Aspects of the black hole horizon in AdS/CFT

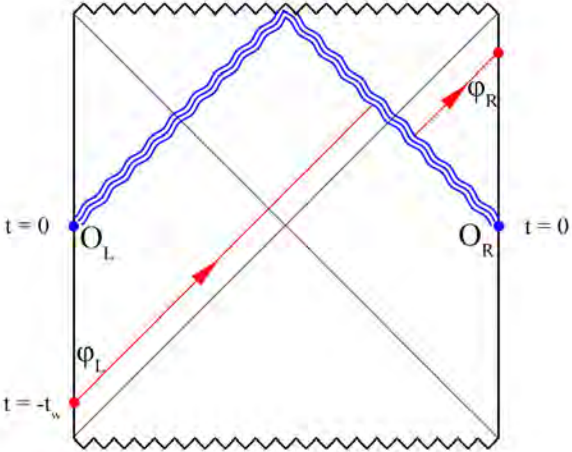
Kyriakos Papadodimas

CERN and University of Groningen

[to appear] + [in progress]

9th regional string meeting, Kolymbari 2017

I will present some additional evidence for the smoothness of the black hole horizon in AdS/CFT, and the need to describe the interior using state dependent operators.



$$|TFD\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E\rangle \otimes |E\rangle$$

[Gao, Jafferis, Wall] [Maldacena. Stanford, Yang]

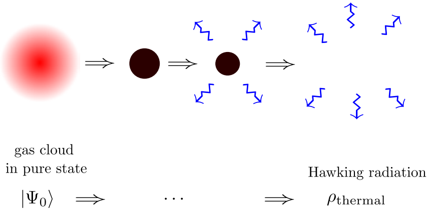
A new class of non-equilibrium states [to appear]

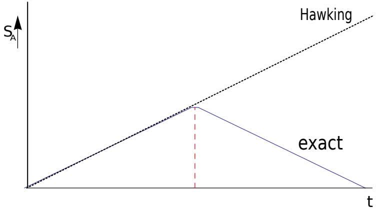
I will present a new natural class of non-equilibrium states, which exist in any chaotic quantum system. In theories with holographic dual these states represent black holes with transient excitations behind the black hole horizon.

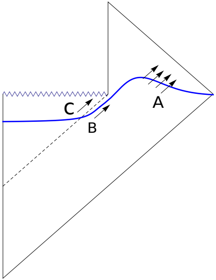
These may be interesting for

- Their role in statistical mechanics
- The existence of these states is evidence for smooth interior
- Using these states, and combined with the GJW protocol, we can probe more directly the interior of a 1-sided black hole [in progress]

The information/firewall paradox







Violation of strong subadditivity of entanglement entropy

$$S_{AB} + S_{BC} \geq S_A + S_C$$

[Mathur], [Almheiri, Marolf, Polchinski, Sully]

Firewall paradox in AdS/CFT

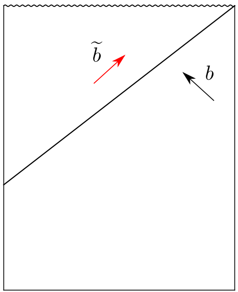
- Large black holes in AdS are holographically dual to QGP states of $\mathcal{N} = 4$ in deconfined phase

- These black holes are in equilibrium with their Hawking radiation and do not evaporate

- Nevertheless the firewall paradox has been formulated even for these stable black holes [Almheiri, Marolf, Polchinski, Stanford, Sully], [Marolf, Polchinski]

- It suggests that big AdS black holes have a singular horizon.

- Most precise formulation of the paradox.



For smooth horizon effective field theory requires:

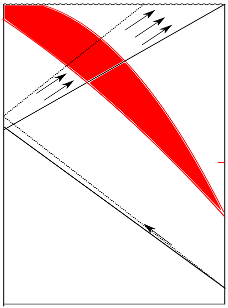
I) $\widetilde{b}$ commute with b

II) $\widetilde{b}$ entangled with b: $(\widetilde{b} - e^{-\frac{\beta\omega}{2}} b^\dagger)|\Psi\rangle = 0$

$$\begin{array}{ccc} b & \Leftrightarrow & \mathcal{O} \\ \widetilde{b} & \Leftrightarrow & ? \end{array}$$

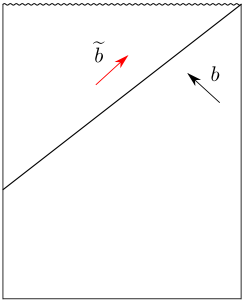
Which CFT operators $\widetilde{\mathcal{O}}$ correspond to $\widetilde{b}$?

Black holes from collapse



- Transplanckian problem
- States formed by collapse form a small subset of typical BH microstates.

Firewall paradox for large AdS black holes



$[b, b^\dagger] = 1$

$[H, b^\dagger] = \omega b^\dagger$

$[\widetilde{b}, \widetilde{b}^\dagger] = 1$

$[H, \widetilde{b}^\dagger] = -\omega \widetilde{b}^\dagger$

- [AMPSS, MP] paradox: if typical CFT states have smooth horizon, using $[H, \widetilde{\mathcal{O}}_\omega^\dagger] = -\omega \widetilde{\mathcal{O}}_\omega^\dagger$ we find

$$Tr[e^{-\beta H} \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega] < 0$$

which is inconsistent

Tomita-Takesaki modular theory

[based on work with S. Raju]

Introduce a "small algebra" $\mathcal{A}$ of simple operators (single trace + small products). H is not an element of the algebra, but if A is then so is $[H, A]$.

Define modular Hamiltonian K for the algebra acting on BH microstate $|\Psi\rangle$

Using large N and the KMS condition for thermal correlators in equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

The mirror operators

Tomita-Takesaki construction:

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta H}{2}}\mathcal{O}_\omega^\dagger e^{\frac{\beta H}{2}}|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]\mathcal{O}....\mathcal{O}|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle$$

These equations define the operators $\widetilde{\mathcal{O}}$ on a subspace $\mathcal{H}_\Psi \subset \mathcal{H}_{\rm CFT}$, which is relevant for EFT around BH microstate $|\Psi\rangle$

$$\mathcal{H}_\Psi = {\rm span}\mathcal{A}|\Psi\rangle$$

Equations admit solution because the algebra $\mathcal{A}$ cannot annihilate the state $|\Psi\rangle$

Reconstructing the interior

$$\phi(t,r,\Omega) = \int_0^\infty d\omega \Big[\mathcal{O}_\omega\, f_\omega(t,\Omega,r) + \widetilde{\mathcal{O}}_\omega g_\omega(t,\Omega,r) + \text{h.c.}\Big]$$

State-dependence

- Interior operators defined by

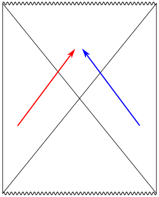
$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}_\omega^\dagger|\Psi\rangle$$

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

- Solution depends on reference state $|\Psi\rangle$
- Operators cannot be upgraded to "globally defined" operators
- Unusual in Quantum Mechanics

Some new evidence in favor of state-dependence



$$H_{\text{total}} = H_L + H_R$$

$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle_L \otimes |E_i\rangle_R$$

Wormhole is centered at $t_L = t_R = 0$.

$$\langle \text{TFD}|\, \mathcal{O}_L(t_L = 0)\ \mathcal{O}_R(t_R = 0)|\text{TFD}\rangle \sim O(1)$$

The $|\mathrm{TFD}\rangle$ has the exact symmetry

$$(H_R - H_L)|\mathrm{TFD}\rangle = 0$$

which implies

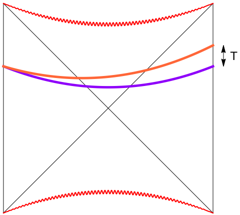
$$e^{i(H_R - H_L)t}|\mathrm{TFD}\rangle = |\mathrm{TFD}\rangle$$

On the other hand

$$|\Psi_T\rangle \equiv e^{iH_R t}|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\frac{\beta E_i}{2}}\, e^{iE_i t}\, |E_i\rangle_L \otimes |E_i\rangle_R$$

is a genuinely new state due to the phases.

In the bulk, the state $|\Psi_T\rangle$ is related to $|\mathrm{TFD}\rangle$ by a large diff (which acts as a time translation on the right boundary, and as identity on the left boundary).

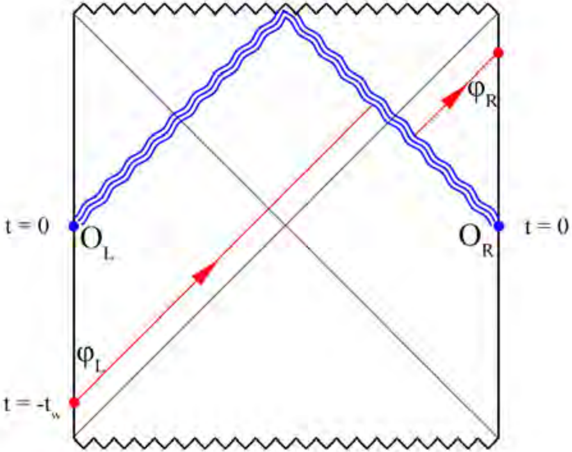


An observer jumps from the left CFT at $t_L = 0$ into the state $|\Psi_T\rangle$. Do they experience a smooth horizon?

Bulk diff symmetry seems to suggest this.

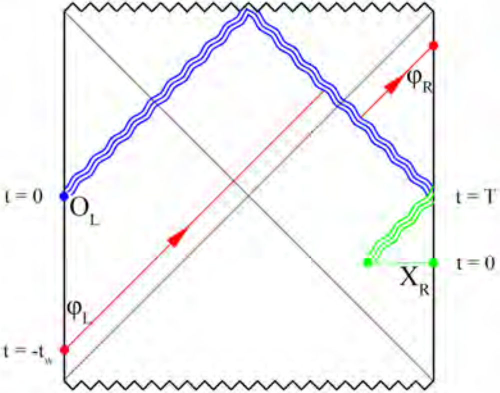
We have a class of states $|\Psi_T\rangle$, which all appear to be smooth to the observer who jumps from the left CFT at $t_L = 0$ for all T, even it if $T \sim e^S$

In previous work [KP + S.Raju] we showed that this can only happen if we allow local operators in the interior to be state-dependent.



$|TFD\rangle$

[Gao-Jafferis-Wall],[Maldacena, Stanford, Yang]



$$|\Psi_T\rangle = e^{iH_RT}|TFD\rangle$$

couple two CFTs at $t = 0$ with

$$U = e^{igO_L(t=0)X_R(t=0)}$$

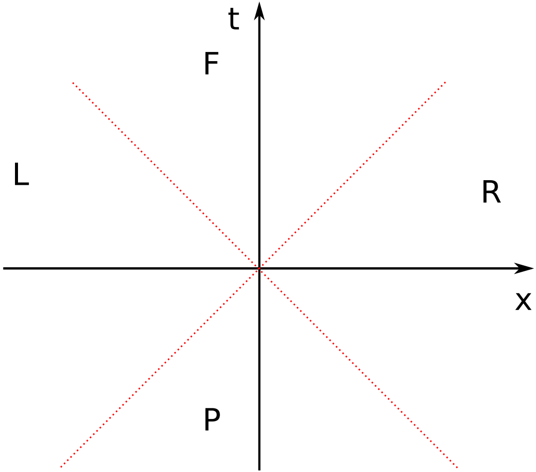
where $X_R \equiv e^{iH_RT}O_Re^{-iH_RT}$
[in progress with Rik van Breuekelen]

To the extent that the computations of GJW and MSY have demonstrated the smoothness of the horizon of the eternal BH, we can prove equally strongly (i.e. exactly in N etc.) that all time-shifted states are smooth.

This is additional evidence that state-dependence may be the correct way to describe the BH interior.

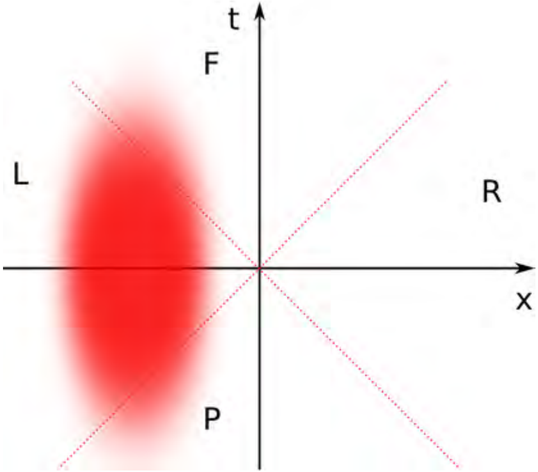
Also interesting to think about state-dependence of local operators after the particle has emerged on the right

States vs operators



Reeh-Schlieder theorem

States vs operators



$$e^{-\pi K} U(\mathcal{O}_R) e^{\pi K} |0\rangle_M$$

Bisognano-Wichmann theorem

A new class of non-equilibrium states

- Motivated by this and the previous construction of the BH interior, we will naturally identify a new class of non-equilibrium states present in any chaotic statistical system. In holographic CFTs these states correspond to excitations behind the black hole horizon.

- The number of such states is in correspondence with possible ways to excite the horizon in EFT

- The existence of these states is motivated by, but logically independent from state-dependent operators.

- Their existence is robust (no subtleties about "generalization of quantum mechanics")

- This shows that the CFT contains in its Hilbert space states which describe excitations of the interior $\Rightarrow$ evidence that the black hole interior is as predicted by GR

Equilibrium states

We define them by demanding that “reasonable observables” are time independent

$$\frac{d}{dt}\langle\Psi|A(t)|\Psi\rangle = 0$$

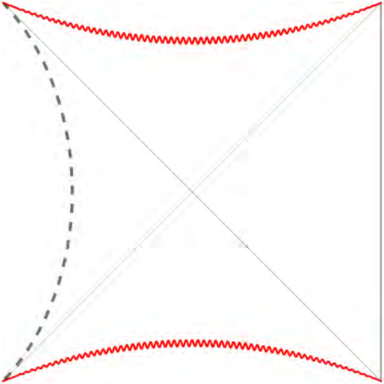
Typical states are equilibrium states

$$|\Psi\rangle = \sum_i c_i|E_i\rangle \qquad c_i \Rightarrow \text{random coefficients with some measure}$$

Atypical states may be time-dependent, hence non-equilibrium.

Unter time evolution even these states will equilibrate, hence an atypical state will start looking like a typical state.

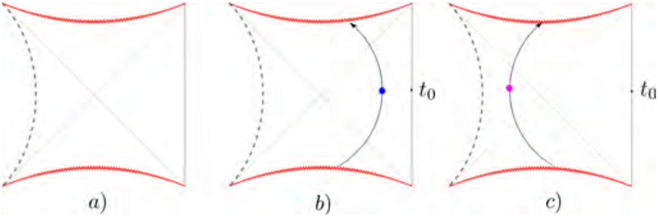
Gravity dual of a typical state



Existence of right exterior region is obvious from typicality and Eigenstate Thermalization Hypothesis

$$\langle\Psi|\mathcal{O}(x_1)...\mathcal{O}(x_n)|\Psi\rangle = Z^{-1}\mathrm{Tr}[e^{-\beta H}\mathcal{O}(x_1)...\mathcal{O}(x_n)] + O(1/N)$$

A new class of non-equilibrium states



- $|\Psi\rangle$ = equilibrium state
- $U(\mathcal{O})|\Psi\rangle$ = standard non-equilibrium state (near equilibrium)
- $U(\widetilde{\mathcal{O}})|\Psi\rangle$ = new type of non-equilibrium state

We will argue that these states exist independent of the $\widetilde{\mathcal{O}}$ construction.

Standard non-equilibrium states of conventional kind

Take equilibrium state $|\Psi_0\rangle$ and excite it as

$$|\Psi\rangle = U(\mathcal{O})|\Psi_0\rangle$$

and example might be $U(\mathcal{O}) = e^{i\theta\mathcal{O}(t_0)}$

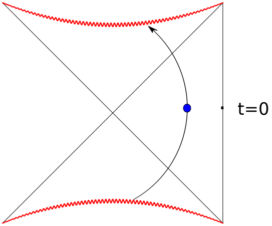
We have

$$\langle\Psi|\mathcal{O}(t)|\Psi\rangle = \langle\Psi_0|U(\mathcal{O})^\dagger\mathcal{O}(t)U(\mathcal{O})|\Psi_0\rangle$$

$$= \langle\Psi_0|\mathcal{O}(t)|\Psi_0\rangle + i\theta\langle\Psi_0|[\mathcal{O}(t), \mathcal{O}(t_0)]|\Psi_0\rangle + \mathcal{O}(\theta^2)$$

The first term is the equilibrium result, the second is time-dependent.

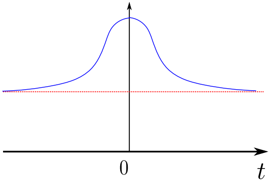
Bulk interpretation of standard non-eq states



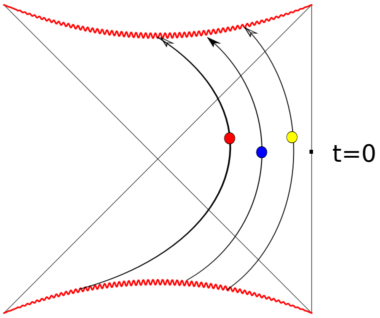
$U(\mathcal{O})|\Psi\rangle$

Excited state $U(\mathcal{O})|\Psi_0\rangle$

$\langle\Psi_0|U^\dagger O(t)U|\Psi_0\rangle = t$-dependent



Space of non-equilibrium states



$$U_1(\mathcal{O})U_2(\mathcal{O})U_3(\mathcal{O})|\Psi_0\rangle$$

New non-equilibrium states

$$|\Psi'\rangle = U(\widetilde{O})|\Psi_0\rangle$$

However, we notice that they can be rewritten as

$$\boxed{|\Psi'\rangle = e^{-\frac{\beta H}{2}} U(O)^\dagger e^{\frac{\beta H}{2}} |\Psi_0\rangle}$$

Remember analogy with Minkowski space.

We will argue

i) Such states appear to be equilibrium when probed by the small algebra $\mathcal{A}$

ii) It can be seen that they are out of equilibrium by including H in correlators

Natural bulk interpretation: black hole with excitations behind the horizon

Notice that operator

$$e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}$$

is not a unitary. However the state

$$|\Psi'\rangle = e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}|\Psi_0\rangle$$

has unit norm. Indeed

$$\langle\Psi'|\Psi'\rangle = \langle\Psi_0|e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}|\Psi_0\rangle$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}] + O(1/S)$$

$$= 1 + O(1/S)$$

Alternatively we can produce the same state via state-dependent unitary operator

$$U(\widetilde{O})|\Psi_0\rangle = e^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}|\Psi_0\rangle$$

State seems to be in equilibrium wrt algebra $\mathcal{A}$

$$\langle\Psi'|A|\Psi'\rangle = \langle\Psi_0|e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}Ae^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}|\Psi_0\rangle$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}e^{\frac{\beta H}{2}}U(O)^\dagger e^{-\frac{\beta H}{2}}Ae^{-\frac{\beta H}{2}}U(O)e^{\frac{\beta H}{2}}] + O(1/S)$$

$$= \frac{1}{Z}\mathrm{Tr}[e^{-\beta H}A] + O(1/S)$$

Including H in correlators. We define $\hat{H} = H - E_0$ and to be concrete consider the state

$$|\Psi\rangle = e^{-\frac{\beta H}{2}} e^{i\theta \mathcal{O}(t_0)} e^{\frac{\beta H}{2}} |\Psi_0\rangle \tag{1}$$

and compute

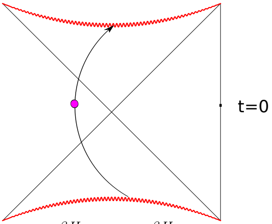
$$\langle\Psi|\mathcal{O}(t)\hat{H}|\Psi\rangle = i\theta \left[\langle\Psi_0|\mathcal{O}(t)\hat{H}\mathcal{O}(t_0 + i\frac{\beta}{2})|\Psi_0\rangle - \langle\Psi_0|\mathcal{O}(t_0 - i\frac{\beta}{2})\mathcal{O}(t)\hat{H}|\Psi_0\rangle \right]$$

$$\langle\Psi|\mathcal{O}(t)\hat{H}|\Psi\rangle \approx \theta \langle\Psi_0|\mathcal{O}(t)\frac{d\mathcal{O}}{dt}(t_0 + i\frac{\beta}{2})|\Psi_0\rangle \tag{2}$$

This correlator decays exponentially as $|t - t_0|$ becomes very large, but it is nonzero and $O(1)$ around the time $t = t_0$.

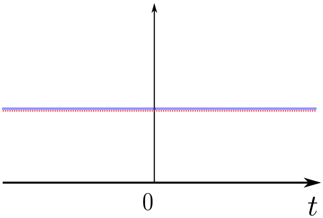
Bulk interpretation of new non-eq states

Excited state $|\Psi'\rangle = U(\widetilde{\mathcal{O}})|\Psi_0\rangle$

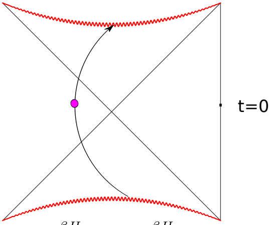


$e^{-\frac{\beta H}{2}}U(\mathcal{O})e^{\frac{\beta H}{2}}|\Psi_0\rangle$

$\langle\Psi'|A(t)|\Psi'\rangle$

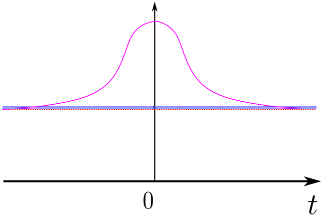


Bulk interpretation of new non-eq states

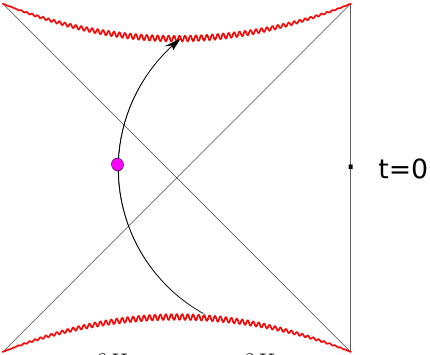


$e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}} |\Psi_0\rangle$

$\langle \Psi' | A(t) H | \Psi' \rangle$



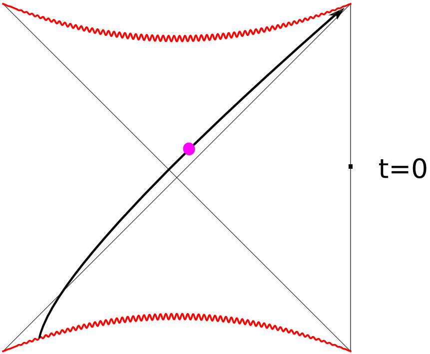
Testing the proposal



$$e^{-\frac{\beta H}{2}} U(\mathcal{O}) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

How does it look to infalling observer? If we use $\widetilde{\mathcal{O}}$ then interpretation turns out to be correct.

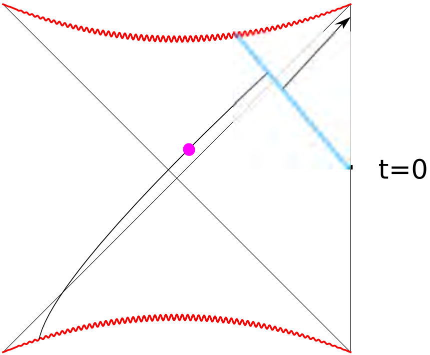
Testing the proposal



Testing the proposal

Following GJW we can try to create a negative energy shockwave by perturbing the CFT with

$$U = e^{ig\mathcal{O}\widetilde{\mathcal{O}}}$$



[based on discussions with R. van Breukelen, J.de Boer, S. Lokhande, E.Verlinde]

Summary

We presented some additional arguments in favor of the smoothness of the interior of big black holes in AdS.

The GJW idea can also be applied to time-shifted wormholes, and we get a large family of states with smooth interior, for which state-dependence is needed.

We identified a canonical class of new non-equilibrium states, of the form

$$e^{-\frac{\beta H}{2}} U(O) e^{\frac{\beta H}{2}} |\Psi_0\rangle$$

which are parametrized in a similar way as perturbations outside horizon (i.e. by unitaries $U(O)$) — yet the perturbations are undetectable by single trace operators.

This indicates the existence of a seemingly causally disconnected region of spacetime in the bulk, whose natural interpretation is the region behind the horizon.

The tilde operators cause transitions between these states.

But the existence of these states is rather robust (no need to use state-dependent operators)

Additional evidence that large AdS black holes have a smooth interior

Further check using GJW.

These states exist in any chaotic statistical system. What is their meaning for the strongly coupled QGP?

Thank you