On $T\bar{T}$ and Black Holes

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Introduction

The purpose of this talk is to comment, expand and generalize on a few points concerning the duality between the current-current worldsheet deformation

$$\delta\mathcal{L} = \lambda J^{-}\overline{J}^{-} \quad \text{of } AdS_3$$

and an irrelevant deformation of the type $\delta L = -tT\bar{T}$ of the block, M, in $\mathrm{Symm}^{\mathrm{p}}(\mathrm{M})$, which is closely related to the dual 2d spacetime theory

Introduction

- We may consider e.g. any superstring on $AdS_3 \times S^1 \times \mathcal{N}$
- For concreteness, we focus on the theory corresponding to the near horizon of a BPS system of k NS5's and w F1's:

$$AdS_3 \times S^3 \times \mathcal{M}^4$$

- And compactify the spatial direction x of the boundary (in Poincare coordinates) on a circle w/ radius R, thus obtaining a massless BTZ black hole
- The deformation `glues' the massless BTZ to an asymptotically flat spacetime w/ a linear dilaton; it amounts to going to the near NS5's regime but NOT to the F1's
- We shall inspect the geometry of black holes in this background, and from their ADM mass, as a function of M and J of the BTZ in their near inner-horizon (the latter amounts to the near NS5+F1 regime), we shall find the eq. for the energy in the spacetime 2d theory, $E(R, h, \bar{h})$, in terms of R and the left and right-handed scaling dimensions of the original CFT_2 spacetime dual

Introduction

- The black holes in the asymptotically linear dilaton spacetime correspond to non-extremal NS5-branes w/ fundamental string winding and momentum charges; we shall describe these 1'st, by inspecting the near NS5 regime of such a system
- We shall then inspect the regime near the F1's – the BTZ black hole w/ mass M and angular momentum J
- Finally, we shall comment on the regime beyond the inner horizon of the black holes

Introduction

- We will translate the geometrical properties of the black holes in the worldsheet background to the dual language, namely, to the spectrum of the dual 2d spacetime theory
- And we shall reproduce the results from the thermodyanamics associated with both the outer horizon and the inner horizon

The non-extremal NS5's w/ F1 charges w, n

Start w/ k near-extremal NS5's on $S^1 \times T^4$,
w/ energy density μ,
and perform a
boost + T-duality + boost on S^1,
thus turning on F1 winding w and momentum n,
given in terms of the boost parameters α_w, α_n,
the radius R of S^1, the volume v of T^4,
and the string coupling g (see next slide).
The geometry is:

The non-extremal NS5's w/ F1 charges w, n

$$ds^2 = \frac{1}{f_w}\left[-\frac{f}{f_n}dt^2 + f_n\left(dx + \frac{\mu \sinh 2\alpha_n}{2f_n r^2}dt\right)^2\right] + f_5\left(\frac{1}{f}dr^2 + r^2 d\Omega_3^2\right) + \sum_{i=1}^{4} dx_i^2$$

$$e^{2\Phi} = g^2 \frac{f_5}{f_w}$$

$$f = 1 - \frac{\mu}{r^2}\ , \qquad f_{w,n,5} = 1 + \frac{r_{w,n,5}^2}{r^2}$$

$$r_{w,n,5}^2 = \mu \sinh^2 \alpha_{w,n,5}$$

$$\frac{1}{2}\mu \sinh 2\alpha_w = \frac{g^2 \alpha' w}{v}\ , \qquad \frac{1}{2}\mu \sinh 2\alpha_n = \frac{g^2 \alpha'^2 n}{R^2 v}\ , \qquad \frac{1}{2}\mu \sinh 2\alpha_5 = \alpha' k$$

The non-extremal NS5's w/ F1 charges w,n

- The asymptotic NS5 and F1 charges, k, w, n, are given in the last line of the previous transparency
- The ADM mass of this non-extremal brane geometry is:

$$M_{adm} = \frac{Rv}{2\alpha'^2}\frac{\mu}{g^2}(\cosh 2\alpha_w + \cosh 2\alpha_n + \cosh 2\alpha_5)$$

- Or, in terms of the numbers of branes, winding and momenta:

$$M_{adm} = \frac{R}{\alpha'}\sqrt{w^2 + \frac{\mu^2 v^2}{4g^2\alpha'^2}} + \frac{1}{R}\sqrt{n^2 + \frac{\mu^2 R^2 v^2}{4g^4\alpha'^4}} + \frac{Rv}{g^2\alpha'}\sqrt{k^2 + \frac{\mu^2}{4\alpha'^2}}$$

The near NS5's geometry

- It is obtained by dropping the 1 in the harmonic function of the NS5's together w/ $g, \mu, r \to 0$ s.t. $k,\ r_0^2 = \mu/g^2$ and $\frac{\mu}{r^2} \to r_0^2/r^2$ are held fixed in the limit; one finds:

$$ds^2 = \frac{1}{f_w}\left[-\frac{f}{f_n}dt^2 + \left(f_n dx + \frac{\alpha'^2}{Rv}\frac{n}{R}\frac{1}{r^2}dt\right)^2\right] + f_5\left(\frac{1}{f}dr^2 + r^2 d\Omega_3^2\right) + \sum_{i=1}^{4} dx_i^2$$

$$e^{-2(\Phi-\Phi_0)} = \frac{f_w}{f_5}$$

with

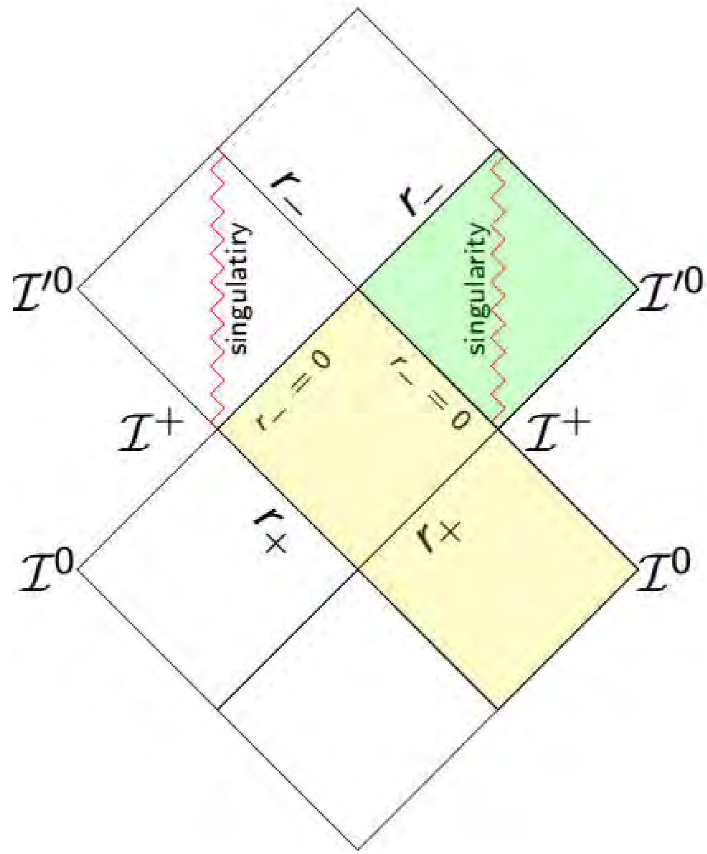
$$f = 1 - \frac{r_0^2}{r^2}\,, \qquad f_{w,n} = 1 + \frac{r_0^2 \sinh^2\alpha_{w,n}}{r^2}\,, \qquad f_5 = \frac{\alpha' k}{r^2}$$

The near NS5's geometry — 2d BH

- This is a 3d charged, rotating black string
- The event horizon is at $r = r_0$
- The inner horizon is at $r = 0$
- Upon reduction to two dimensions, this is a 2d black hole (BH) w/ F1 charges w, n

2d BH

Penrose diagram of the 2d BH



The near NS5's geometry

- From the value of the ADM mass and charges in terms of the boost parameters, the compactification sizes, R, v, and the non-extremal parameter, one finds that the energy of the black hole above the extremal background of the NS5's and F1's, and the charges, w, n, are given in the near NS5's limit by:

$$E = \left(\frac{Rvr_0^2}{2\alpha'^2}\right)(\cosh 2\alpha_w - \sinh 2\alpha_w + \cosh 2\alpha_n)$$

$$\left(\frac{Rvr_0^2}{2\alpha'^2}\right)\sinh 2\alpha_w = \frac{wR}{\alpha'}\ , \qquad \left(\frac{Rvr_0^2}{2\alpha'^2}\right)\sinh 2\alpha_n = \frac{n}{R}$$

The near NS5's+F1's geometry

- It is obtained by taking the near inner-horizon limit: $\frac{r}{g}$, $g \to 0$, keeping $\frac{\mu}{r^2}$ and w fixed
- (namely, dropping the 1 also from the harmonic function f_w)
- Rescaling $x = R\phi$, and rescaling t, as well as shifting and rescaling r^2, one finds a BTZ black hole background (Argurio)

$$ds^2_{btz} = -N^2 dt^2 + N^{-2} d\rho^2 + \rho^2 (d\phi + N_\phi dt)^2$$

$$N^2 = \frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\alpha' k \rho^2} = \frac{\rho^2}{\alpha' k} - 8G_3 M_{btz} + \frac{(4G_3 J)^2}{\rho^2} \,, \qquad N_\phi = -\frac{4G_3 J}{\rho^2}$$

The near NS5's+F1's geometry

- The 3d Newton constant and the locations of the outer and inner horizons are given in terms of the parameters of the near extremal NS5-F1-n system by:

$$G_3 = \frac{\alpha'^{3/2}}{4vk^{1/2}r_0^2 \sinh^2 \alpha_w}$$

$$\rho_+^2 = \frac{\cosh^2 \alpha_n}{\sinh^2 \alpha_w} R^2 \ , \qquad \rho_-^2 = \frac{\sinh^2 \alpha_n}{\sinh^2 \alpha_w} R^2$$

- The dimensionless mass, $M \equiv \sqrt{k} l_s M_{btz}$
- And the angular momentum, J, of the BTZ black hole

are given in terms of R, v, r_0^2, n by:

The near NS5's+F1's geometry

$$M = h + \overline{h} - \frac{c}{12} = \left(\frac{Rvr_0^2}{2\alpha'^2}\right) R\cosh 2\alpha_n$$

$$J = n = h - \overline{h} = \left(\frac{Rvr_0^2}{2\alpha'^2}\right) R\sinh 2\alpha_n$$

Here we also expressed the the mass, M, and angular momentum, J, of the BTZ BH, in the AdS_3/CFT_2 context, namely, in terms of the left and right dimensions,

$h, \overline{h},$ and the central charge

$$c = 6kw$$

Combining near NS5's w/ near NS5's+F1's

- Now, combining the eqs. for E, w, n, M, J, we find:

$$\left(E+\frac{wR}{\alpha'}\right)^2-\left(\frac{wR}{\alpha'}\right)^2=\frac{2w}{\alpha'}(h+\overline{h}-c/12)+\left(\frac{h-\overline{h}}{R}\right)^2+\ldots$$

- The `dots' are corrections in $1/w$
- Identifying

$$t\equiv\frac{\pi\alpha'}{w}$$

this is the energy of a CFT_2 w/ $c=6kw$ plus $-tT\bar{T}$

Interpretation

Assuming an $\frac{(M_{6k})^w}{S_w}$ structure in the spacetime dual theory,
and dividing $E, h, \bar{h}$ equally among the blocks M
(which is the maximal contribution to black holes states),
namely, $E \to wE$, $h = wh_1$ etc., we get:

$$\left(E + \frac{R}{\alpha'}\right)^2 - \left(\frac{R}{\alpha'}\right)^2 = \frac{2}{\alpha'}\left(h_1 + \bar{h}_1 - \frac{k}{2}\right) + \left(\frac{n}{R}\right)^2$$

with $n = h_1 - \bar{h}_1$; this is the eq. for $E\left(R, h_1, \overline{h_1}\right)$ in a
$\delta L = -tT\bar{T}$ deformed CFT_2 w/ $c = 6k$ and $t = \pi\alpha'$

$E(R,h,\bar{h})$ from the entropy

- Following the logic in \`$T\bar{T}$ in LST' (in David's talk), which is compatible w/ the geometric picture described here, we know that the thermodynamics associated with the horizon should lead to the same results obtained from the geometry
- Concretely, by comparing the entropy of the black hole in the asymptotically flat spacetime w/ a linear dilaton background to the entropy of the BTZ black hole in its strong coupling regime, we should reproduce the eq. for $E(R,h,\bar{h})$

$E(R,h,\bar{h})$ from the entropy – S(BTZ)

The entropy of the BTZ black hole, in terms of the dual CFT_2 language, is the Cardy entropy:

$$S_c(M,J) \simeq 2\pi \left(\sqrt{\frac{c}{6}(h - c/24)} + \sqrt{\frac{c}{6}(\bar{h} - c/24)} \right)$$

where

$$M = h + \bar{h} - \frac{c}{12}\,, \qquad J = h - \bar{h}$$

$$c = 6kw$$

and the `$\simeq$' stands for corrections in $1/w$

$E(R,h,\bar{h})$ from the entropy – S(2d BH)

On the other hand, the entropy of the 2d black hole (the reduction of the 3d black string) with mass

$$m = E + \frac{wR}{\alpha'}$$

is:

$$S(E,n) \simeq 2\pi\sqrt{\frac{\alpha' k}{4}}\left(\sqrt{m^2 - q_L^2} + \sqrt{m^2 - q_R^2}\right)$$

where

$$q_{L,R} = \frac{n}{R} \pm \frac{wR}{\alpha'}$$

$E(R,h,\bar{h})$ from the entropy

By equating $S(E,n) = S_c(M,J)$, we find

$$\left(E + \frac{wR}{\alpha'}\right)^2 - \left(\frac{wR}{\alpha'}\right)^2 = \frac{2w}{\alpha'}(h + \bar{h} - c/12) + \left(\frac{n}{R}\right)^2$$

which is precisely what we derived from the geometry

$t < 0$ and the regime beyond the inner-horizon

- We will next argue that the ADM masses, associated w/ the asymptotic regime beyond the singularity of the 2d black holes, give rise to $E(R, h, \bar{h})$ in the case of $\delta L = -tT\bar{T}$ w/ $t < 0$
- Then we will show that this result is reproduced from the thermodynamics associated w/ the inner horizon

$t < 0$ and the regime beyond the inner-horizon

The regime beyond the singularity is obtained by the continuation

$$r^2 \to -r^2$$

This is equivalent to taking

$$r_0^2 \to -r_0^2$$

Namely, $M \to -M$

and $E(M) \to -E(-M)$

This leads to the eq. for $E(R, h, \bar{h})$ w/ $w > 0 \to -w < 0$

Namely, $t \to -t$

$t<0$, comments:

$t < 0$ amounts to $\lambda \to -\lambda$ in the worldsheet deformation

$$\delta\mathcal{L} = \lambda J^- \overline{J}^-$$

- This indeed takes the 1 in the harmonic function $f_w(r^2)$ to -1, equivalently, $r^2 \to -r^2$ (w/ $x \leftrightarrow t$)
- Hence, it describes the regime beyond the inner horizon, which includes the singularity at $f_w(r^2) = 0$

More comments:

- The worldsheet CFTs of the black holes sigma models can be described by exact quotient CFTs $\frac{SL(2)\times U(1)}{U(1)}$
- An axial gauging gives the exact CFT description of the regime outside the outer horizon, while vector gauging describes the regime beyond the inner horizon
- In the extremal case it is convenient to consider instead the gauging of $SL(2) \times R^{1,1}$ by a null current (see David's talk)
- Axial gauging amounts to $\lambda, t > 0$
- Vector gauging amounts to $\lambda, t < 0$

Thermodynamics

- The thermodynamics associated w/ the horizons is obtained by defining the entropy to be $S^{\pm} = \frac{A_{\pm}}{4G}$, where $A_{\pm}$ is the area of the outer horizon (for $+$) and inner horizon (for $-$), and the temperature, $T^{\pm} > 0$, is defined by e.g. the asymptotic radius of Euclidean time in the regime outside the black hole (for $+$) and beyond the singularity (for $-$)
- One finds that for either BTZ or the 3d black string, $S^{\pm} = S_L \pm S_R$, where $S_{L/R}$ are the contributions of the left/right moving modes

$t<0$, thermodynamics

For the inner horizon in the BTZ case we have

$$S_{inner-btz}(M,J) \simeq 2\pi\sqrt{\frac{kw}{2}}\left(\sqrt{M+J}-\sqrt{M-J}\right)$$

and from $\left(\frac{\partial S^{\pm}_{btz}}{\partial M}\right)_J = \pm\frac{1}{T^{\pm}_{btz}}$

we find that the thermodynamics mass is $M^{\pm}_{btz} = \pm M$

Thermodynamics

Similarly, for the black string we have $\left(\frac{\partial S_{bs}^{\pm}}{\partial m}\right)_{q_{L,R}(\pm w,\pm n)} = \pm\frac{1}{T_{bs}^{\pm}}$

which imply that the thermodynamics mass associated w/ the asymptotically linear dilaton regimes outside and beyond the singularity is $m_{bs}^{\pm} = \pm m$

Combining the above we find that the mass obtained from the thermodynamics associated w/ the regime beyond the singularity of the black string is $-m(-M,-w) = -E(-M) + wR/\alpha'$.

which is identical to the ADM mass derived from the geometry

Summary

- We derived the equation for the spectrum of a CFT_2 dual to a superstring on AdS_3, w/ an irrelevant deformation of the type $\delta L = -tT\bar{T}$ (like in the block, M, of $\mathrm{Symm}^{\mathrm{p}}(M)$), from the geometry of black holes in the asymptotically linear dilaton background of the dual, deformed AdS_3, for both positive and negative t
- $t > 0$ corresponds to the geometry outside the black hole
- $t < 0$ corresponds to the regime beyond the inner horizon
- We also reproduced these results from the thermodynamics associated with the outer and inner horizons, respectively

Summary

- This provides, in particular,
  a non-gravitational,
  boundary dual
  that may shed light on the
  fate of black hole's singularities
  and/or horizons in string theory