# Theta angle in holographic QCD

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[arXiv:1609.08922 with D. Arean,
I. Iatrakis, and E. Kiritsis]

# Outline

1. Axial anomaly in QCD at large $N$
2. V-QCD and the axial symmetry
3. Results for CP-odd observables

# The $U(1)_A$ problem

One light mode missing in the experimentally observed spectrum:
The mass of $\eta'$ is 958 MeV, not small

- The would-be pion for the breaking of $U(1)_A$
- Does not obey the GMOR relation

Solution: $U(1)_A$ broken by quantum effects:
axial (triangle) anomaly

- Measure of the path integral not invariant under $U(1)_A$
  $\Rightarrow$ the Goldstone theorem does not apply

# The $U(1)_A$ problem

It is known exactly how $U(1)_A$ is broken:

$$\partial_\mu J_A^\mu = \frac{N_f}{16\pi^2} \mathbb{T}\mathrm{r}\, G\tilde{G} - 2i \sum_i m_i\, \bar{\psi}_i \gamma_5 \psi_i$$

Connection to the $\theta$-angle – CP-odd terms in QCD

$$Z_{\mathrm{QCD}} \sim \int D\bar{\psi} D\psi\, e^{i \int d^4x \left[ \frac{\theta}{32\pi^2} \mathbb{T}\mathrm{r}\, G\tilde{G} - \sum_i m_i e^{i\phi_i} \bar{\psi}_R^i \psi_L^i - \sum_i m_i e^{-i\phi_i} \bar{\psi}_L^i \psi_R^i + \cdots \right]}$$

- $U(1)_A$ transformation can remove $\sum_i \phi_i$, but shifts $\theta$ (due to noninvariance of $D\bar{\psi} D\psi$)
- $\bar{\theta} \equiv \theta + \sum_i \phi_i$ invariant physical parameter, unless some of the $m_i$ vanishes (experimentally $\lesssim 10^{-10}$)

# The $U(1)_A$ anomaly at large $N$

- In the 't Hooft (probe) limit ($g^2 N_c$ and $N_f$ fixed, $N_c \to \infty$) anomalous contribution absent at leading order, appears at NLO: anomaly $\sim \mathcal{O}(N_f/N_c)$
- In the Veneziano limit ($g^2 N_c$ and $N_f/N_c \equiv x$ fixed, $N_c \to \infty$ and $N_f \to \infty$) anomaly present at leading order

Witten-Veneziano formula for the mass of $\eta'$:

$$m_{\eta'}^2 \simeq m_\pi^2 + x \frac{\chi}{\bar{f}_\pi^2} = m_\pi^2 + \frac{N_f}{N_c} \frac{\chi}{\bar{f}_\pi^2}$$

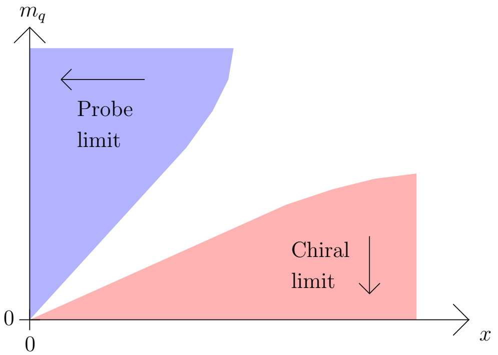
where $\chi =$ the topological susceptibility $\bar{f}_\pi = \mathcal{O}(1)$, and $x$ small

# Order of limits and $U(1)_A$

Tricky issue with order of limits: chiral ($m_q \to 0$) and probe ($x = N_f/N_c \to 0$) limits do not commute

1. Chiral limit at finite $x$: light $\pi$'s and heavy $\eta'$ with suppressed couplings to heavy states
2. Probe limit at finite $m_q$: degenerate $\pi$'s and $\eta'$, enhanced chiral symmetry $SU \to U$

- QCD at Veneziano limit interpolates between the two regimes
- Implement this in a holographic model?



# 2. V-QCD and axial symmetry

# Holographic V-QCD: the fusion

The fusion:

1. IHQCD: bottom-up model for glue by using 5d dilaton gravity

   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via space filling $D4$-$\bar{D}4$ branes

   [Klebanov,Maldacena]
   [Bigazzi,Casero,Cotrone,Kiritsis,Paredes hep-th/0505140,0702155]

Consider 1. + 2. in the Veneziano limit with full backreaction
$\Rightarrow$ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

# Dictionary

In the flavor/CP-odd sector

1. The tachyon: $T^{ij} \leftrightarrow \bar{\psi}^i_R \psi^j_L$; $(T^\dagger)^{ij} \leftrightarrow \bar{\psi}^i_L \psi^j_R$
   - Source: the (complex) quark mass matrix $M^{ij}$
     Note: the phase of the tachyon sources the phase of the mass
2. The gauge fields $A^{ij}_{\mu,L/R} \leftrightarrow \bar{\psi}^i_{L/R} \gamma_\mu \psi^j_{L/R} \equiv J^{(L/R)}_\mu$
   - Sources: chemical potentials and background fields (not turned on in this study)
3. The bulk axion $\mathfrak{a} \leftrightarrow \mathbb{T}\mathrm{r}\, G \wedge G$
   - Source: (normalized) theta angle $\theta / N_c$

In the glue sector

1. The dilaton $\lambda \leftrightarrow \mathbb{T}\mathrm{r}\, G^2$
   - Source: the 't Hooft coupling $g^2 N_c$

# The CP-odd term in V-QCD

CP-odd action $\mathcal{S}_a$ arises from the WZW term

- Couples the RR axion $C_3$ to the $U(1)_A$ gauge field $A = (A_L - A_R)/2$ and the tachyon phase $\xi = \arg\det T/N_f$
- Dualizing $C_3 \to$ axion field $\mathfrak{a}$ gives

$$S_a = -\frac{M^3 N_c^2}{2} \int d^5x \sqrt{-\det g}\, Z(\lambda) \times \left[\partial_M \mathfrak{a} - x\,(2V_a\, A_M - \xi\, \partial_M V_a)\right]^2$$

[Casero, Kiritsis, Paredes]

Symmetry

$$A_\mu \to A_\mu + \partial_\mu \epsilon\ , \qquad \xi \to \xi - 2\epsilon\ , \qquad a \to a + 2x\, V_a \epsilon$$

implies the axial anomaly in QCD (with $\epsilon = \epsilon(x_\mu)$)

# Full V-QCD action

Complete V-QCD model

$$\mathcal{S}_{\mathrm{V-QCD}} = \mathcal{S}_g + \mathcal{S}_f + \mathcal{S}_a$$

1. $\mathcal{S}_g$: 5D dilaton-gravity (gluon sector, IHQCD)
2. $\mathcal{S}_f$: Generalized tachyon DBI action for flavor ($T$, $F_{\mu\nu}$)
3. $\mathcal{S}_a$: The CP-odd action

- Fine details of $\mathcal{S}_g$ and $\mathcal{S}_f$ not important for this talk
- Full backreaction between all sectors $\Rightarrow$ solve saddle points numerically
  - Analytic results (in chiral limit) implied by symmetry

# 3. Results

# Free energy and topological susceptibility

Using symmetry and "linearization" near the UV

$$\mathcal{E}(\bar\theta)-\mathcal{E}(0) \approx \min_{\xi}\left[-\langle\bar\psi\psi\rangle\big|_{m_q=0}\, m_q\,(1-\cos\xi) + \frac{\chi_{\mathrm{YM}}}{2}\left(N_f\xi-\bar\theta\right)^2\right]$$

- Agrees precisely with chiral Lagrangians!

For a single branch of vacua $\mathcal{E} = N_c^2 f(\bar\theta/N_c)$
Imposing the $2\pi$ periodicity of $\bar\theta$:

$$\mathcal{E}(\bar\theta) = \frac{1}{2}\chi\min_k\left(\bar\theta+2\pi k\right)^2 = \mathcal{O}\left(N_c^0\right)$$

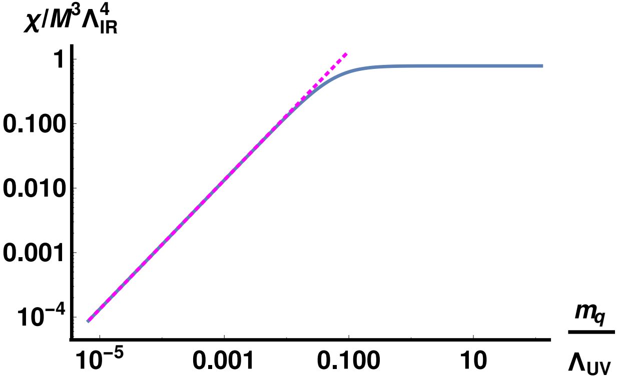
Topological susceptibility:

$$\chi = \left.\frac{d^2\mathcal{E}}{d\bar\theta^2}\right|_{\bar\theta=0} \propto \int d^4x\langle \mathbb{T}\mathrm{r}\, G\tilde G(x)\ \mathbb{T}\mathrm{r}\, G\tilde G(0)\rangle$$
$$\simeq \frac{1}{\chi_{\mathrm{YM}}^{-1} - N_f^2/m_q\langle\bar\psi\psi\rangle}$$

Small $m_q$ means $m_q\langle\bar\psi\psi\rangle \ll x\,\Lambda_{\mathrm{IR}}^4$ – order of limit issue!

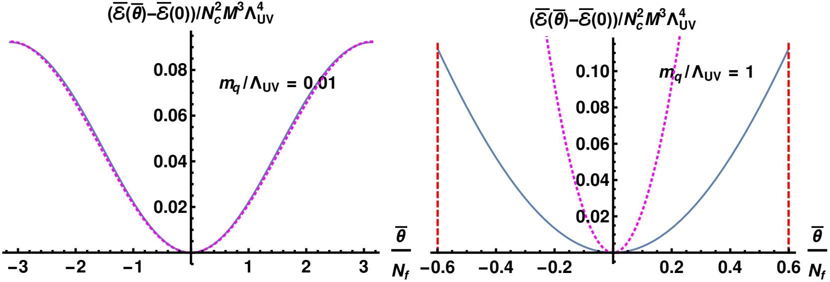
# Topological susceptibility

Dependence on $m_q$ at $x = 2/3$




Interpolates between expected results at small and large $m_q$

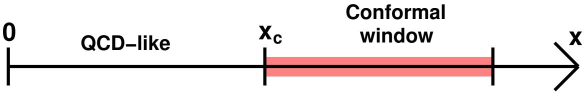
# The free energy




- Small $m_q$: agreement with chiral Lagrangians
- Large $m_q$: finite range of $\bar{\theta}$

# Topological susceptibility in the conformal window

So far I discussed only QCD the like phase, but the model also includes the conformal window




- IR fixed point perturbed by a small quark mass
- "Hyperscaling" of meson masses can by computed from tachyon flow

[Evans, Scott; MJ]

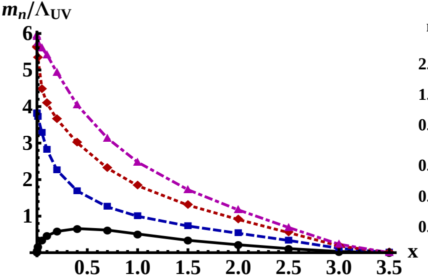
The same approach gives the scaling of the topological susceptibility

$$\chi \propto m_q^{\frac{4}{1+\gamma_*}} \quad \text{as} \quad m_q \to 0\,,$$

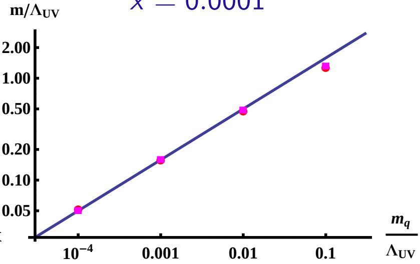
where $\gamma_*$ is the anomalous dimension of the quark mass at the IR fixed point.

# The mass of $\eta'$ in V-QCD

PS masses at $m_q = 0$




$\pi$ and $\eta'$ masses at $x = 0.0001$




Agreement with Veneziano-Witten (can also be checked analytically)

$$m_{\eta'}^2 \simeq m_\pi^2 + x\,\frac{\chi}{f_\pi^2} \simeq -\frac{\langle\bar{\psi}\psi\rangle\, m_q}{f_\pi^2} + x\,\frac{N_f N_c \chi}{f_\pi^2}$$

# Modified "complex" Efimov spirals

- Complex tachyon violates the Breitenlohner-Freedman bound $\Rightarrow$ oscillates near the fixed point
- The complex quark mass and the VEV $\langle\bar{\psi}\psi\rangle$ have spiral-like behavior

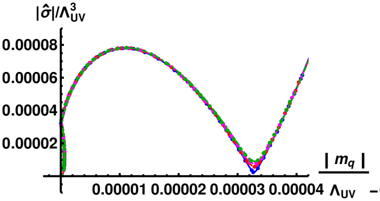
$$\frac{m_q e^{i\xi_0}}{\Lambda_{\rm UV}} = \frac{K_{\rm IR} e^{ik_{\rm IR}}}{K_m} \frac{\sin(\phi_{\rm IR}-\phi_\sigma-\nu u)\cosh\varphi_{\rm IR} + i\cos(\phi_{\rm IR}-\phi_\sigma-\nu u)\sinh\varphi_{\rm IR}}{\sin(\phi_m-\phi_\sigma)} e^{-2u}$$

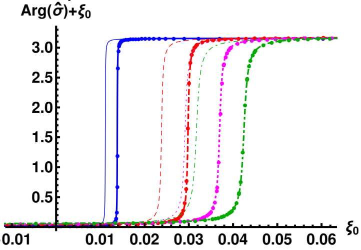
$$\frac{\hat{\sigma} e^{i\xi_0}}{\Lambda_{\rm UV}^3} = \frac{K_{\rm IR} e^{ik_{\rm IR}}}{K_\sigma} \frac{\sin(\phi_{\rm IR}-\phi_m-\nu u)\cosh\varphi_{\rm IR} + i\cos(\phi_{\rm IR}-\phi_m-\nu u)\sinh\varphi_{\rm IR}}{\sin(\phi_\sigma-\phi_m)} e^{-2u}$$

(Real) spirals common in holographic computations

[E.g. Iqbal,Liu,Mezei 1108.0425]

Comparison to numerical data ($x = 2.5$) with various $\bar{\theta}$:



# Conclusions

- We studied the implementation of the axial anomaly with full backreaction (finite $N_f/N_c$) in V-QCD
- Explored CP-odd physics over the parameter space ($\bar{\theta}$, $m_q$, $x$)
  - Results complement those obtained in effective theories and in Witten-Sakai-Sugimoto model [Bartolini,Bigazzi,Bolognesi,Cotrone,Manenti,Sisca]
- Extensive agreement with known results in specific limits (chiral, probe, large $m_q$)

# Extra Slides

# V-QCD action

Degrees of freedom

- The tachyon $\tau$ , and the dilaton $\lambda$
- $\lambda = e^{\phi}$ is identified as the 't Hooft coupling $g^2 N_c$
- $\tau$ is dual to the $\bar{q}q$ operator

$$\mathcal{S}_{\rm V-QCD} = N_c^2 M^3 \int d^5x \sqrt{g} \left[ R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] \\ - N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)}$$

$$V_f(\lambda, \tau) = V_{f0}(\lambda) \exp(-a(\lambda)\tau^2) \; ; \qquad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu} x^\mu x^\nu)$$

Need to choose $V_{f0}$, $a$, and $\kappa$ ...

# CP-odd action

CP-odd action $\mathcal{S}_a$ arises from the WZW term [hep-th/0702155, 1309.2286]

$$\mathcal{S}_a = \mathcal{S}_{\rm open} + \mathcal{S}_{\rm closed}\,; \qquad \mathcal{S}_{\rm closed} = -\frac{M^3}{2}\int d^5x\sqrt{-\det g}\,\frac{|dC_3|^2}{Z(\lambda)}$$

$$\mathcal{S}_{\rm open} = i\int C_3 \wedge d\Omega_1\,; \qquad \Omega_1 = iN_f\left[2V_a A - \xi dV_a\right]$$

- Couples the RR axion $C_3$ to the $U(1)_A$ gauge field $A = (A_L - A_R)/2$ and the tachyon phase $\xi = \arg\det T/N_f$

Dualizing $C_3 \to$ axion field $\mathfrak{a}$

$$Z^{-1}dC_3 = {}^*(N_c d\mathfrak{a} + i\Omega_1)$$

gives

$$\mathcal{S}_a = -\frac{M^3 N_c^2}{2}\int d^5x\,\sqrt{-\det g}\,Z(\lambda)\left[\partial_M\mathfrak{a} - x\,(2V_a\,A_M - \xi\,\partial_M V_a)\right]^2$$

Bottom-up: potentials $(Z,\ V_a)$ to be determined...

# $U(1)_A$ symmetry and the $\theta$-angle

$$\mathcal{S}_a = -\frac{M^3\,N_c^2}{2}\int d^5x\,\sqrt{-\det g}\,Z(\lambda)\,[\partial_M \mathfrak{a} - x\,(2V_a\,A_M - \xi\,\partial_M V_a)]^2$$

The $U(1)_A$ gauge transformation is

$$A_M \to A_M + \partial_M \epsilon\ , \qquad \xi \to \xi - 2\epsilon\ , \qquad \mathfrak{a} \to \mathfrak{a} + 2x\,V_a \epsilon$$

reflecting the axial anomaly in QCD (with $V_a \to 1$ at boundary)

- Implies $\partial_\mu J_A^\mu = N_f \mathbb{T}\mathrm{r}\, G\,\tilde{G}/16\pi^2 - 2m_q i\bar{\psi}\gamma_5\psi$
- Gauge invariant $\mathfrak{a} + xV_a\xi$ sources $\bar{\theta}/N_c = (\theta + \arg\det M)/N_c$

How about periodicity of $\bar{\theta}$?

- For diagonal tachyon $T = \tau e^{i\xi}\mathbb{I}$, $\xi \mapsto \xi + 2\pi$ implies $\bar{\theta} \mapsto \bar{\theta} + 2\pi N_f$ (!)
- Configurations with $T = \tau\mathbb{I}$ and $T = \tau e^{2\pi i/N_f}\mathbb{I}$, corresponding to $\bar{\theta} \mapsto \bar{\theta} + 2\pi$, both have real $\det T$ $\Rightarrow$ different branch of $\xi = \arg\det T/N_f$
- $2\pi$ peridiocity appears through branch structure (as it should)