The Eight Field Way

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Technion

Kim, SSR, Vafa, Zafrir – to appear
SSR, Vafa, Zafrir 1610.09178
Bah, Hanany, Maruyoshi, SSR, Tachikawa, Zafrir 1702.04740

Regional meeting

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Field theory curious observation

- Consider one of the most beloved SCFTs, $su(2)$ SQCD with four flavors

- This theory has many, 72, self-dual frames

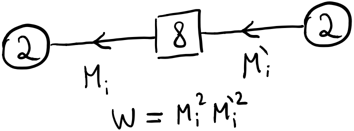
- One original frame, One Intriligator-Pouliot dual ($\mathbf{8} \to \overline{\mathbf{8}}$ and all the M_iM_j flipped)

- 35 Seiberg duals (split $\mathbf{8}$ to $\mathbf{4}$ and $\overline{\mathbf{4}}$ and flip the mesons)

- 35 Csaki, Schmaltz, Skiba, Terning duals (split $\mathbf{8}$ to $\mathbf{4}$ and $\overline{\mathbf{4}}$ and flip the baryons)

Field theory curious observation cont.

- $72 = \frac{2903040}{8!}$. 2903040 is the dimension of the Weyl group of E_7.
- The self-dualities form an orbit of the action of the Weyl group of E_7 (include the trivial $SU(8)$ transformations) (Rains; Spiridonov, Vartanov)
- Actually, (Dimofte, Gaiotto 2012), we can construct a theory which is invariant under the action of the duality group and can be argued to have E_7 flavor symmetry!!



- Computing the protected spectrum we find that all the states fall into E_7 representations. (The E_7 surprise)

A question

- This is a very neat observation
- Why does the symmetry enhance to E_7 ?
- What is the physics (geometry) responsible for such enhancement?

The answer

- The answer turns out to be very interesting leading us to new observations of the same kind and to connection between six dimensions and four dimensions

- The E_7 surprise theory is a simple deformation of a theory one obtains compactifying a six dimensional $(1,0)$ theory with E_8 global symmetry (E - string theory) on a torus with flux breaking E_8 to $E_7 \times u(1)$

- In the rest of the talk we will derive and extend this geometric explanation of the surprise

The strategy

- Start from reviewing general expectations from six dimensions
- Concentrate on case of E-string
- Will go to four dimensions and discuss theories obtained by compactifying on a torus
- Will understand the surprise as part of a geometric scheme and will discover (infinity of) many other surprises

Six dimensions

- We have variety of six dimensional superconformal theories with $(1, 0)$ supersymmetry
- In general $(1, 0)$ theories have some global symmetry G
- Eg: E - string theory
- The flavor symmetry is E_8

Six dimensions on $\mathcal{C}_g$

- We consider the $(1,0)$ theory on a Riemann surface $\mathcal{C}_g$ and we consider compactifications which preserve $\mathcal{N}=1$ supersymmetry

- In addition to the choice of the surface (genus g) can turn on background configuration for flavor symmetry G
  - flat G bundles, continuous parameters *(holonomies around the cycles)*
  - flux for abelian subgroup $\mathbf{L}$ of G through $\mathcal{C}_g$, discrete parameters *($c_1(L)$)*

- The symmetries are given by a subgroup of G, G_{max} (commuting with the fluxes)

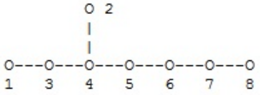
Six dimensions on $\mathcal{C}_g$: Predictions for 4d

- Compactifications labeled by choice of $\mathcal{C}_g$ and choice of flux, $\mathcal{F}$: discrete choices (symmetry G_{max})

- The 't Hooft anomalies for all the symmetries in 4d can be inferred from 6d by integrating the anomaly polynomial over the Riemann surface

Six dimensions on $\mathcal{C}_g$: E - string (A)

- Take E - string then $G = E_8$
- There is rather rich variety for G_{max}
- With vanishing flux symmetry is E_8
- Flux for one $u(1)$



$u(1)e_7$	$u(1)so(14)$	$u(1)su(2)e_6$	$u(1)su(3)so(10)$	$u(1)su(8)$

$u(1)su(2)su(7)$	$u(1)su(2)su(3)su(5)$	$u(1)su(4)su(5)$

- For every choice of g and $\mathcal{F}$ we obtain in general different symmetries in four dimensions and different conformal manifolds

Six dimensions on $\mathcal{C}_g$: E – string (B)

- The 't Hooft anomalies of the four dimensional theory can be obtained by integrating the anomaly eightform of the six dimensional theory over the Riemann surface, $\int_{\mathcal{C}_g} \mathcal{I}_8(\mathcal{F}) = \mathcal{I}_6^{(g,\mathcal{F})}$

(The anomaly polynomial for E - string: Ohmori, Shimizu, Tachikawa in 2014)

- Example, genus one with no punctures $G_{max} = u(1)e_7$,

$$a = 2z\,, \qquad c = \frac{5}{2}z$$

- Example, $G_{max} = u(1)G'$ with flux z,

$$a = 2\sqrt{\xi_{G'}}z\,, \qquad c = \frac{5}{2}\sqrt{\xi_{G'}}z$$

Here $\xi_{G'}$ is the embedding index of the $U(1)$ in E_8

$$e_7 \to 1\,, \qquad e_6 \to 3\,, \qquad so(14) \to 2\,, \qquad su(8) \to 4\,,$$

$$su(7) \to 7\,,\; so(10) \to 6\,,\; su(4)su(5) \to 10\,,\; su(3)su(5) \to 15$$

Four dimensions

- How can we find the theories in four dimensions arising in the compactifications?

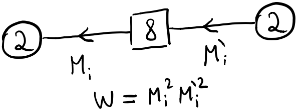
- Theories might not have a description using standard Lagrangian

- We need some idea to take us into the space of possibilities

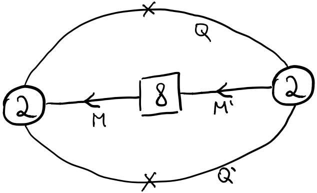
- The surprise of Dimofte-Gaiotto will be that idea

Upgraded surprise

- Try to find a compactification of E-string leading to the surprise



- Anomalies do not match to any simple compactification
- Anomalies do match with a simple generalization



- $c = \frac{5}{2}\,, \quad a = 2$
  consistent with e_7 compactification with flux one

Checks

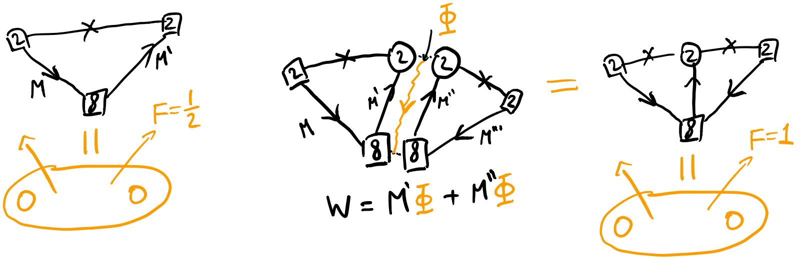
- Anomalies match
- The index exhibits E_7 representations

$$\mathcal{I} = 1 + (pq)^{\frac{2}{3}}(3z^2 + \frac{1}{z}\chi[\mathbf{56}]) - 2pq + (pq)^{\frac{2}{3}}(p+q)(2z^2 + \frac{1}{z}\chi[\mathbf{56}])$$
$$+(pq)^{\frac{4}{3}}(6z^4 + z\chi[\mathbf{56}] + \frac{1}{z^2}(\chi[\mathbf{1463}] - \chi[\mathbf{133}] - 1)) + \dots$$

- Deformation giving vacuum expectation value to the flippers gives the surprise

The tube and the gauging

Think of the torus theory as two tubes with two punctures with half unit of flux glued together

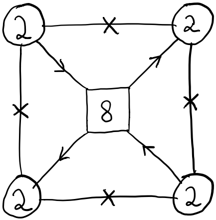


- Symmetry of puncture is $su(2)$, when glue punctures we add fundamental octet of fields Φ

- E-string to five dimensions, $su(2)$ gauge with eight fund. hyper; Mnemonic rule: puncture symmetry in four is the gauge symmetry in five and hyper become chiral

Torus with more flux

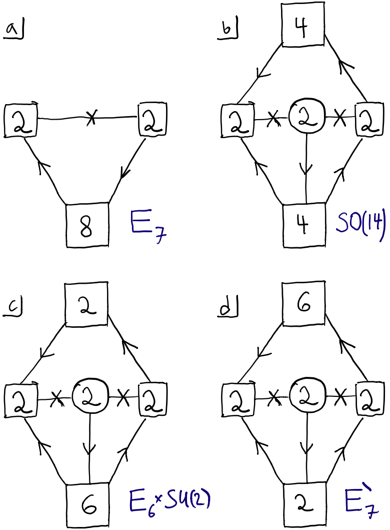
- Combine tubes together to form torus with higher value of flux
- Take $2z$ tubes get flux z (triangulation of a circle)



- This is a non trivial check of the gauging and identification of a tube

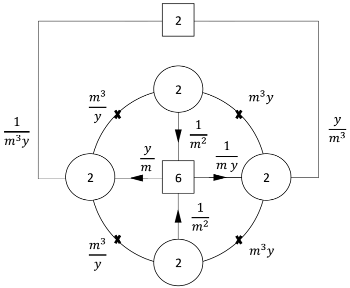
Tubes for other $U(1) \subset E_8$

We can now try to find tubes for other types of flux



Example: $u(1)su(2)e_6$ theories

Combine two tubes to obtain a theory with flux one



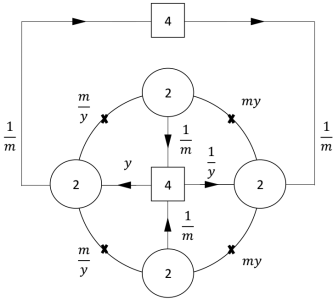
$c = \frac{5}{2}\sqrt{3} \qquad a = 2\sqrt{3}$

$$\chi[\mathbf{2},\mathbf{1}] = y^2 + \frac{1}{y^2}\,,\;\; \chi[\mathbf{1},\overline{\mathbf{27}}] = \chi[\mathbf{2},\mathbf{6}]_{SU(2)\times SU(6)} + \chi[\mathbf{1},\overline{\mathbf{15}}]_{SU(2)\times SU(6)}$$

$$\begin{aligned}\mathcal{I} \quad &= 1 + \frac{3}{m^6}\chi[\mathbf{2},\mathbf{1}](pq)^{\frac{1}{3}} + \frac{2}{m^4}\chi[\mathbf{1},\overline{\mathbf{27}}](pq)^{\frac{5}{9}} + \frac{3}{m^6}\chi[\mathbf{2},\mathbf{1}](pq)^{\frac{1}{3}}(p+q)\\ &+ \frac{3}{m^{12}}(1+2\chi[\mathbf{3},\mathbf{1}])(pq)^{\frac{2}{3}} + \frac{1}{m^2}\chi[\mathbf{2},\mathbf{27}](pq)^{\frac{7}{9}} + \frac{6}{m^{10}}\chi[\mathbf{2},\overline{\mathbf{27}}](pq)^{\frac{8}{9}}\\ &+ pq\frac{2}{m^{18}}(4\chi[\mathbf{4},\mathbf{1}] + 3\chi[\mathbf{2},\mathbf{1}]) + \dots .\end{aligned}$$

Example: $u(1)so(14)$ models

Combine two tubes to obtain a theory with flux one

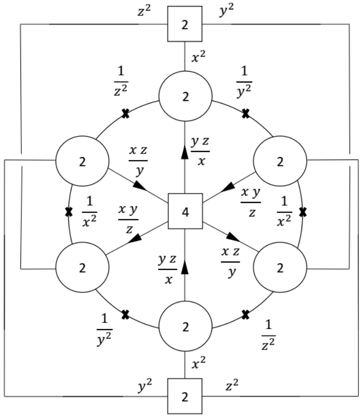


$$c = \frac{5}{2}\sqrt{2} \quad a = 2\sqrt{2}$$

$$\chi[\mathbf{14}] = y^2 + \frac{1}{y^2} + \chi[\mathbf{6},\mathbf{1}] + \chi[\mathbf{1},\mathbf{6}]\,,\;\; \chi[\mathbf{64}] = y(\chi[\mathbf{4},\overline{\mathbf{4}}] + \chi[\overline{\mathbf{4}},\mathbf{4}]) + \frac{1}{y}(\chi[\mathbf{4},\mathbf{4}] + \chi[\overline{\mathbf{4}},\overline{\mathbf{4}}])$$

$$\begin{aligned}\mathcal{I} \quad &= 1 + \frac{2}{m^2}\chi[\mathbf{14}](pq)^{\frac{1}{2}} + \frac{1}{m}\chi[\mathbf{64}](pq)^{\frac{3}{4}} + \frac{2}{m^2}\chi[\mathbf{14}](pq)^{\frac{1}{2}}(p+q) \\ &+ pq(m^4 + \frac{1}{m^4}(3\chi[\mathbf{104}] + \chi[\mathbf{91}] - 1)) + \dots .\end{aligned}$$

Example: $u(1)su(3)so(10)$ model



The symmetry is implied by group theory and can be checked with index. The anomalies are $c = \frac{5}{2}\sqrt{6}\,,\ a = 2\sqrt{6}$

Comments

- Compactification on general genus
- Flows relating different models with different compactifications
- Some of the constructions here have a generalization to higher rank E-string
- For example the torus theories for higher rank can be related to similar models with $su(2) \to usp(2Q)$ and adding a field in antisymmetric for every gauging (Q is the rank)

Summary

- Surprises can be not surprising
- The E_7 surprise can be imbedded in a geometric program
- The program undergoes large number of consistency checks
- Leading to many new surprises

Thank You