A Solvable Irrelevant Deformation of AdS_3 / CFT_2

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Introduction

QFT is usually thought of as an RG flow connecting a UV fixed point to an IR fixed point (which may be empty). It is defined by specifying the UV fixed point (a CFT) and a finite set of parameters that can be thought of as coefficients of relevant and marginal operators, that describe the particular RG trajectory. This is known as flowing down the RG.

From the perspective of the long distance theory, the RG trajectory can often be described as a perturbation of the IR fixed point by a set of irrelevant operators. However, this flow up the RG is in general on a very different footing – it does not correspond to a well defined QFT.

The basic reason is that there is an infinite number of different RG trajectories that look at long distances like a particular IR fixed point perturbed by a finite set of irrelevant operators, so we have to supply an infinite amount of data to specify the particular RG trajectory we are interested in. (This is related to the irreversibility of RG flow)

All this has an analog in the context of holography. Relevant perturbations of string theory in an asymptotically AdS spacetime correspond in the bulk to perturbations that do not modify the UV AdS asymptotics, and are therefore specified by a finite number of parameters, while irrelevant perturbations change the geometry in the UV and thus one in general needs an infinite number of parameters to specify the new geometry.

In this talk we will discuss a variation on this theme. It was inspired by two papers, arXiv 1608.05534, 1608.05499, which discussed a perturbation of a CFT_2 by an irrelevant operator that is supposed to be better behaved than the general case, and moreover in a certain sense be solvable.

I will next briefly review their results and then discuss a holographic analog of their system.

Our main interest in these systems is that they interpolate between a CFT in the IR and a system with a Hagedorn entropy in the UV. Understanding holography for this case better is thus a useful step towards extending it to spacetimes with other asymptotics, such as flat spacetime.

$T\bar{T}$ deformation of CFT_2

We start with a two dimensional CFT, and add to the Lagrangian the deformation

$$\delta\mathcal{L} = tT\bar{T}$$

where T and $\bar{T}$ are the holomorphic and anti-holomorphic components of the stress tensor.

The perturbing operator has dimension four, therefore the coupling t has units of length squared. At distances $\gg \sqrt{t}$ the theory approaches the original CFT, and at short distances one in general expects the description to break down.

When we turn on t, the theory breaks conformal symmetry. The authors of arXiv 1608.05534, 1608.05499 show that if one defines the perturbation $T\,\bar{T}$ at a generic point along the RG trajectory as

$$T\bar{T}(y) = \lim_{x \to y} \left(T(x)\bar{T}(y) - \Theta(x)\Theta(y) \right)$$

one can say a lot about the theory. In particular, one can compute exactly the energies of states in the original CFT as a function of the coupling t, and they are insensitive to the UV completion.

For example, starting with a state with $L_0 = \bar{L}_0 = h$ in the original CFT, which corresponds on a cylinder with circumference R to a state with energy

$$E(R,0) = \frac{4\pi}{R}(h - \frac{c}{24})$$

gives rise at finite t to

$$E(R,t) = -\frac{R}{2t} + \sqrt{\frac{R^2}{4t^2} + \frac{4\pi}{t}(h - \frac{c}{24})}$$

The result for general $(h, \bar{h})$ is also known.

Consider, for example, states with $h \geq c/24$.

For states with $h - \frac{c}{24} \ll \frac{R^2}{t}$ one has $E(R,t) \simeq E(R,0)$, i.e. their energies are not influenced by the perturbation.

On the other hand, in the opposite regime $h - \frac{c}{24} \gg \frac{R^2}{t}$, one has instead

$$E \simeq \sqrt{\frac{4\pi}{t}\left(h - \frac{c}{24}\right)}$$

The crossover between the two regimes is at $E \sim \frac{R}{t}$.

It is interesting to calculate the entropy of the $T\bar{T}$ deformed CFT. Since above we only computed the energies of the states of the original CFT after deformation, calculating their entropy only gives a lower bound on the total entropy – there could be additional states in the deformed theory that are not present in the original CFT, e.g. states whose energies go like $1/\sqrt{t}$, which decouple in the limit $t \to 0$.

Anyway, keeping track only of the states visible in the IR CFT gives an entropy that behaves like

- $$S_c = \sqrt{\frac{2\pi c}{3} E R} \quad \text{for } E \ll \frac{R}{t}$$
- $$S_H = \sqrt{\frac{2\pi c t}{3}} E \quad \text{for } E \gg \frac{R}{t}$$

which exhibits a crossover from 2d CFT (Cardy) behavior at low energies to Hagedorn behavior at high energies, $S = \beta_H E$, with

$$\beta_H = \sqrt{\frac{2\pi c t}{3}}$$

The fact that the high energy entropy is Hagedorn implies that the UV behavior is not governed by a fixed point of the RG. It is an open question to what extent the description above, as an irrelevant deformation of an IR CFT, gives rise to a well defined quantum theory, and if not, where and how it breaks down.

Partly with these issues in mind, we will ask the question whether theories of the sort described above can be realized in the context of holography. In particular, if the low energy CFT has an AdS dual, can we realize the deformed theory by perturbing the AdS dual?

Holographic perspective

One way to realize the AdS dual of the $T\bar{T}$ deformed CFT described above is to start with an AdS_3/CFT_2 dual pair, construct the stress tensor of the CFT in the AdS language, and perform the deformation. Since the stress tensor is a ``single trace'' operator, this is a double trace deformation. It corresponds to changing the boundary conditions for bulk fields on AdS_3.

I will not pursue this direction here. Instead, I will show that there is a single trace deformation that shares many elements with the above discussion. I will next describe it and comment on some of its properties.

One of the interesting properties of the QFT work reviewed above is that the story is universal. Indeed, every CFT_2 can be deformed in the way we described.

It is natural to ask whether there is a similarly universal deformation of string theory on AdS_3 (with NS B-field). The answer is yes – any theory with an AdS_3 factor contains an operator, $D(x)$, constructed in KS (1999), which is a quasi-primary of the spacetime Virasoro, and has the same OPE with the spacetime stress tensor as the operator $T\bar{T}$ that figured in our previous discussion.

This operator is constructed as follows:

The left-moving SL(2,R) worldsheet currents can be combined into the single current

$$J(x;z) = 2xJ_3(z) - J^+(z) - x^2J^-(z)$$

Where x=position on the boundary, z=position on the worldsheet, and (J^-, J^3, J^+) give rise to (L_{-1}, L_0, L_1) in the spacetime CFT.

The left-moving spacetime stress tensor takes the form (in the bosonic string)

$$T(x) = \frac{1}{2k}\int d^2z(\partial_x J\partial_x\Phi_1 + 2\partial_x^2 J\Phi_1)\bar{J}(\bar{x};\bar{z})$$

The operator D(x) takes the form

$$D(x) = \int d^2z(\partial_x J \partial_x + 2\partial_x^2 J)(\partial_{\bar{x}} \bar{J} \partial_{\bar{x}} + 2\partial_{\bar{x}}^2 \bar{J})\Phi_1$$

This operator has spacetime scaling dimension (2,2). Hence it is a supergravity field. It is essentially the massive dilaton on AdS_3. In analogy to the $T\bar{T}$ deformation story, one can ask what happens when one adds to the Lagrangian of the spacetime CFT the irrelevant operator $\delta L = \lambda D(x)$.

This corresponds from the worldsheet point of view to adding to the worldsheet action the vertex operator $\lambda D(x)$. One finds:

$$\int d^2 x D(x,\bar{x}) \simeq \int d^2 z J^-(z) \bar{J}^-(\bar{z})$$

Thus, the irrelevant deformation of the spacetime CFT corresponds on the worldsheet to a marginal deformation. Moreover, this is a current-current deformation, which one expects to be exactly solvable.

To understand the physics associated with this deformation, we next make a few comments:

(1) A useful description of this theory is as a null coset of a 10+2 dimensional background. For example, if the background we start with is $AdS_3 \times S^3 \times T^4$, the deformed theory can be thought of as follows. We start with the background

$$AdS_3 \times S^3 \times T^4 \times R^{1,1}$$

The extra $R^{1,1}$ is parametrized by the coordinates $x^{\pm} = y \pm \tau$, while the AdS_3 is parametrized by the Poincare coordinates $(\phi, \gamma, \bar{\gamma})$, where $(\gamma, \bar{\gamma})$ are coordinates on the boundary.

We now gauge the $U(1)^2$ symmetry that acts on the coordinates as

$$x^- \to x^- + \alpha \; ; \qquad \gamma \to \gamma + \epsilon\alpha,$$

$$x^+ \to x^+ + \overline{\alpha} \; ; \qquad \overline{\gamma} \to \overline{\gamma} + \epsilon\overline{\alpha},$$

The background associated with this coset can be obtained using standard techniques:

The Lagrangian of the gauged model is

$$\mathcal{L} = k\left[\partial\phi\overline{\partial}\phi + e^{2\phi}(\overline{\partial}\gamma + \epsilon\overline{A})(\partial\overline{\gamma} + \epsilon A)\right] + (\partial x^{+} + A)(\overline{\partial}x^{-} + \overline{A}).$$

Integrating out the gauge fields gives the background

$$\mathcal{L} = k\partial\phi\overline{\partial}\phi + \frac{k}{k\epsilon^2 + e^{-2\phi}}\overline{\partial}(\gamma - \epsilon x^{-})\partial(\overline{\gamma} - \epsilon x^{+})$$

with a dilaton that goes like $e^{-\Phi} \sim 1 + k\epsilon^2 e^{2\phi}$. This geometry interpolates between AdS_3 as $\phi \to -\infty$ (where it is convenient to choose the gauge $x^{\pm} = 0$), and $R_\phi \times R^{1,1}$ as $\phi \to +\infty$, where it is convenient to set $\gamma = \bar{\gamma} = 0$.

(2) The resulting background $M_3 \times S^3 \times T^4$ has a very simple physical interpretation. It is the geometry created by k NS5-branes and p fundamental strings in the near-horizon geometry of the fivebranes. It interpolates between the near-horizon geometry of both the fivebranes and the strings in the IR region $\phi \to -\infty$, and that of just the fivebranes in the UV region $\phi \to \infty$.

Thus, the irrelevant deformation $\delta L = \lambda D(x)$ takes us out of the near-horizon of the strings. The parameter ϵ sets the scale at which this happens.

(3) Spectrum: one can use the coset description to study the spectrum of perturbative strings in the background

$$M_3 \times S^3 \times T^4$$

One finds a continuum of states labeled by the momentum and winding around the y circle, (n, w). The mass shell formula is:

$$\omega^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{wR}{\alpha'}\right)^2 + \frac{2}{\alpha'}\left(-\frac{2j(j+1)}{k} + \Delta_{\mathcal{O}} + \overline{\Delta}_{\mathcal{O}} - 1\right)$$

where $j = -\frac{1}{2} + is$, with $s \in R$ the radial momentum of the state, and the Δ's are (left and right) internal excitation levels.

Consider for example the winding one case (w=1), and write

$\omega = \frac{R}{\alpha'} + E$ (measure the energy above the BPS one)

In the undeformed limit $t \to 0$ one has

$$h - \frac{k}{4} = -\frac{j(j+1)}{k} + \Delta_{\mathcal{O}} - \frac{1}{2}$$
$$\overline{h} - \frac{k}{4} = -\frac{j(j+1)}{k} + \overline{\Delta}_{\mathcal{O}} - \frac{1}{2}$$

a relation between the worldsheet quantum numbers and the dimension of the corresponding state for long strings in the spacetime CFT (MO, 2000).

Putting all this together, one finds the mass-shell condition

$$\left(E+\frac{R}{\alpha'}\right)^2-\left(\frac{R}{\alpha'}\right)^2=\frac{2}{\alpha'}\left(h+\bar{h}-\frac{k}{2}\right)+\left(\frac{n}{R}\right)^2$$

which is precisely the spectrum for a $T\bar{T}$ deformed CFT with central charge $c = 6k$, with $t = \pi l_s^2$. This theory is nothing but the theory on a long string, so we conclude that the supergravity deformation $\delta L = \lambda D(x)$ acts on that theory as a $T\bar{T}$ deformation.

For $w > 1$ one finds the spectrum of the Z_w twisted sector of the theory M^w/S_w on w strings.

(4) Black holes: as one increases the energy, the above long strings move towards the boundary, where their coupling increases (SW 1999) and eventually they cross over to black holes (GKRS 2005).

Therefore, we would expect that the entropy of black holes in the background $M_3 \times S^3 \times T^4$ should agree with that of the symmetric product CFT M^p/S_p describing the strings. This is indeed the case:

In the boundary theory, the entropy is dominated by states in which we distribute the available energy equally between the p factors of M, and one has

$$S(\mathcal{E}) = pS_{\mathcal{M}}(\mathcal{E}/p) = 2\pi\sqrt{2kp\mathcal{E} + kb\mathcal{E}^2},$$

In the bulk the entropy comes from the black hole

$$ds^2 = -\frac{f_{\mathcal{E}}}{f_1}(dx^0)^2 + \frac{1}{f_1}(dx^1)^2 + \frac{f_5}{f_{\mathcal{E}}}dr^2,$$
$$e^{-2(\Phi-\Phi_0)} = \frac{f_1}{f_5},$$

with

$$f_1 = 1 + \frac{r_1^2}{r^2}, \quad f_{\mathcal{E}} = 1 - \frac{r_0^2}{r^2}, \quad f_5 = \frac{k l_s^2}{r^2}, \quad r_1^2 = 8pk^{3/2} l_s G_3$$

$$r_0^2 = \frac{16\pi^2 G_3 k^{3/2} l_s^3 \mathcal{E}}{R^2}$$

The Bekenstein-Hawking entropy of this black hole is

$$S_{BH} = \frac{R\sqrt{f_1}}{4G_3 f_5} = 2\pi\sqrt{\frac{4k\pi^2 l_s^2 \mathcal{E}^2}{R^2} + 2pk\mathcal{E}}$$

It agrees with the boundary formula if we set the coupling to

$$t = \pi l_s^2.$$

Future work

There is clearly a lot to do. E.g. :

➢ In the $T\bar{T}$ deformed CFT:

- Understand the symmetries of the deformed theory.
- Calculate correlation functions on the plane.
- Understand the UV limit of the theory.

➢ In deformed AdS_3 :

- Understand the operator $D(x,\bar{x})$ in the boundary CFT.
- Calculate correlation functions on the plane and compare to the boundary analysis.
- Understand the symmetries of the deformed theory.
- Understand the double trace $T\bar{T}$ deformation.