Dedicated to the memory of Iaonnis Bakas

QFT Corrections to Black Holes

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July 9, 2017

Targets

- Understanding of QFT fluctuations on background geometry.-

- by Applying it to back-reaction effects to the BH solutions , and studying its consequences to geometric and thermal of BH's , collapse process , Hawking Radiation.

Setup

Semi classical approximation

Classical Gravity + Quantum Field Theory + Minimal coupling Lagrangian.

Semi classical approximation

$$G_{\mu\nu} = T_{\mu\nu} + \langle T_{\mu\nu} \rangle$$
$$\nabla_\mu F^{\mu\nu} = \nabla_\mu * F^{\mu\nu} = 0$$

where $\langle T_{\mu\nu} \rangle$ is the the expectation value of $T_{\mu\nu}$ resulting from the quantum field fluctuations. In a general curved background it is a quadratic function of the curvature, trace of which has been exactly calculated.

History

Semiclassical backreaction and black hole formation:

A. Paranjape and T. Padmanabhan, Phys.Rev. D80, 044011 (2009), arXiv:0906.1768 [gr-qc].

V. P. Frolov, in Proceedings, 18th International Seminar on High Energy Physics (Quarks 2014) (2014) arXiv:1411.6981 [hep-th] .

J. M. Bardeen, (2014), arXiv:1406.4098 [gr-qc] .

L. Mersini-Houghton, (2014), 10.1016/j.physletb.2014.09.018, arXiv:1406.1525 [hep-th] .

L. Mersini-Houghton and H. P. Pfeiffer, (2014), arXiv:1409.1837 [hep-th] .

S. Chakraborty, S. Singh, and T. Padmanabhan, (2015), arXiv:1503.01774 [gr-qc] .

H. Kawai, Y. Matsuo, and Y. Yokokura, Int. J. Mod. Phys. A28, 1350050 (2013), arXiv:1302.4733 [hep-th] .

H. Kawai and Y. Yokokura, Int. J. Mod. Phys. A30, 1550091 (2015), arXiv:1409.5784 [hep-th] .

Quantum effects derived through conformal anomaly lead to an inflationary model that can be either stable or unstable:

$\langle T_{\mu\nu} \rangle$

The backreaction of QFT fluctuations corresponds to an effective energy momentum tensor $\langle T_{\mu\nu} \rangle$;

Effective backreaction

The trace of this effective tensor is known as trace(conformal) anomaly:

$$\langle T^{\rho}_{\rho} \rangle = \frac{\hbar}{32\pi}\{(c_A + c'_A)(\mathcal{F} + \frac{2}{3}\Box\mathcal{R})c'_A\mathcal{E} + c_A''\Box\mathcal{R}\}$$

$$c_A = \frac{1}{90\pi}(n_0 + \frac{7}{4}n^M_{\frac{1}{2}} + \frac{7}{2}n^D_{\frac{1}{2}}13n_1 + 212n_2)$$

$$c'_A = \frac{1}{90\pi}(\frac{1}{2}n_0 + \frac{11}{4}n^M_{\frac{1}{2}} + \frac{11}{2}n^D_{\frac{1}{2}} + 31n_1 + 243n_2)$$

n_s is the number of spin s particles and M, D respectively stand for Majorana and Dirac spinors.

Symmetries + trace anomaly $\Rightarrow \langle T_{\mu\nu} \rangle$

Going beyond trace is not a straight forward, In certain highly symmetric conditions it becomes possible. We are looking for corrections to RN and Schwarzschild solutions;

Step 1: We limit our study to stationary spherical charged BHs .

Step 2: T_{μ}^{ν} is diagonal $\Rightarrow T_2^2 = T_3^3$

Step 3: Invariance of the Riemann tensor under radial boost $\Rightarrow T_{\nu}^{\mu}$, would also remain unchanged. Which further implies $T_r^r = T_t^t$.

Symmetries + trace anomaly $\Rightarrow \langle T_{\mu\nu} \rangle$

Step 4: Another useful constraint is that T^{μ}_{ν} is divergence free. To implement it we expand T^{μ}_{ν} in terms of inverse power of r. Each term must be divergent free and for the stationary Rotationally invariant background, it implies that each term has to satisfy the constarint. Hence

$$T^{\mu}_{\nu} = \sum_{p \in I} T_p \begin{pmatrix} I_2 & 0 \\ 0 & (-\frac{p}{2}+1) I_2 \end{pmatrix} r^{-p}$$

This is a powerful constraint imposed on the form of T^{μ}_{ν} such that we can uniquely find T^{μ}_{ν} from the r-expansion of its trace.

Modified perturbative equations for RN background

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\mathcal{R} = 8\pi \left\langle T_{\mu\nu} \right\rangle + T_{\mu\nu} \tag{1}$$

Substituting the zeroth expansion component of metric into the right hand side of equation (1) one can find the next order of the LHS. Up to first order in terms of l_p. The smallest perturbative component of $\left\langle T^{\mu}_{\nu} \right\rangle$ takes the following form:

$$\left\langle T^{\nu}_{\mu} \right\rangle = -\frac{3c_A l_p^2}{4\pi} \begin{pmatrix} I_2 & 0 \\ 0 & -2I_2 \end{pmatrix} \frac{M^2}{r^6} + \frac{c_A l_p^2}{2\pi} \begin{pmatrix} 2I_2 & 0 \\ 0 & -5I_2 \end{pmatrix} \frac{MQ^2}{r^7} - \frac{(6c_A + c'_A) l_p^2}{16\pi} \begin{pmatrix} I_2 & 0 \\ 0 & -3I_2 \end{pmatrix} \frac{Q^4}{r^8}$$

and so the modified metric takes the following form: $\left(\alpha = \frac{3}{5} + \frac{c'_A}{10 c_A}\right)$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + c_A l_p^2 \left(\frac{2M^2}{r^4} - \frac{2MQ^2}{r^5} + \frac{\alpha Q^4}{r^6} \right)$$

Two different modes of modified solution, small $\frac{QM}{l_P^2}$ and large $\frac{QM}{l_P^2}$

Modified horizons ($\frac{l_P}{M} \ll \frac{Q}{l_P} \ll 1$):

$$\tilde{r}_\pm = M \pm \sqrt{M^2 - Q^2} - \frac{c_A l_p^2}{M\beta_\pm^2}\left(1 + \frac{\alpha}{2}\frac{(\beta_\pm - 2)^2}{\beta_\pm - 1}\right)$$

where $\beta_\pm = 1 \pm \sqrt{1 - (Q/M)^2}$.

Modified horizons ($\frac{Q}{l_P} \ll \frac{l_P}{M} \ll 1$):

$$\tilde{r}_+ = 2M - \frac{c_A l_P^2}{4M} - \frac{Q^2}{2M}$$

$$\tilde{r}_- = c_A^{\frac{1}{3}} l_P^{\frac{2}{3}} M^{\frac{1}{3}} - \frac{6Q^2}{M}$$

Extremal black holes ($QM \gg l_P^2$)

For classical RN solution without quantum corrections f approaches $f_0(r, M, Q) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$, resulting in the extremal values of Q and $r_\pm$,

$$r_\pm = Q = M$$

Quamtum correction modifies extremal solutions as following:

$$\tilde{Q} = M - c_A l_p^2 \frac{\alpha}{2M}$$

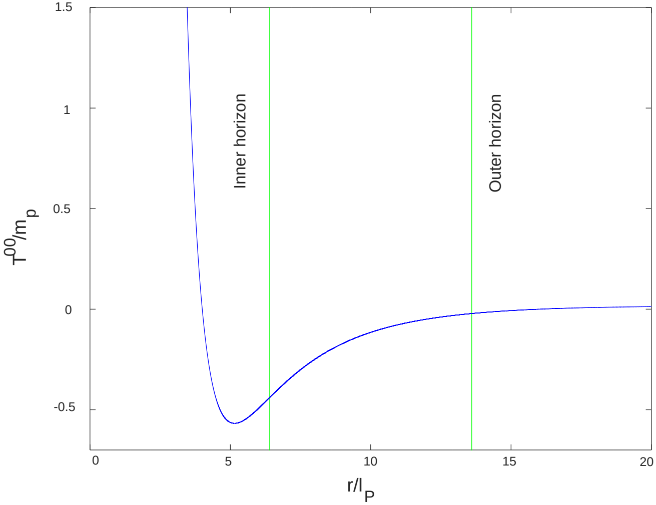
$$\tilde{r}_\pm = M + c_A l_p^2 \frac{2\alpha - 1}{M}$$

Extremal black holes ($QM \ll l_P^2$)

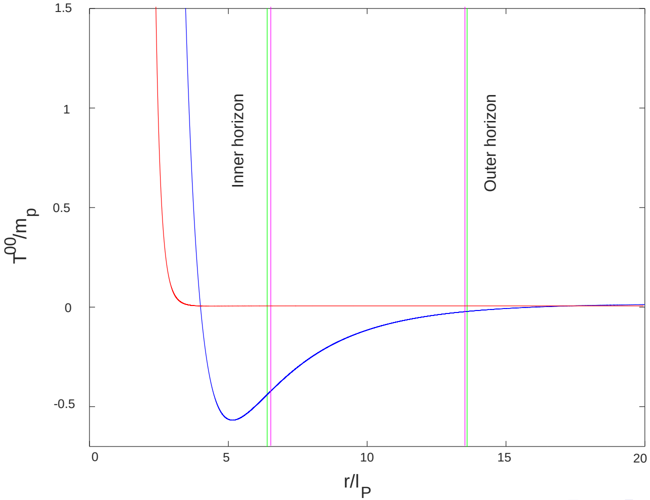
For $QM \ll l_P^2$, we should expand in terms of $\frac{Q}{l_P}$ rather than $\frac{l_P}{M}$. This changes the extremal solutions as following:

$$\tilde{M}_{ext} = \sqrt{\frac{32}{27} c_A l_p} + \sqrt{\frac{3}{32}} \frac{Q^2}{\sqrt{c_A} l_p}$$
$$\tilde{r}_{\pm} = \sqrt{\frac{8}{3} c_A l_p} + \frac{\sqrt{3}}{\sqrt{512}} \frac{Q^2}{\sqrt{c_A} l_p}$$

Energy density of EM field vs Quamtum fluctuations



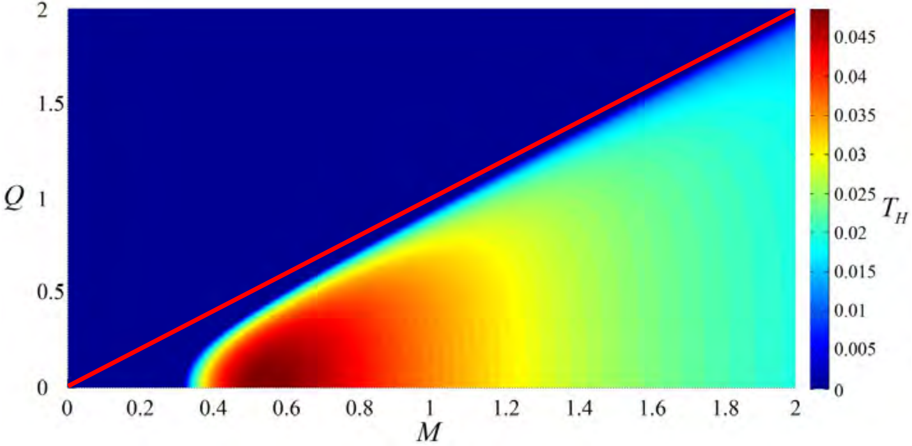
Energy density of EM field vs Quamtum fluctuations



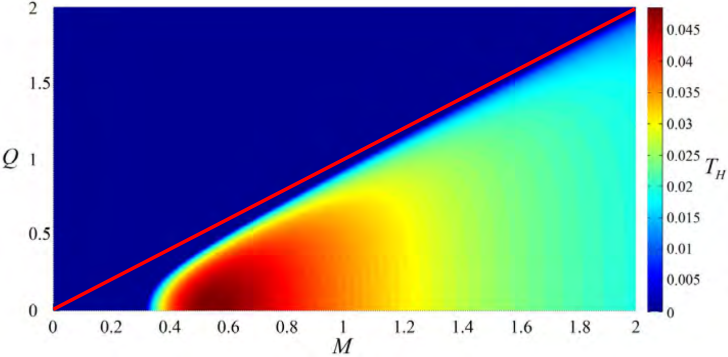
Modification on thermal properties, temperature

The Hawking temperature takes quantum correction as well,

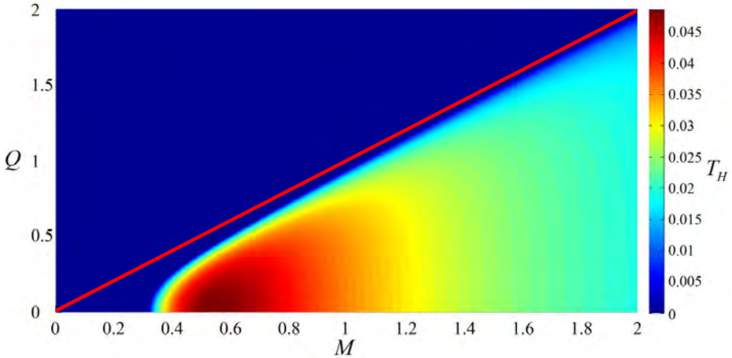
$$T_H = T_H^{RN} - c_A l_p^2 \frac{\beta - 1}{\pi M^3 \beta^5}\left(1 + \frac{3\alpha}{4}\frac{(\beta-2)^2(\beta-2/3)}{(\beta-1)^2}\right)$$



Modification on thermal properties, temperature

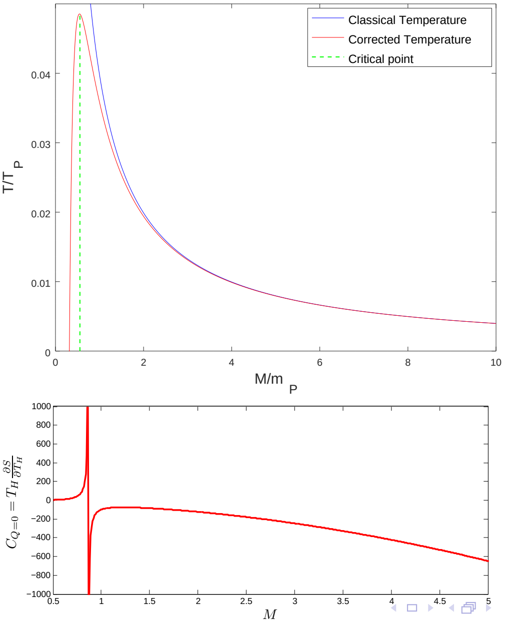


Where $T_H^{RN} = (\beta - 1) / (2\pi M \beta^2)$ is the classical temperature. Note that the line $Q = M$, or $\beta = 1 + \sqrt{1 - (Q/M)^2} = 1$ no longer represents the $T = 0$ state; the first term of the correction vanishes at this line but the second term diverges negatively. This is another indication that the classically extremal case has moved to a nonphysical region with naked singularity as the result of quantum correction.



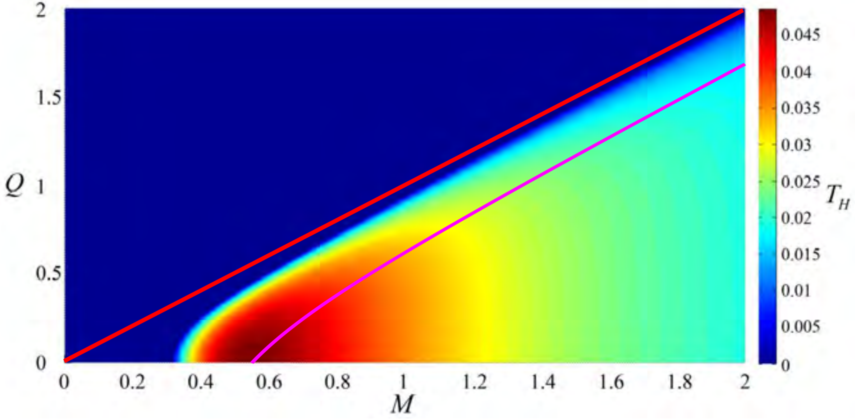
The $T = 0$ curve deviates from its asymptote as the mass moves down from large values. Hence the deviation from the classical extremality condition becomes larger for small mass and charge, i.e. the charge to mass ratio becomes smaller than the classical case to the extent that the zero charge limit acquires a finite mass. The finite mass of the zero charge limit of the extremal case is of the order of Planck's mass.

Modification on thermal properties, phase transition



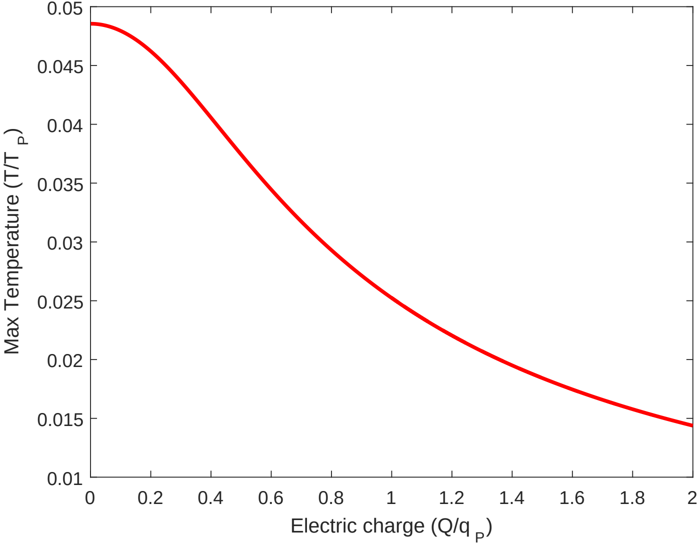
Modification on thermal properties, phase transition

Phase transition≈ maximum temperature



Modification on thermal properties, maximum temperature

Increasing the $Q \Rightarrow$ decreasing of T_{Max}



On the outer horizon, $g_{00} = f(Q, M, r)$ vanishes so we have,

$$\left.\frac{\partial f}{\partial M}\right)_{\tilde{r}_+,Q} \mathrm{d}M + \left.\frac{\partial f}{\partial Q}\right)_{\tilde{r}_+,M} \mathrm{d}Q + \left.\frac{\partial f}{\partial \tilde{r}_+}\right)_{M,Q} \mathrm{d}\tilde{r}_+ = 0$$

$$\Rightarrow dM = -\frac{\left.\frac{\partial f}{\partial Q}\right)_{r_+,M}}{\left.\frac{\partial f}{\partial M}\right)_{r_+,Q}} dQ - \frac{T_H}{\left.\frac{\partial f}{\partial M}\right)_{r_+,Q}} 4\pi d\tilde{r}_+$$

Comparison with the standard thermodynamic relation $dE = TdS + \phi dQ$ gives the expression for dS ,

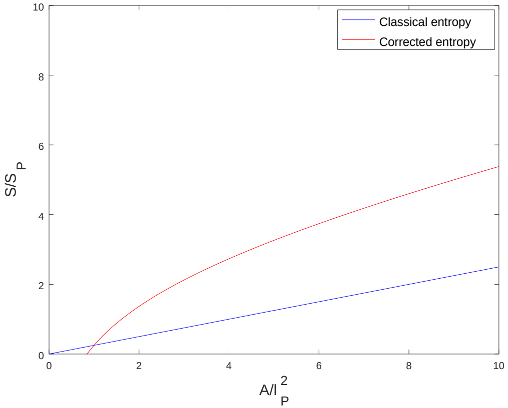
$$dS = -\left(\frac{\partial f}{\partial M}\right)^{-1} 4\pi dr_+ = \frac{2\pi r_+ dr_+}{\left(1 - c_A l_p^2 \left(\frac{2M}{r_+^3} - \frac{Q^2}{r_+^4}\right)\right)}$$

Modification on thermal properties, entropy

Modified entropy

First order quantum corrections gives logarithmic correction to the entropy.

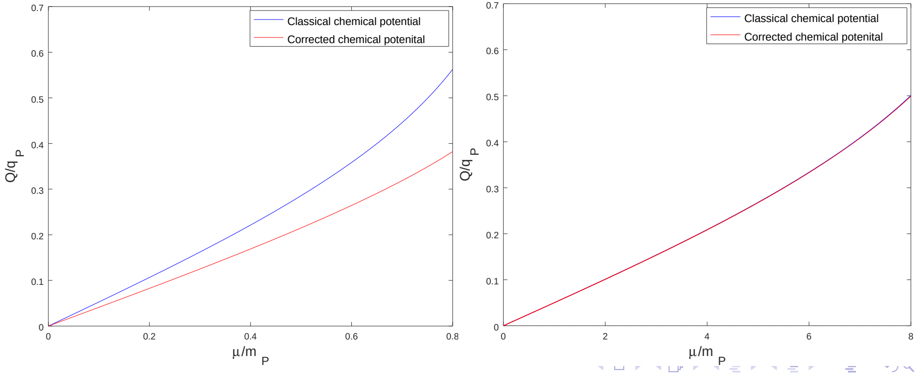
$$S_{Classic} = \frac{A}{4} \Rightarrow S_{Quantum} = \frac{A}{4} + \pi c_A l_p^2 \ln(A)$$



Modification on thermal properties, chemical potential

The chemical potential is defined to be the work needed to adiabatically increase the ADM mass, M, per unit of eletric charge, Q. In the classical case the μ is given by $\frac{Q}{r_+}$. After applying quantum correction, entropy gets modifed and so the adiapatic paths get correction.

Figure: $M = 1m_P$ (Left) and $M = 10m_P$ (Right)



Modification on thermal properties, chemical potential

Physics behind this correction

After applying quantum correction, entropy gets modifed and so the adiapatic paths get correction. Hence in order to keep the entropy unchanged under falling of a charged particle with electric charge dQ and rest mass dm, dm cannot be zero anymore, like the classical case. This nonzero restmass results in a gravitational work appearing in the chemical potental.

$$\mu = \frac{Q}{r_+}\left[1 - c_A l_p^2 \left(\frac{2}{r_+^2} + \frac{(1-2\alpha)Q^2}{r_+^4}\right)\right]$$

- Third law of Thermodynamics states that reaching zero temperature in finite time should be impossible.

- Before studying the quantum corrections, blackholes were belived to evaporate in finite time. But now we know the fate of any black hole will be extremal, even if it's not electrically charged!

-So the "Third Law" states that the lifetime of all black holes must be infinite.

$\Rightarrow$ **Any small quantum correction results in such a strong change**

Hawking radiation, third law

Here is the consistency check

Let us take the black body radiation for black hole,

$$-\frac{dM}{dt} = \sigma A_H T_H^4 = \sigma(4\pi r_+^2)\left(\frac{\partial f}{\partial r}\bigg|_{r=r_+}\right)^4 (4\pi)^{-4} \tag{2}$$

Using the above equation , we shall find lifetime of black hole τ:

$$\tau = -(4\pi)^{-3}\sigma \int_{r_+^0}^{r_{ext}} \frac{dM}{dr_+} \frac{dr_+}{f'^4(r_+)r_+^2} \tag{3}$$

This integral diverges and so the lifetime of black holes is infinite, as it was supposed to be.

Remnant, extremal Schwarzschild black hole, Candidate for dark matter

Conclusion

Conclusions:
1)Change in Extermality Condition
2) Change in the radii of the Horizons
3) Minimum mass for black holes
4)Logarithmic correction to Entropy
5)Correction to thermal quantities such as Chemical potential and specific heat
6)Maximum temperature for Black Holes
7) evasion of singularity in the process of collapse
.
.
8)Existence of a remnant after the evaporation
with zero temperature which can be a candidate for dark matter

Thank you