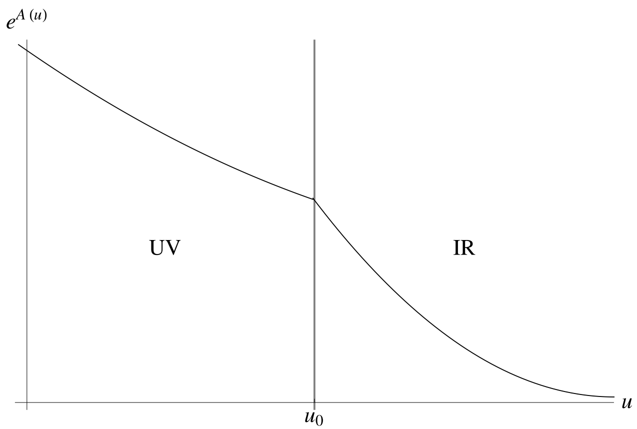
V, W^B and W^{IR} are *fixed functions* of φ.

1. Solve for φ_0
2. Solve superpotential equation for $W_{UV}(\varphi)$ with initial condition:

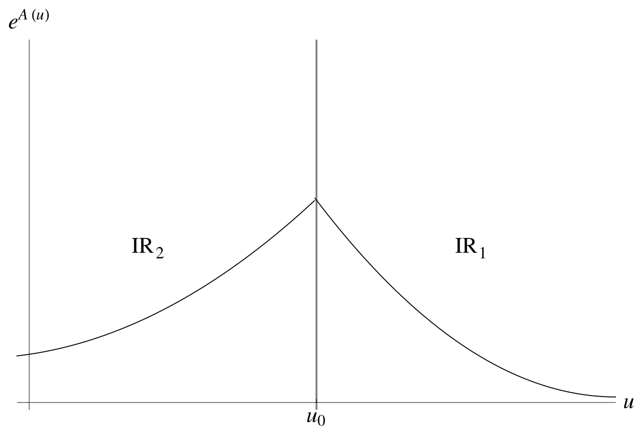
$$W^{UV}(\varphi_0) = W^{IR}(\varphi_0) - W^B(\varphi_0)$$

Consistent self-tuining

Two possibilities:



$W_{UV} > 0$



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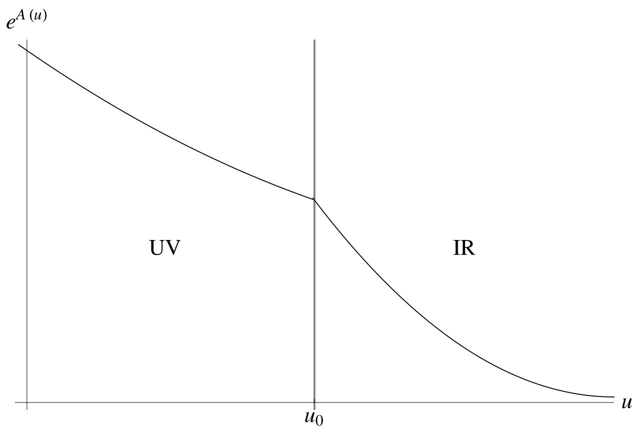

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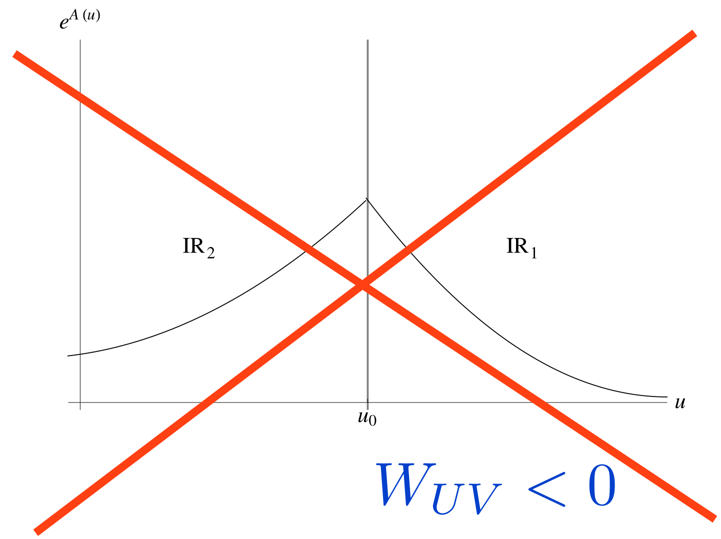


$W_{UV} < 0$

Needs fine tuning of the brane potential to join two “special” solutions

Cfr. Randall-Sundrum setup

Genericity

As we will see, it is desirable (but not strictly necessary) that $W_B(\varphi_0) > 0$, i.e. $0 < W_{UV}(\varphi_0) < W_{IR}(\varphi_0)$ (in this case, the solution is manifestly ghost-free).
It turns out that that for such solutions to exist, it is enough that

$$W(\bar{\varphi}) = 0, \qquad W'(\bar{\varphi}) > 0$$

for some $\bar{\varphi}$. Then the equations are solved, with $W_B(\varphi_0) > 0$, for:

$$\varphi_0 \approx \bar{\varphi} + \frac{\partial_\varphi(W_{IR}^2)}{4|V|}\Bigg|_{\varphi=\bar{\varphi}}$$

provided:

$$\frac{W_B(\varphi_0)}{W_{IR}(\varphi_0)} \ll 1$$

Relating scales

- We can relate bulk parameters M, e^{A_0}, ℓ_{UV} to those of the dual field theory N, g_0, Δ:

$$e^{A_0} \propto (\ell_{UV} g_0)^{1/(d-\Delta)}, \quad (M\ell_{UV})^3 \propto N^2$$

- Bulk superpotentials set the scale of the bulk curvature scale: $W(\varphi(u)) \propto \mathcal{R}(u)$

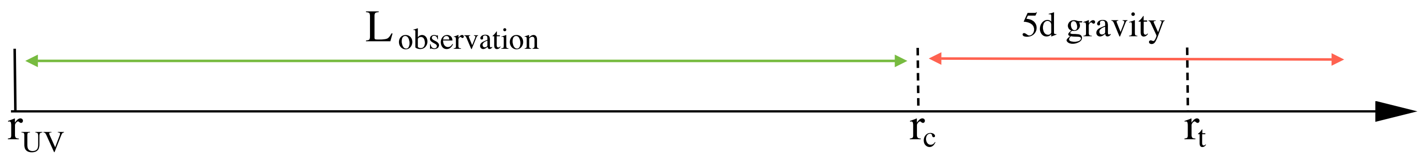
$$\Rightarrow \qquad \frac{M}{\mathcal{R}_0} \sim \frac{N^{2/3}}{\ell_{UV} W_{UV}(\varphi_0)}$$

- The scale of brane potentials is set by the UV cut-off Λ:

$$W_B \sim \frac{\Lambda^4}{M^3}, \qquad U_B \sim \frac{\Lambda^2}{M^3}$$

DGP scenario

Requires $r_t > r_c$



- Gravity must be modified at cosmological distances:

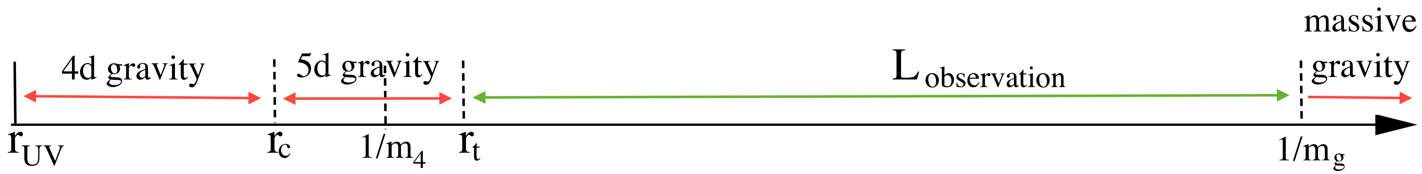
$$M_p r_c = \left(\frac{MU_0}{4}\right)^{3/2} \approx \left(\frac{\Lambda}{M}\right)^3 u^3(\varphi_0) \approx 10^{60}$$

- The assumption $r_t > r_c$ translates into:

$$e^{-A_0} U_0 \mathcal{R}_0 \lesssim 1 \quad \Rightarrow \quad \left(\frac{\Lambda}{M}\right)^2 u(\varphi_0) \, \frac{\ell_{UV} W_{UV}(\varphi_0)}{N^{2/3} \left(\ell_{UV} g_0\right)^{\frac{1}{(d-\Delta)}}} \lesssim 1$$

Massive gravity scenario 1

Requires $r_t > r_c$



- Large distance modification (graviton mass) must be at cosmological scales

$$\frac{m_g}{M_p} \lesssim 10^{-60} \quad \Rightarrow \left(\frac{M}{\Lambda}\right)^2 \frac{(\ell_{UV} W_{UV}(\varphi_0))^{1/2}}{u(\varphi_0)} \frac{1}{N^{1/3}} \quad < \quad 10^{-60}$$

- Short distance modification must be below (tenths of) mm:

$$r_t M_p \lesssim 10^{-30} \quad \Rightarrow \quad \left(\frac{M}{\Lambda}\right) \frac{\ell_{UV} W_{UV}(\varphi_0)}{u^{1/2}(\varphi_0)\,(\ell_{UV} g_0)^{\frac{1}{(d-\Delta)}}\, N^{2/3}} \quad > \quad 10^{-30}$$

Massive gravity scenario 2

Alternatively, $r_t < r_c$ (no DGP regime)

- Same large scale condition:

$$\frac{m_g}{M_p} \lesssim 10^{-60} \quad \Rightarrow \left(\frac{M}{\Lambda}\right)^2 \frac{(\ell_{UV} W_{UV}(\varphi_0))^{1/2}}{u(\varphi_0)} \frac{1}{N^{1/3}} \quad < \quad 10^{-60}$$

- No short distance modification until the UV cut-off.

Scalar-mediated interaction

Define metric and dilaton sources:

$$T_{\mu\nu} = -\frac{2}{\sqrt{\gamma}}\frac{\delta S_m}{\gamma^{\mu\nu}}, \qquad O = \frac{\delta S_m}{\delta\varphi}.$$

Interaction between brane-localized sources:

$$S_{int} = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\mathcal{T}^\dagger(q)G_s(q)\mathcal{T}(-q), \qquad \mathcal{T} \equiv \left(T^\mu_\mu,\ O\right)$$

$$G_s(q) \equiv \frac{1}{2M^3}P\left[\Sigma\left(\Gamma_1 + q^2\Gamma_2\right) + \mathcal{D}^{-1}(r_0;q)\right]^{-1}P^\dagger$$

$$P \equiv -\frac{z_{IR}z_{UV}}{[z]}\begin{pmatrix} \frac{1}{z_{IR}} & -\frac{1}{z_{UV}} \\ 1 & 1 \end{pmatrix}.$$

- Modes coupling to O can be parametrically heavy, $m \simeq \mathcal{M}$.
- Modes coupling to T remain light.

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Localized term ⟷ Bulk contribution

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localized mass → Γ_1; localized kinetic term → $q^2\Gamma_2$

$$P \equiv -\frac{z_{IR} z_{UV}}{[z]} \begin{pmatrix} \frac{1}{z_{IR}} & -\frac{1}{z_{UV}} \\ 1 & 1 \end{pmatrix}.$$

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DGP regime (short distance)

Overall interaction for light modes in the 4d regime:

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Something interesting happens if

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$$\Rightarrow \quad \mathcal{V}(q) \simeq \frac{1}{q^2}\left[\frac{1}{2M_p^2}\left(T_{\mu\nu}(q)T^{\mu\nu}(-q) - \frac{1}{2}T^{\mu}_{\mu}(q)T^{\nu}_{\nu}(-q)\right)\right], M_p^2 = M^3U_0$$

- Tensor Structure becomes that of a 4d massless graviton !
- Leftover interaction is light scalar with ultra-weak coupling
- Warning: need to check explcitly about ghosts