Traversable wormholes

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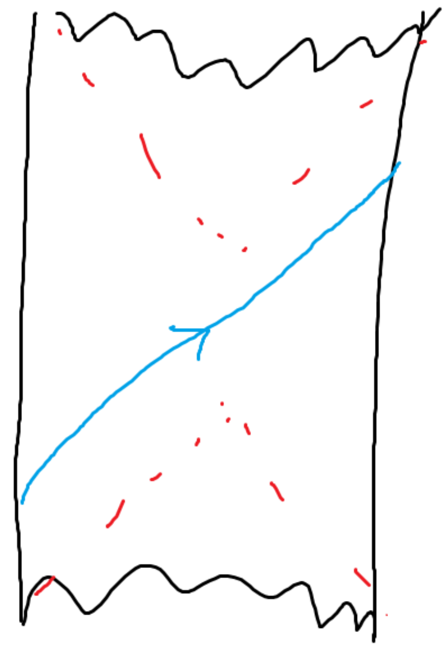
Introduction

- Present a traversable wormhole solution in a consistent, UV completable theory of gravity. The solution is supported by a quantum matter stress tensor with negative null energy. Direct interaction between the ends is crucial to avoid the usual no-go theorems.

- Traversing the wormhole is a gravity dual of quantum teleportation. The construction also gives a method to operationally verify the ER=EPR relation of entanglement and geometry. In some situations, it explains how quantum information can be extracted from black holes without violating locality.

Traversable wormholes

- A causal connection between distant locations. The following seems to be the first example that is definitely consistent in a UV completable gravity theory.



Require negative ANE

- Consider a null geodesic that traverses the wormhole. On the ingoing side, there will be contraction of neighboring light rays.

- Thus traversability requires defocusing if the null ray is to reach the other boundary.

- By Rachaudhuri's equation, this requires negative null energy.

$$\int_{-\infty}^{+\infty} T_{\mu\nu} k^{\mu} k^{\nu} \, d\lambda < 0$$

[Morris Thorne Yurtsever]

Negative energy in QFT

- Although physically reasonable classical theories have positive null energy (otherwise the vacuum would be unstable), it is violated by certain states in any quantum field theory (no local operator can vanish in the vacuum and be positive in all states).

$$|\xi\rangle = \exp(\tfrac{1}{2}\xi^* a_0^2 - \tfrac{1}{2}\xi a_0^{\dagger 2})|0\rangle$$

- A typical example are squeezed states, which oscillate between positive and negative energy.

Ruled out in some contexts

- If the null geodesic is achronal, there are strong arguments that the ANEC is satisfied in QFT.

  [Graham Olum, Faulkner Leigh Parrikar Wang, Hartman Kundu Tajdini]

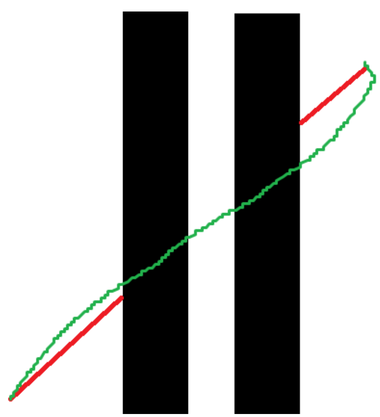
- Also, the generalized second law disallows the future horizon of a traversing worldline from having infinite area in the past.

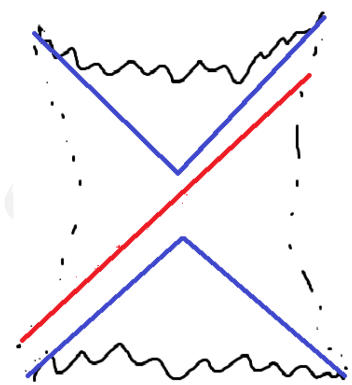
  [Wall]

- Moreover, there is a risk of constructing time machines by applying a relative boost.

Direct interaction

- The key point is that if there is a direct path between the two ends, then a traversing null ray will not be achronal.





- A simple model for this is eternal AdS Schwarzschild with an inter-boundary coupling.

Eternal AdS black hole

- Start with the eternal AdS black hole, dual to the thermofield double state in the decoupled product of two CFTs.

$$|\mathrm{tfd}\rangle = \sum e^{-\beta E/2}|E_L\rangle\ |E_R\rangle$$

[Maldacena]

- The existence of the Killing symmetry implies that there is a bifurcation surface, and the spacetime is on the edge of traversability even including quantum corrections.

Non-traversable in any state

- It would contradict the lack of interaction between the boundary quantum systems if the Einstein-Rosen bridge were traversable in any state. $H = H_L + H_R$, since in gravity the Hamiltonian is equal to a boundary term.

- One can show that the bulk field vacuum is an eigenstate of the ANE with the lowest eigenvalue, 0.

$$\int_{-\infty}^{\infty} dU T_{UU} |\text{tfd}\rangle = 0 \qquad \delta \text{ANE} = i\langle [A, \text{ANE}] \rangle = 0$$

Couple the two CFTs

- Start in the eternal black hole state in the decoupled system and turn on an interaction at some time.

$$S_{\text{int}} = \int dt\, d^{d-1}x\ h(t,x)\mathcal{O}_L(t,x)\mathcal{O}_R(t,x)$$

- This only affects the configuration in the future.

- Now there need not be any cancelation, so the ANE can change at linear order. Adjust the sign to make it negative.

Double trace deformations

- These change the boundary conditions for the dual scalar. The expectation value on the left acts as the source on the right, and vice versa. In other words, there is a transmission amplitude of h.

- Need an operator with dimension less than d/2 to make it relevant.

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{(\tfrac{d}{2})^2 + m^2}$$

$$\varphi(r \to \infty_R) \to \alpha_R(t,\phi) r^{-\Delta} + \beta_R(t,\phi) r^{-2+\Delta}, \;\; \beta_L(t,\phi) = h(t,\phi)\alpha_R(t,\phi)$$

$$\varphi(r \to \infty_L) \to \alpha_L(t,\phi) r^{-\Delta} + \beta_L(t,\phi) r^{-2+\Delta}, \;\; \beta_R(t,\phi) = h(t,\phi)\alpha_L(t,\phi)$$

[Witten, Aharony Berkooz Katz]

An BTZ example

- For simplicity consider a scalar field in the BTZ black hole.

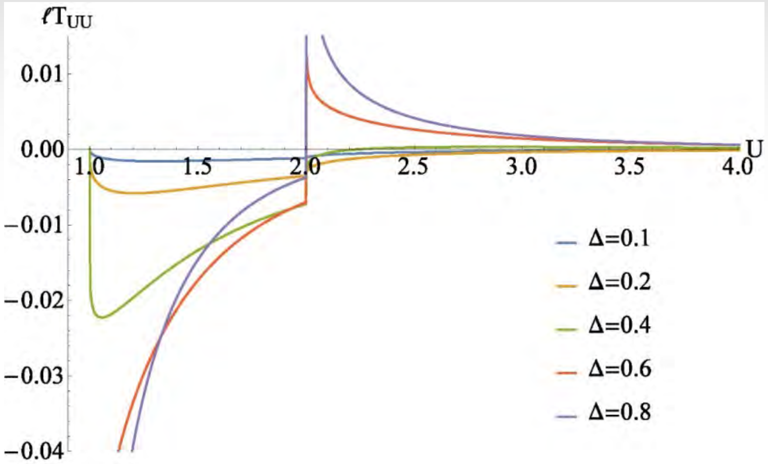
$$ds^2 = -\frac{r^2-r_h^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2-r_h^2}dr^2 + r^2 d\phi^2$$

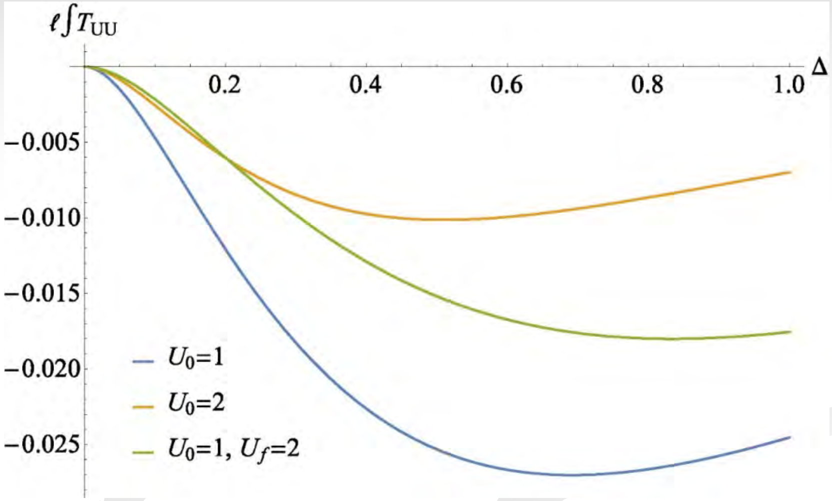
- The modified boundary conditions change the bulk 2-point function at linear order

$$G_h = 2\sin\pi\Delta \int dt_1 h(t_1) K_\Delta(t'+t_1-i\beta/2) K^r_\Delta(t-t_1) + (t \leftrightarrow t')$$

$$K_\Delta(r,\phi,t) = \langle \varphi(r,\phi,t)\mathcal{O}(0,0)\rangle = \frac{r_h^\Delta}{2^{\Delta+1}\pi}\left(-\frac{(r^2-r_h^2)^{1/2}}{r_h}\cosh r_h t + \frac{r}{r_h}\cosh r_h\phi\right)^{-\Delta}$$

The result





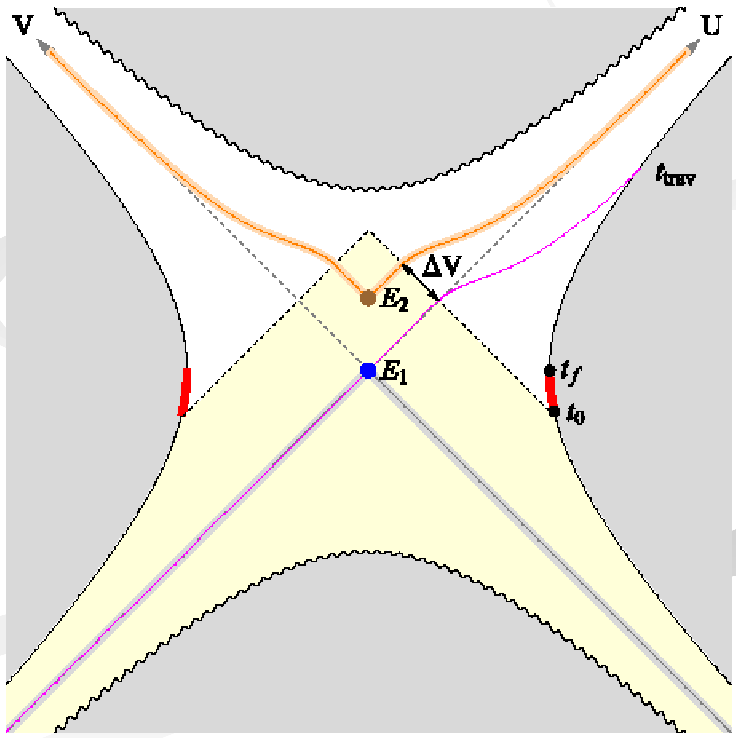
$$\int_{U_0}^{\infty} dU T_{UU} = -\frac{h\Gamma(2\Delta+1)^2}{2^{4\Delta}(2\Delta+1)\Gamma(\Delta)^2\Gamma(\Delta+1)^2\ell} \times \frac{{}_2F_1\left(\frac{1}{2}+\Delta, \frac{1}{2}-\Delta; \frac{3}{2}+\Delta; \frac{1}{1+U_0^2}\right)}{(1+U_0^2)^{\Delta+1/2}}$$

Features of the wormhole

- The perturbation reduces the energy of the state, hence the horizon shrinks.
- One must jump in early enough.

$$\Delta V \sim \frac{hG_N}{R^{D-2}}$$

- For a non-test particle, there is a collision with the negative energy squeezed state $\sqrt{s} \sim \sqrt{\frac{R^{D-4}}{hG_N}}$, but there is a time window in which the Eikonal approximation is valid, ie. the dominant effect is the Shapiro time advance.



- Total amount of information that can get through is bounded by backreaction.

[Maldacena Stanford Yang]

Connecting two black holes in a single spacetime

- There will be a tiny Casimir energy associated to the cycle that threads the wormhole. This can be negative and result in traversability.
- Stronger if Hawking radiation from one maximally entangled black hole is actively sent into the other.
- The generalized second law implies that no speed up can occur over asymptotically long distances.

ER=EPR

- Entangled systems can be described as being connected by a non-traversable wormhole.

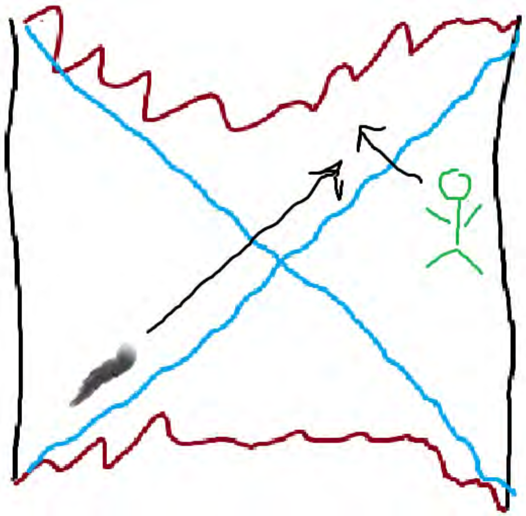
[Maldacena Susskind]

- The most novel feature is that if observers jump into each of the entangled systems, they can meet in the connected interior.

- However, their fate then appears sealed
  and they die in the singularity.

$$e^{iH_L t}|\mathrm{BH}\rangle = \sum e^{-\beta E/2}|E\rangle_L|E\rangle_R$$

[Papadodimas Raju]



No topology operator

- A duality between connected geometries and entangled sums of disconnected geometries might seem strange, since entangled states are a non-linear subspace.
- It is possible only because the naïve topology operator is not non-perturbatively diffeomorphism invariant.
- In terms of the gravitational wavefunctional $\Psi(g) = \langle g|\psi\rangle$, the lack of independence of the $|g\rangle$ implements the Hamiltonian constraints. Kets with different topology also fail to be orthogonal.

Verification in the bulk semiclassical regime

- One might have argued that this interior geometry is impossible to probe from the outside or the CFT.
- In particular, it is not obvious how to determine if the meeting indeed took place.
- Now we see that by coupling the two systems after the observers jump in, we can see inside.

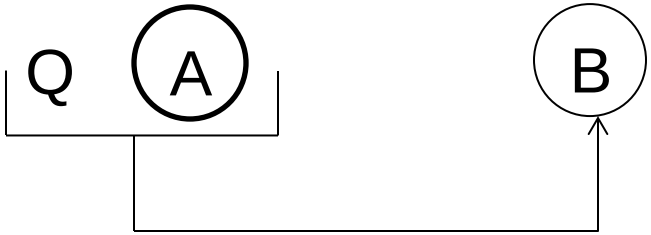
Quantum information interpretation

- Suppose that a qubit Q is sent from the left to right boundary. Although the boundaries are coupled, Q is not being sent directly via the coupling. For example, by causality, it commutes with the interaction Hamiltonian to exponential accuracy.

- The initial entanglement is crucial to the transmission. Similar to teleportation. In fact, when viewed from the outside, ie. for observers effectively out of the gravity well of the black holes, the transmission through the wormhole looks like quantum teleportation.

Quantum teleportation

- Suppose Alice and Bob shared an entangled pair of qubits, A and B. Alice wants to sent another qubit, Q, to Bob, but they are connected only by a classical channel.



- By making a combined measurement on QA, and sending the resulting 2 bits to Bob, the pattern of Q is transferred to B, after application of a unitary by Bob.

As a dynamical process

- Thought of as a strict projection, B had the pattern of Q from the beginning – this is what was learned during Alice's measurement.

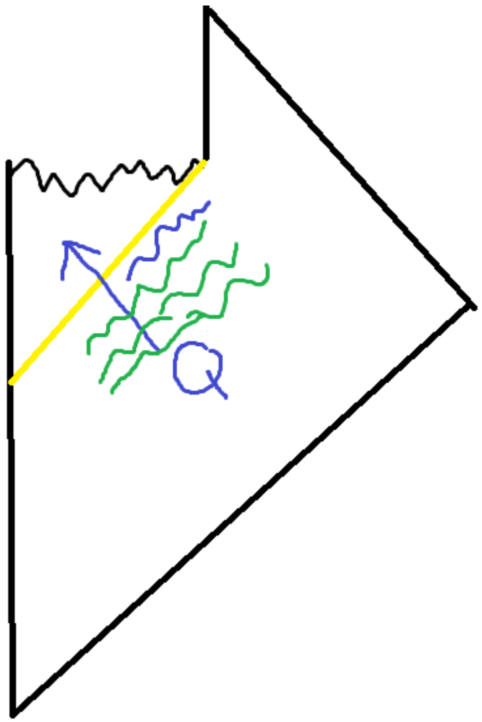
- However, the fully quantum description of the measurement is given by a unitary evolution:

$$V = \sum_i P_i^{QA} U_i^B, \; \int dx \; \delta(\mathcal{O}_R - x) e^{i\lambda x \mathcal{O}_L} = e^{i\lambda \mathcal{O}_L \mathcal{O}_R}$$

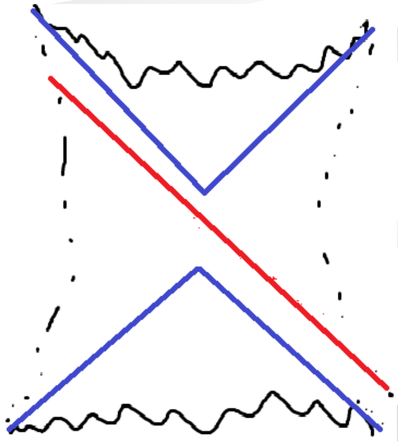
- This is exactly of the type which can lead to a traversable wormhole.

Black hole information extraction

- Hayden-Preskill showed that if one knows the initial state of a black hole, which then half evaporates to be maximally entangled with its emitted Hawking quanta, that any new qubit thrown in can be extracted using only a few more Hawking quanta.



Collapse old Hawking radiation into a black hole with short wormhole



[Maldacena Stanford Yang]

Summary

- Traversable wormholes can exist in consistent gravity theories. But they cannot be used to violate causality.
- Provides a way to operationally detect the connected interior in ER=EPR. Allows causal extraction of black hole information.
- Traversing the wormhole is dual to quantum teleportation.