Local bulk physics from Intersecting Modular Hamiltonians

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Introduction

- How does bulk physics arise from the CFT?

- How do the degrees of freedom of the CFT arrange into bulk degrees of freedom?

- We are interested in local bulk objects

- Q: what are the CFT operators whose CFT correlation function reproduce (in some approximation) bulk theory correlation functions.

Bulk reconstruction

We want to find CFT operators that obey

$$< \phi(z_1, x_1) \cdots \phi(z_n, x_n) >_{CFT} = bulk\ result$$

In perturbation theory in 1/N we start with 2-point functions.
Then we can write

$$\Phi^{(0)}(x, z) = \int dx' K(x, z|x')\mathcal{O}(x')$$

To find $K(x, z|x')$ we need to know the free equation of motion, i.e the bulk metric.

There are many representations of the CFT bulk operator. One of them for empty d=3 AdS in Poincare coordinates (HKLL)

$$\Phi^{(0)}(Z,X,T) = \frac{\Delta-1}{\pi}\int_{y'^2+t'^2<Z^2} dt'dy' \left(\frac{Z^2-y'^2-t'^2}{Z}\right)^{\Delta-2} \mathcal{O}(T+t',X+iy')$$

$$m^2 = \Delta(\Delta-d)$$

$$< \Phi^{(0)}(Z_1,X_1,T_1)\Phi^{(0)}(Z_2,X_2,T_2) >_{CFT} = bulk - bulk\ two\ point\ function$$

Transforms as a bulk scalar under conformal transformations

Bulk causality encoded in the two point function

Inserting $\Phi^{(0)}$ into 3-point functions gives an expression that does not obey bulk locality, which we know from the 2-point function.

To cure this we redefine the bulk operator (KLL 2011)

$$\phi_i(x,z) = \int d^d x' K_{\Delta_i}(x,z|x')\mathcal{O}_i(x') + \frac{1}{N}\sum_n a_n^{CFT} \int d^d x' K_{\Delta_n}(x,z|x')\mathcal{O}_n(x')$$

where $\mathcal{O}_n(x')$ are a tower of higher dimension double trace operators, smeared appropriately.

The coefficients a_n^{CFT} are chosen such that inserted in the 3-point function bulk locality is restored.

This gives the correct equation of motion for the bulk operator, and includes the freedom of bulk field re-definitions (KL 2015)

Question:

K(z,x|x') is a basic building block in this program. Can we get it from the CFT. Is there a meaning for $\Phi^{(0)}$ in the CFT ?.

1)OPE blocks (Czech, Lamprou, McCandlish, Mosk, Sully; Carneiroda Cunha, Guica;de Boer, Hael, Heller,Myers)

$$\mathcal{O}(y_1)\mathcal{O}(y_2) = \sum_k c_k \int_\gamma ds \Phi_k^{(0)}$$

can try to invert, but need to know the appropriate transform

2)Boundary states (Miyaji, Numasawa, Shiba, Takayanagi, Watanabe; Verlinde; Nakayama, Ooguri)
Use symmetries to give an expression for the bulk operator at the center of AdS

$$[Q_0 - P_0, \Phi^{(0)}] = [Q_1 + P_1, \Phi^{(0)}] = [M_{01}, \Phi^{(0)}] = 0$$

Possible in very symmetric situations and uses bulk information
Can we do better ?.

Modular Hamiltonian

Given a density matrix one can define a modular hamiltonian which generates a modular flow

$$H_{mod} = -\log \rho$$

For a region A in the CFT we can define a density matrix by tracing over the compliment region, so we get $H_{mod,A}$

One can similarly do for the compliment region of A, and get

$$H_{mod,\bar{A}}$$

One defines the total modular hamiltonian as

$$\tilde{H}_{mod} = H_{mod,A} - H_{mod,\bar{A}}$$

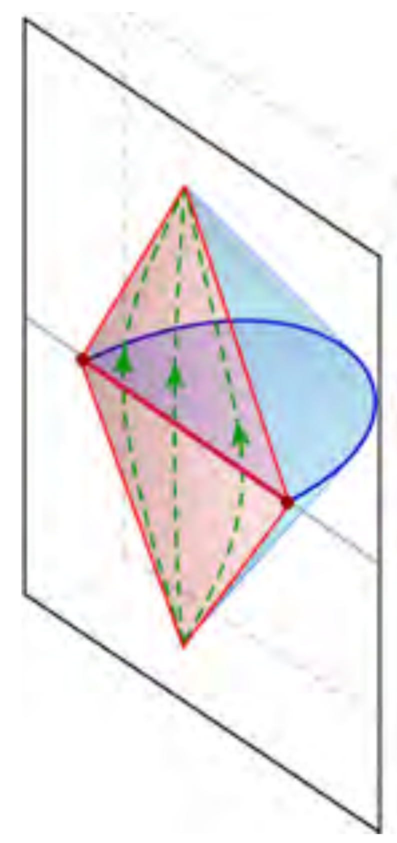
For spherical regions in the CFT ground state the expression for the modular hamiltonian is known

Given a gravity dual and a region A on the boundary, one has an RT surface in the bulk which is the minimal surface in the bulk whose boundary is A.

The RT surface separates the bulk into two and one can define a bulk modular hamiltonain by tracing over one of the bulk regions.

In this way a bulk total bulk modular hamiltonian can be constructed.

The action of the bulk and boundary modular hamiltonians should be identified (JLMS)



The RT surface serve as a horizon for the modular evolution which is a fixed point of the modular flow, so one has for bulk objects on the RT surface

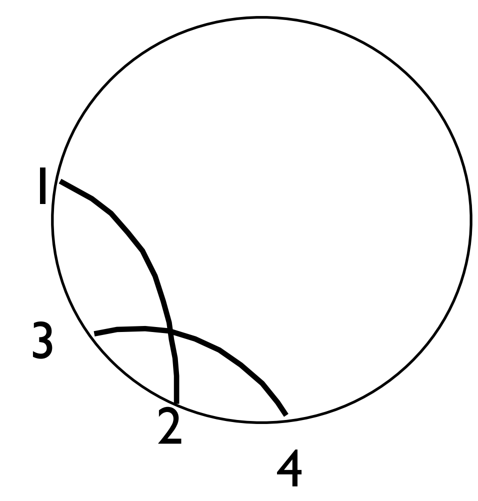
$$[\tilde{H}^{bulk}_{mod}, \Phi] = 0$$

Thus a CFT operator which represents a local bulk objects inside the bulk should obey

$$[\tilde{H}^{CFT}_{mod}, \Phi] = 0$$

For any modular hamiltonian whose RT surface goes through that point.

Lets look at a concrete example



For a CFT on a line in d=2 the total modular hamiltonian for a region $[y_1, y_2]$ is given by (Casini, Huerta, Myers)

$$\tilde{H}_{mod,12} = 2\pi \int_{-\infty}^{\infty} \frac{(w-y_1)(y_2-w)}{y_2-y_1} T_{ww}(w) + 2\pi \int_{-\infty}^{\infty} \frac{(\bar{w}-y_1)(y_2-\bar{w})}{y_2-y_1} T_{\bar{w}\bar{w}}(\bar{w})$$

The action on a primary scalar operator is

$$[\tilde{H}_{mod}, \mathcal{O}(\xi,\bar{\xi})] = \frac{2\pi i}{y_2-y_1} \Big((\bar{\xi}-\xi)\Delta - y_1 y_2(\partial_{\bar{\xi}} - \partial_{\xi}) + (y_1+y_2)(\xi\partial_{\xi} - \bar{\xi}\partial_{\bar{\xi}}) + \bar{\xi}^2\partial_{\bar{\xi}} - \xi^2\partial_{\xi}\Big)\, \mathcal{O}$$

We are looking for a CFT object that obeys

$$(y_2-y_1)[\tilde{H}^{12}_{mod}, \Phi(\xi,\bar{\xi})] = 0, \quad (y_4-y_3)[\tilde{H}^{34}_{mod}, \Phi(\xi,\bar{\xi})] = 0$$

We take an ansatz

$$\Phi(X) = \int dt' dy' g(p,q) \mathcal{O}(q,p)$$

$$q = X - t' + iy', p = X + t' + iy'$$

The conditions of commuting with the two different modular hamiltonians become upon integration by parts two first order partial differential equations for g(p,q). The solution depends on two parameters build from the points labeling the two boundary regions,

$$c_\Delta \left(Z^2 + (p - X_0)(q - X_0)\right)^{\Delta - 2}$$

$$X_0 = \frac{y_1 y_2 - y_3 y_4}{y_1 + y_2 - y_3 - y_4} \qquad Z^2 = (y_1 + y_2)X_0 - y_1 y_2 - X_0^2$$

Validity of integration by parts fix the integration region

$$\Phi(Z, X_0) = c_\Delta \int_{t'^2 + y'^2 < Z^2} dt' dy' (Z^2 - t'^2 - y'^2)^{\Delta - 2} \mathcal{O}(t', X_0 + iy').$$

We see that we get just from the CFT the bulk operator up to a coefficent.

We also have that the solution depends on 2 parameter made out of the 4 variables we started with.

$$X_0 = \frac{y_1 y_2 - y_3 y_4}{y_1 + y_2 - y_3 - y_4} \qquad Z^2 = (y_1 + y_2) X_0 - y_1 y_2 - X_0^2$$

These can be identified as the coordinate of the bulk intersection point, and the relationship between them as the bulk geodesic equation.

Using this and the Hadamard form of bulk 2-point function we can fix the coefficient of the bulk operator and the bulk metric.

Further results

Working in Fourier space we recover bulk modes

Taking CFT at finite temperature we recover bulk operators in Rindler/AdS

Check that modular hamiltonian acts correctly also on operators not on the RT surface

Conclusions and Outlook

- Smearing functions can be computed just from CFT data independently of special symmetric cases.
- This also gives the un-parametrized bulk geodesics and make it possible to find the bulk metric.
- Thus bulk reconstruction can be done just from the CFT.
- Issues of dressings.
- Issues with gauge redundancies.
- Maybe will give a new perspective on bulk reconstruction ?

Looking at the bulk-boundary 3-point function one gets

$$\int d^d x' K_i(x,z|x')\langle \mathcal{O}_i(x')\mathcal{O}_j(y_1)\mathcal{O}_k(y_2)\rangle = \frac{1}{(y_1-y_2)^{2\Delta_j}}\left[\frac{z}{z^2+(x-y_2)^2}\right]^{\Delta_k-\Delta_j} I_{ijk}(\chi)$$

$$I_{ijk}(\chi) = c_{ijk}\left(\frac{1}{\chi-1}\right)^{\Delta_*} F(\Delta_*,\, \Delta_* - \frac{d}{2}+1,\, \Delta_i - \frac{d}{2}+1,\, \frac{1}{1-\chi})$$

$$\Delta_* = \tfrac{1}{2}(\Delta_i+\Delta_j-\Delta_k) \qquad \chi = \frac{[(x-y_1)^2+z^2][(x-y_2)^2+z^2]}{(y_1-y_2)^2 z^2}$$