Dipole CFTs and integrability

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Introduction

- AdS/CFT : examples (IIB on $AdS_5 \times S^5$ / $\mathcal{N}=4$ SYM)
  - precision matching
- universal approach: generic CFT (large N, large gap) =

  = approximately local QFT in AdS (for low-mass fields)

i) solutions to CFT bootstrap in $1/N$ $\leftrightarrow$ all possible local bulk interactions

(Heemskerk et. al '09)

ii) ~ local bulk operator $\Phi(z,x) = \int d^dx' \, K(z,x|x')\,\mathcal{O}(x') + O\left(\frac{1}{N}\right)$

(Banks, Kabat-Lifschytz)

dictated by large N factorization + conformal symmetry

Motivation

- Would like to study a holographic correspondence that is also universal, but whose low-lying sector is less fixed by symmetry than AdS/CFT

- i.e. dynamical information from the field theory should be needed to derive the low-lying dictionary $\mapsto$ insight into emergence of bulk locality

Dipole CFTs

- holographic duals to gravity on Schrödinger$_{d+1}$ space-times

$$ds^2 = -\frac{\mu^2 (dx^+)^2}{z^4} + \frac{2dx^+dx^- + (dx^i)^2 + dz^2}{z^2} \qquad i \in \{1, \dots, d-2\}$$

(a.k.a "non-relativistic CFTs" in context of AdS/cold atom)

- d - dimensional field theories non-local along x^-

- non-relativistic conformal invariance in $d-1$ dimensions

$$x^+ \to \lambda^2 x^+ \quad , \quad x^i \to \lambda x^i \quad , \quad x^- \to x^-$$

- low-lying holographic dictionary does not follow from NR conformal symm alone

- e.g. for free field of mass m $\quad (\Box - m^2)\Phi = 0$

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 + \mu^2 M^2} \qquad M = P_-$$

bulk locality

Q: How can we understand this formula from the field theory point of view?

This talk

- consider an explicit example of a dipole CFT
  "null dipole-deformed $\mathcal{N}=4$ SYM" (Bergman, Ganor '00; Alishahiha, Ganor '03)
  - $*$-product deformation of $\mathcal{N}=4$ SYM (β, non-commut.)
  - integrable in planar limit
  - dual to IIB on $Sch_5 \times S^5$

non-trivial anomalous dim in sugra!

$$\Delta \simeq 2 + \sqrt{4 + m^2 + \lambda \frac{L^3 M^3}{4\pi^2}}$$

t' Hooft coupling

- we compute the one-loop anomalous dim. in field theory (operator mixing) & compare

Plan

- Schrödinger space-times and non-relativistic (NR) CFTs
- Schrödinger space-times in string theory (TsT)

review

- dipole-deformed $\mathcal{N}=4$ SYM
- integrability setup
- one-loop anomalous dimensions from twisted $SL(2)$ spin chain
- discussion & future directions

Schrödinger space-times and non-relativistic CFTs

Schrödinger backgrounds

- $$ds^2_{d+1} = -\mu^2 \frac{(dx^+)^2}{z^4} + \frac{2dx^+dx^- + \overbrace{dx^i dx^i}^{d-2} + dz^2}{z^2} \qquad A = \mu \frac{dx^+}{z^2}$$

- geometrically realises the $\underbrace{\text{NR conformal group}}$ in $d-1$ dimensions (u, x^i)

- $P_- = P_v \to$ particle number N commutes

Balasubramanian, McGreevy '08
Son '08

- $SL(2,\mathbb{R})$ subalgebra : $D = D_{rel} + M_{+-}$, $\mathcal{H} = P_+$, $C = \frac{1}{2} K_-$

- D : non-relativistic scale transf: $x^+ \to \lambda^2 x^+$, $x^i \to \lambda x^i$, $z \to \lambda z$

- C, $G_i = M_{i-}$ lower the D eigenvalue ; P_i, $\mathcal{H}$ raise it

Non-relativistic CFTs

- primary operators $[C, \mathcal{O}] = [G_i, \mathcal{O}] = 0$ Nishida, Son '07

- labeled by Δ, M $[D, \mathcal{O}] = i\Delta\mathcal{O}$, $[\underline{P}, \mathcal{O}] = iM\mathcal{O}$

  NR dimension (of Δ); number op. (of M)

- two-point function fixed by NR conf. invar.

$$\langle \mathcal{O}(t,\vec{x})\,\mathcal{O}^{+}(0,0)\rangle = \frac{1}{t^{\Delta}}\, e^{-\frac{iM\vec{x}^2}{2t}}$$

- three-point function fixed up to an arbitrary function of the

  NR 3-point invariant cross ratio y

- NR CFTs dual to Schrödinger space-times (dipole CFTs)

  bulk locality : $\Delta(\mu M)$, tree 3-pt. f $F(y, \mu M)$

Schrödinger spacetimes in string theory

Generating Schrödinger space-times

- start from $AdS_p \times S^q$ & perform TsT — Alishahiha, Ganor '03

Hopf, Cartan

- $y \in AdS_p$ (null) and $\psi \in S^q$ = $U(1)$ isometry directions

$$\text{TsT}: \begin{cases} T_y \,,\ \text{shift } \psi \to \psi + \frac{L}{2\pi}\tilde{y} \,,\ T_{\tilde{y}} \\ T_\psi \,,\ \text{shift } y \to y + \frac{L}{2\pi}\tilde{\psi} \,,\ T_{\tilde{\psi}} \end{cases}$$

- not a symmetry of string theory if $\frac{L}{2\pi} \notin \mathbb{Z}$
- AdS_{d+1} vac + TsT = Sch_{d+1} + B-field
- more generally, can combine TsT w/ other dualities (S-duality)
- $d=4$: supergravity spectrum only depends on $\mu M = \sqrt{\lambda}\,\frac{L}{2\pi} M$

Spinning strings in $\mathrm{SoAdS}_5 \times S^5$

- string spinning along the ϕ direction (pointlike, global coord) $\vec{x} = 0$

$$\phi = \omega\tau \qquad T = \kappa\tau \qquad V = \mu^2 m\tau \qquad z = \sqrt{\frac{\kappa}{m}}$$

- conserved charges $\Delta = \sqrt{\lambda}\,\kappa$, $M = \sqrt{\lambda}\, m$, $J = \sqrt{\lambda}\,\omega$

- Virasoro constraint: $\Delta = \sqrt{J^2 + \frac{\lambda}{4\pi^2} L^2 M^2}$ ← combination $\lambda L^2 M^2$

- undo TsT in the worldsheet action

$$\boxed{\begin{array}{c}\text{closed string SoAdS} \leftrightarrow \text{open string in AdS} \\ \text{w/ twisted boundary conditions: } \left\{ \begin{array}{l} \Delta\phi = -LM \\ \Delta V = LJ \end{array} \right.\end{array}}$$

Frolov '05

Dipole - deformed $\mathcal{N}=4$ SYM

Dipole-deformed $\mathcal{N}=4$ SYM

Bergman, Ganor '00

- $\forall\ \Phi \to$ dipole length $L^\mu_\phi = q_\phi L^\mu$

  q_ϕ: charge, ∂_ψ ; L^μ: fixed vector

  $L^\mu = \mu\, \delta^\mu_y$ (null)

- $S_{dipole} = S_{N=4\,SYM}$ $\quad \cdot \to *$

- star product:

$$(\phi_1 * \phi_2)(x) = \phi_1\left(x^\mu - \frac{L_2^\mu}{2}\right)\phi_2\left(x^\mu + \frac{L_1^\mu}{2}\right)$$

$$= e^{i\frac{L}{2}(P_{y_1} q_2 - P_{y_2} q_1)}\, \phi_1(x_1)\,\phi_2(x_2)\Big|_{x_1 = x_2 = x}$$

- very similar to non-commutative (P_x, P_y)

  & to β-deformation (q_A, q_B)

Properties

- gauge transf. $\phi(x) \to \mathcal{U}(x-\frac{L}{2})\,\phi(x)\,\mathcal{U}^\dagger(x+\frac{L}{2})$

$\Rightarrow$ $\phi(x) =$  $\hat{\phi}(x)$ local

- Seiberg-Witten map $\phi(x) = e^{iq\int_{x-L/2}^{x} A}\,\hat{\phi}(x)\,e^{iq\int_{x}^{x+L/2} A}$ + plug in

- $S_{dip\,\mathcal{N}=4} = S_{\mathcal{N}=4}(\hat{\phi}) + \int L^\mu \mathcal{O}_\mu + \int L^\mu L^\nu \mathcal{O}_{\mu\nu} + \dots$

CFT + infinite # of irrelevant, $\underbrace{\text{Schrödinger-invar.}}_{L^\mu \text{ lightlike}}$ operators

- can be modelled by a NR CFT

Planar equivalence

- Feynman rules: kinetic terms unaffected
  interactions phases $e^{i k^\mu L_\mu}$

- Planar equivalence: phases cancel in planar diagrams or can be moved to the external lines

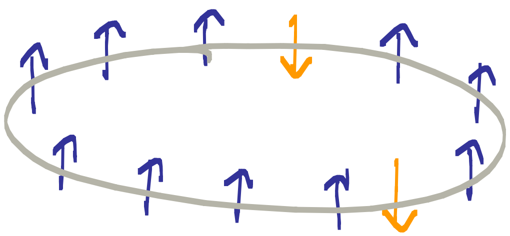
Filk '96

same as in $\mathcal{N}=4$ SYM up to $e^{i \sum k_i^\mu L_\mu^i}$

external lines

Map to a spin chain

- gauge-invar. operators, e.g. $\mathcal{O} = \mathrm{tr}[\underbrace{Z Z Z \ldots X Z \ldots X \ldots Z}_{J}]$



- map to spin chain

- operator mixing $\leftrightarrow$ spin chain Hamiltonian $\Gamma = \frac{\lambda}{8\pi^2} \sum_{l=1}^{J} h_{l,l+1}$

(integrable)

- dipole-deformed $\mathcal{N}=4$

$\mathcal{O} = \mathrm{tr}(\hat{\phi}_1 \ldots \hat{\phi}_n)$ + planar equiv $h_{ab} \to \tilde{h}_{ab} = \underbrace{F_{ab}}_{e^{\frac{iL}{2}(P_-^a R^b - P_-^b R^a)}} h_{ab} F_{ab}$

cyclic

$\mathcal{O} = \mathrm{tr}(\phi_1 * \ldots * \phi_n * \mathcal{U}_{-\Sigma L_i})$ h_{ab} same, but we have twisted bnd. cond (Beisert, Roiban '05)

Deformation triality

$AdS_5 \times S^5$	$\mathcal{N}=4$ SYM	Spin chain
TsT φ_1 φ_2 commuting isometries	$A * B =$ $= e^{i \frac{\gamma_{ij}}{2} Q^i_A Q^j_B} AB$ commuting charges	Drinfeld – Reshetikhin twist (integrable) $\tilde{h}_{ab} = F_{ab} h_{ab} F_{ba}$ $F_{ab} = e^{i \frac{\gamma_{ij}}{2} Q^i_a Q^j_b}$ Cartan gen.
closed string Schr open string AdS twisted b.c.	CFT + irrelev def. usual gauge inv star product → planar equiv dipole gauge invar	modified h_{ab} periodic spin chain $tr(\hat{\phi}_1 .. \hat{\phi}_n)$ same h_{ab} as $\mathcal{N}=4$, twisted periodicity $tr(\phi_1 * \phi_2 * \, .. \, \phi_n * \mathbb{1}_{-\gamma})$

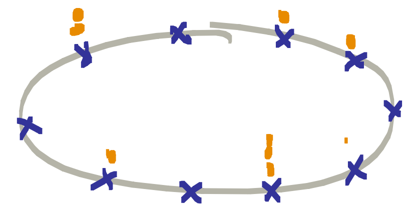
Integrability in planar

null dipole-deformed $\mathcal{N}=4$ SYM

Setup

- smallest closed subsector where twist acts non-trivially
- SL(2) sector (closed) - Heisenberg $XXX_{-\frac{1}{2}}$

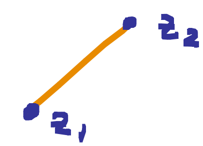
$$\mathcal{O} = \mathrm{tr}\,[\underbrace{D_-^{S_1} Z \; D_-^{S_2} Z \; \dots \; D_-^{S_J} Z}_{J \text{ sites} \,:\, \underbrace{\text{twist}}_{D+M_{+-}} \, J \; @ \; \lambda = 0}]$$



- $\mathcal{H} \sim \Gamma_{NR}$ = non-relativistic anomalous dimension
- natural to fix total momentum, M
- $L = 0$ ($\mathcal{N}=4$ SYM) → only operators w/ same $\sum_{i=1}^{J} S_i$ mix
  $L \neq 0$: $\sum S_i$ not conserved ⇐ broken Lorentz invar. $SL(2) \searrow U(1)$

Example : $J=2$

- $\mathcal{O} = \mathrm{tr}\,(Z D_-^S Z) \longleftrightarrow$ lightray operators $\mathcal{O}(z_1,z_2) = \mathrm{tr}\, Z(z_1)\, Z(z_2)$

$\mathcal{O} =$ 

- operator mixing $\quad \Gamma = \frac{\lambda}{8\pi^2} \sum_1^J h_{\ell,\ell+1}$

$$h_{ab}\, \mathcal{O}(z_a,z_b) = \int_0^1 \frac{du}{u} \Big(2\,\mathcal{O}(z_a,z_b) - \mathcal{O}(z_a + u z_{ba}, z_b) - \mathcal{O}(z_a, z_b + u z_{ab}) \Big)$$

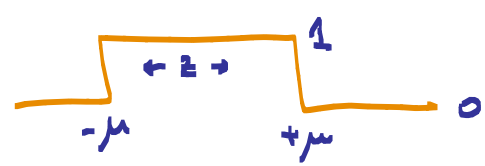
Balitsky, Braun '89
Belitsky, Derkachov, Korchemsky, Manashov '04

- add Drinfeld - Reshetikhin twist $\quad h_{ab} \to \tilde{h}_{ab}$

$$\tilde{h}_{ab}\, \mathcal{O}(z_a,z_b) = \int_0^1 \frac{du}{u} \Big(2\,\mathcal{O}(z_a,z_b) - \mathcal{O}(z_a + u z_{ba} + uL) - \mathcal{O}(z_a, z_b + u z_{ab} - uL) \Big)$$

- operators of definite scaling $O_\psi = \int dz_1 dz_2\, \psi(z_1,z_2)\, O(z_1,z_2)$
- fix total momentum, M $\quad \psi(z_1,z_2) = \chi(z)\, e^{\frac{i}{2}M(z_1+z_2)}$ $\qquad z = z_2 - z_1$
- eigenvalue eqn. for $\chi(z)$ + dep. on M, L
- $M=0$, pert. is upper-triangular => spectrum unchanged

$\Delta = \Delta(M,L)$

$\mu = 0$	$\mu \neq 0$
$\chi(z) = \left(\frac{d}{dz}\right)^s \delta(z)$	$\chi(z) = P_s(z/\mu) \times$  Legendre pol
eigenst: $\mathrm{tr}\, Z D_-^s Z$ local op	eigenst: non-local ops, arbitrary # D_-

eigenval: $h(s)$

$M \neq 0$ (and J arbitrary)

- ground state energy is lifted. $E_0(LM)$ $(\mathrm{tr}\, Z^J)$

- compute perturbatively in LM (Yang-Baxter)

$$\Delta_0 = J + \frac{\lambda}{4\pi^2}\left(\frac{L^2M^2}{2J+2} + \frac{L^4M^4}{24(J+1)^2} + \cdots\right)$$

- matches w/ BMN string energy for $J \gg 1$, $\frac{\lambda}{J^2}$, μM fixed

$$\Delta_{BMN} = \sqrt{J^2 + \frac{\lambda}{4\pi^2}L^2M^2} \simeq J + \frac{\lambda}{8\pi^2}\frac{L^2M^2}{J} + O(\lambda^2)$$

- reproduced T_{NR} @ large J via coherent states w/ twisted bnd. cond.

- at small J, weak coupling expansion $\neq$ strong coupling one (λL^2M^2)

Conclusions & future directions

- one-loop NR anomalous dimension of $\mathrm{tr}\, Z^J$ in null dipole-deformed $\mathcal{N}=4$ SYM matches strong coupling prediction at large J, $\frac{\lambda}{J^2}$ fixed

$$\text{bulk locality (gravity)} \overset{\Delta(M)}{\longleftrightarrow} \text{planar equivalence + strong coupling (field theory)}$$

- higher loops? three-point functions $F(y)$?
- general constraints on $\Delta(M)$, $F(y)$ from bulk locality
- is there a universal field theory mechanism that reproduces them?
- special properties of dipole CFT_2s & connection to Kerr/CFT?

Thank you!