Holographic RG flow in curved spacetime

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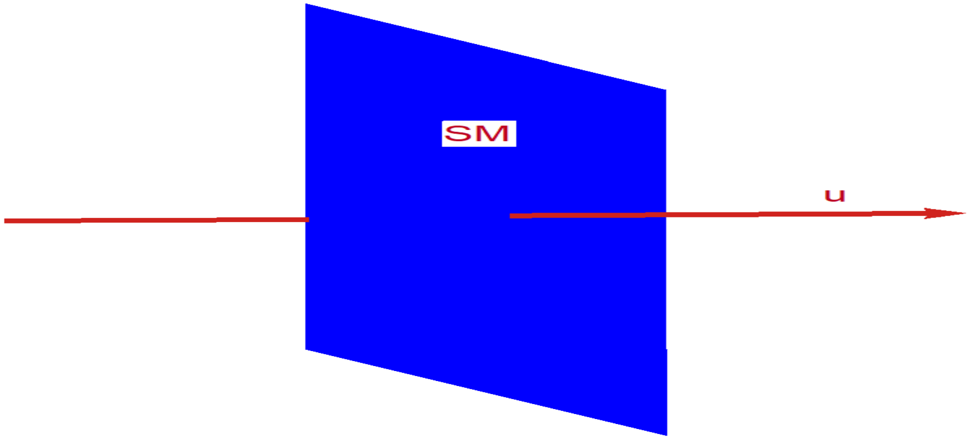
Work with Elias Kiritsis, Francesco Nitti and Lukas Witkowski

Introduction

- Conflict between GR and QFT $\Longrightarrow$ Cosmological Constant Problem
- Flat-self tuning $\Longrightarrow$ Vacuum energy curves the bulk but leaves 4d-brane flat (Talk by F. Nitti)



- Study RG flow in cuved spacetime using holography

Plan of the talk

- Holographic Renormalization Group Flows
- Holographic Renormalization Group Flows in Maximally Symmetric Spacetime
- Examples
- Conclusions and outlook

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Holographic RG flow

- Gauge Theory $\leftrightarrow$ Gravity Theory/ String Theory
- Large N and large coupling

$$\text{String Theory} \Rightarrow \text{Classical supergravity}$$

- Undeformed CFT $\leftrightarrow$ Gravity in AdS

- Adding relevant deformation breaks conformal invariance

$$S_{\mathrm{QFT}} = S_* + \lambda \int d^d x \mathcal{O}_\Delta(x)$$

$$S_* \to \text{UV CFT}$$

$$\frac{d\lambda(\mu)}{d\ln\mu} = \beta(\mu)$$

- How to describe it holographically?

Holographic RG flow

- Action is:

$$S[g,\phi] = \int du d^d x \sqrt{-g}\left(R^{(g)} - \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi)\right) + S_{GHY}$$

- Ansatz:

$$\phi = \phi(u), \quad ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu$$

$$A(u) \rightarrow \text{Scale factor}$$

$$\zeta_{\mu\nu} \rightarrow \text{Maximally symmetric metric: flat, dS, AdS}$$

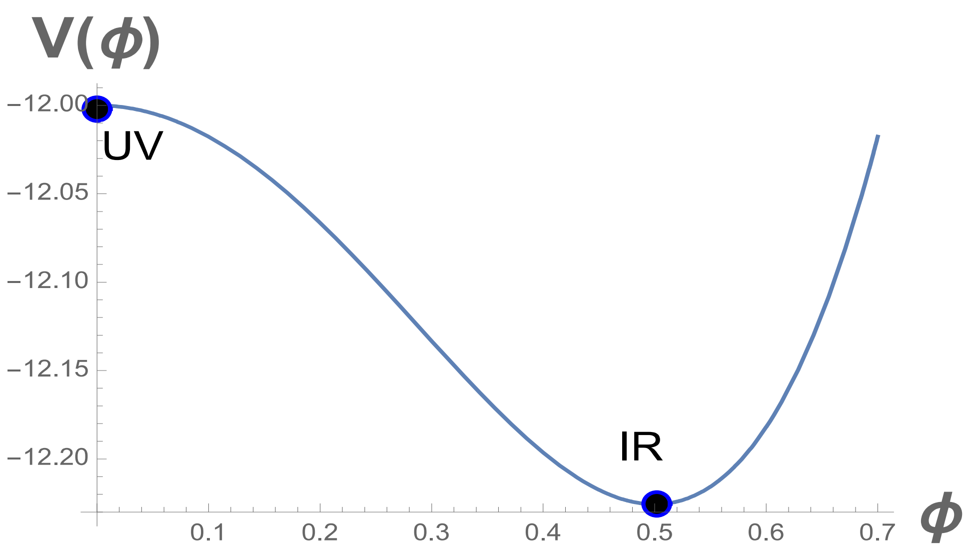
Holographic RG flow (flat case)

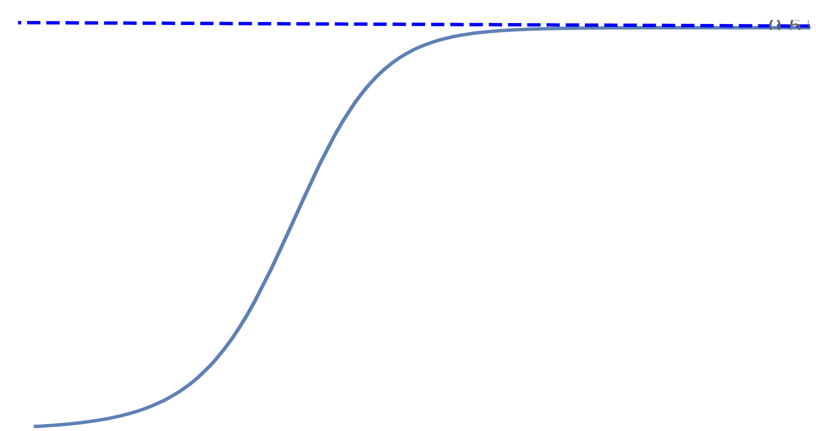
$$\text{Near maximum}, \phi = 0$$

$$V(\phi) \approx -\frac{d(d-1)}{\ell^2} - \frac{m^2}{2}\phi^2, \quad \text{where } m^2 > 0$$

Mass and scaling dimension are related by:

$$\Delta = \frac{d}{2} + \frac{d}{2}\sqrt{1 - \frac{4m^2\ell^2}{d^2}}$$





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Equation of Motion

- Notation: $\dot{} = \frac{d}{du}\ , \quad ' = \frac{d}{d\phi}$

- Equations of motion

$$2(d-1)\ddot{A} + \dot{\phi}^2 + \frac{2}{d}e^{-2A}R^{(\zeta)} = 0$$

$$d(d-1)\dot{A}^2 - \frac{1}{2}\dot{\phi}^2 + V(\phi) - e^{-2A}R^{(\zeta)} = 0$$

$$\ddot{\phi} + d\dot{A}\phi\phi - V'(\phi) = 0$$

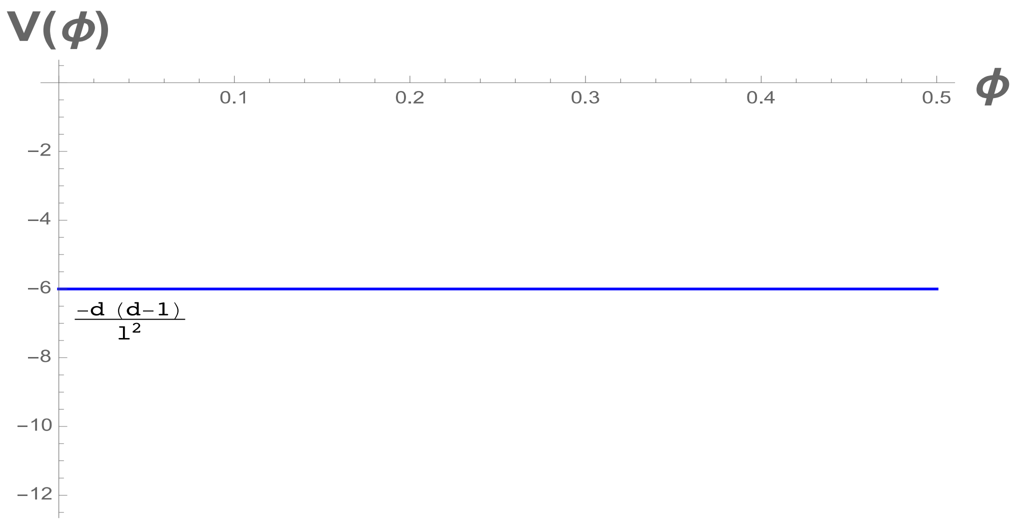
First two: Einstein's equations

Third one: Klein-Gordon equation

- In this project: $R^{(\zeta)} \neq 0$ ($R^{(\zeta)} = 0$, Talk by L. Silva Pimenta)

Pure AdS

$$V(\phi) = -\frac{d(d-1)}{\ell^2}, \quad \dot{\phi} = 0$$



$$A(u) = \begin{cases} -\frac{u}{\ell} & \text{for flat} \\ \ln\left[-\frac{\ell}{\alpha}\sinh\left(\frac{u+c}{\ell}\right)\right] & \text{for dS} \\ \ln\left[\frac{\ell}{\alpha}\cosh\left(\frac{u+c}{\ell}\right)\right] & \text{for AdS} \end{cases}$$

$$\text{where}: \quad R^{(\zeta)} = \pm\frac{d(d-1)}{\alpha^2}, \quad c = \ell\ln\left(\frac{\ell}{2\alpha}\right)$$

First Order Formalism

Superpotentials

$$\dot{A}(u) = -\frac{1}{2(d-1)} W(\phi)$$

$$\dot{\phi}(u) = S(\phi)$$

$$e^{-2A(u)} = \frac{1}{R^{(\zeta)}} T(\phi)$$

- To make contact with RG flows
- Junction conditions are written in terms of superpotentials [Talks by F. Nitti and L. Witkowski]

First Order Formalism

EOMs

$$S^2 - SW' + \frac{2}{d}T = 0$$

$$\frac{d}{2(d-1)}W^2 - S^2 - 2T + 2V = 0$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0$$

- Eliminate T:

$$\frac{d}{2(d-1)}W^2 + (d-1)S^2 - dSW' + 2V = 0$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0$$

Properties of Superpotentials(dS slicing)

- Two branches (one with W’>0, another W’<0)

$$SW' = S^2 + \frac{2}{d}T > 0$$

- W is bounded by critical curve

$$\frac{d}{2(d-1)}W^2 = S^2 + 2T - 2V \geq -2V$$

$$\Rightarrow |W| \geq \sqrt{-\frac{4(d-1)}{d}V(\phi)} = B(\phi)$$

UV boundary

- Scale factor and warp factor diverge

$$A(u) \to \infty$$
$$e^{A(u)} \to \infty$$

- Maximum of potential at $\phi = 0$ corresponds to UV

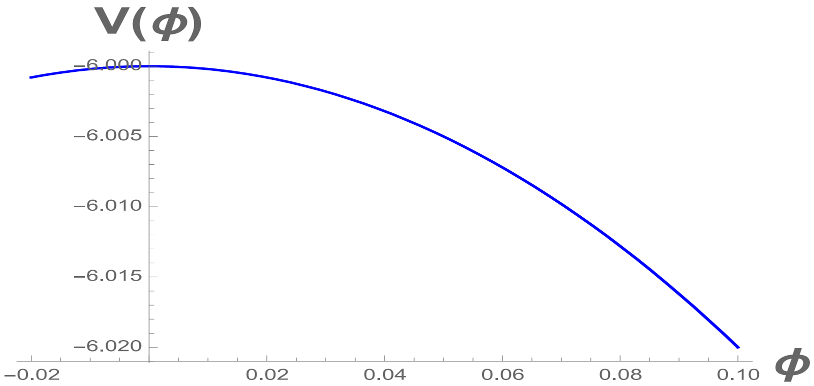
- In u coordinate UV happens at $u = -\infty$

UV Expansions

Near maximum, $\phi = 0$

$$V(\phi) \approx -\frac{d(d-1)}{\ell^2} - \frac{m^2}{2}\phi^2$$

where $m^2 > 0$



$$W(\phi) = \frac{2(d-1)}{\ell} + \frac{\Delta_-}{2\ell}\phi^2 + \frac{\ell R}{d}\left(\frac{\phi}{\phi_-}\right)^{2/\Delta_-} + C\phi^{d/\Delta_-} + \cdots$$

$$S(\phi) = \frac{\Delta_-}{\ell}\phi + C\frac{d}{\Delta_-}\phi^{\frac{d}{\Delta_-}-1} + \cdots$$

$$T(\phi) = R\left(\frac{\phi}{\phi_-}\right)^{2/\Delta_-} + \cdots$$

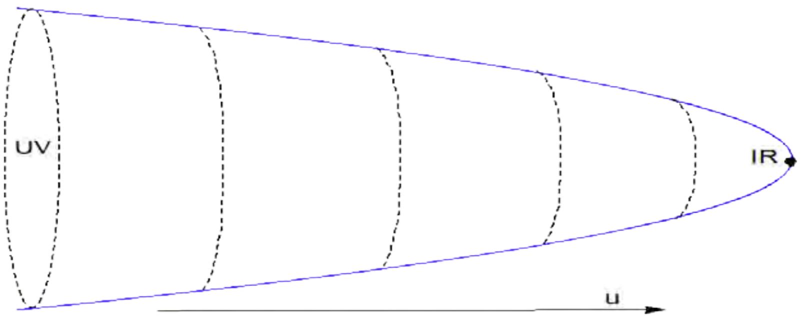
where, $\Delta_- = \frac{d}{2} - \frac{d}{2}\sqrt{1 - \frac{4m^2\ell^2}{d^2}} = d - \Delta$

$\phi_- \leftrightarrow$ source

$C \leftrightarrow$ vev

Positive curvature: IR endpoints

IR: $e^{A(u)} \to 0$



- Square root expansion is the solution

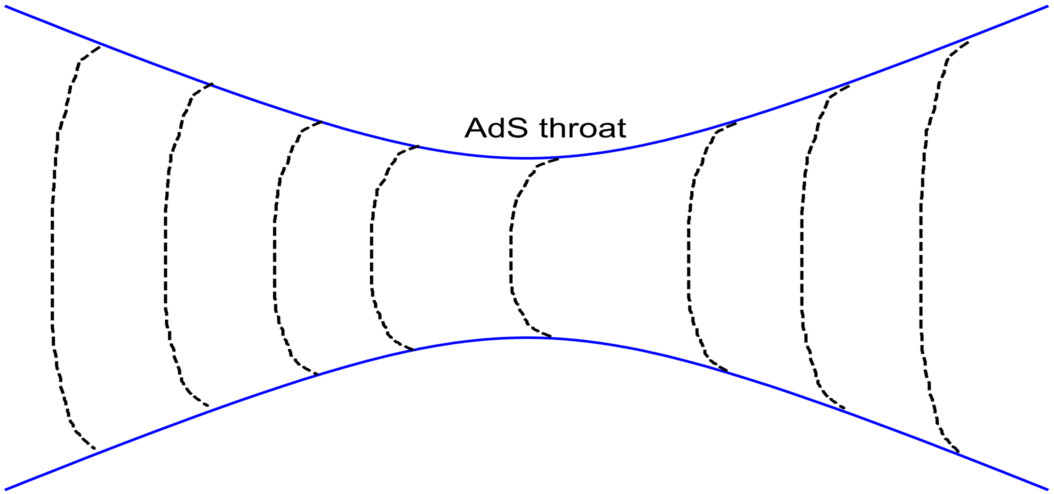
$$W(\phi) = \frac{W_0}{\sqrt{\phi - \phi_0}} + \cdots$$

$$S(\phi) = S_0 \sqrt{\phi - \phi_0} + \cdots$$

$$T(\phi) = \frac{T_0}{\phi - \phi_0} + \cdots$$

- Flow stops here
- Geometry is asymptotically AdS near IR

Negative Curvature: AdS throat



$$\text{AdS “IR”:}\ \dot{A} = 0, e^{A} = \text{constant} \neq 0$$

$$W = W_2\sqrt{\phi - \phi_0} + \cdots$$

$$S = S_0\sqrt{\phi - \phi_0} + \cdots$$

$$T = \text{constant} + \cdots$$

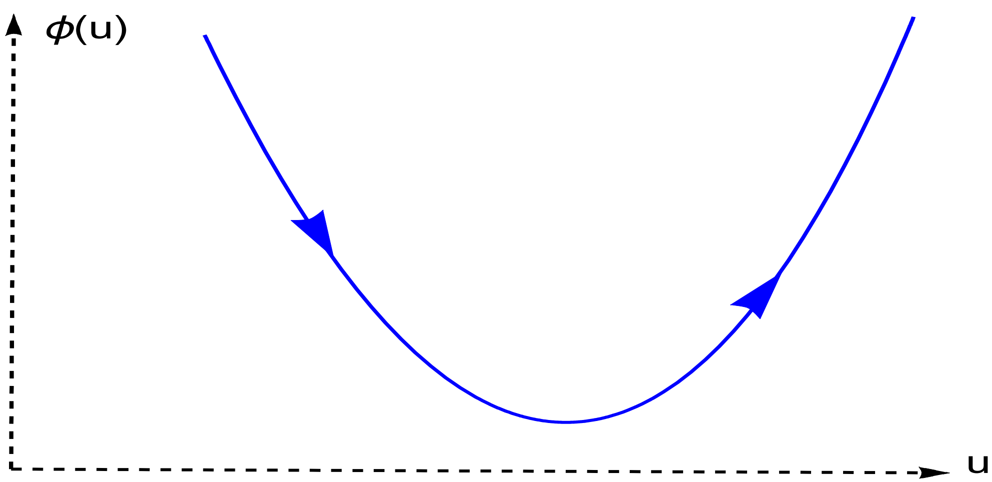
- Flow stops here
- Geometry is asymptotically AdS

Bouncing Solution

- Warp factor neither diverges, nor takes the minimum value

$$e^A \neq 0, \dot{A} \neq 0, \dot{\phi} = 0, \ddot{\phi} \neq 0$$

- Flow does not stop here, continues and reaches IR.



- All curvature invariants regular

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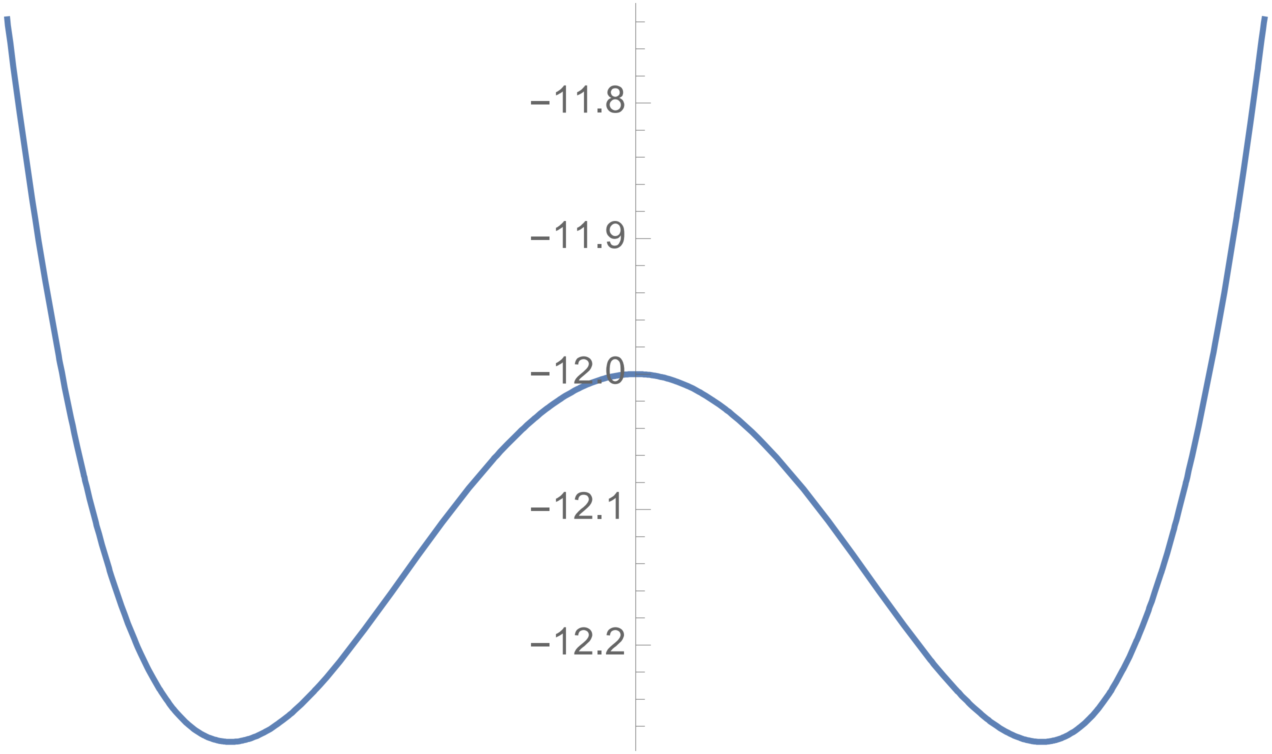
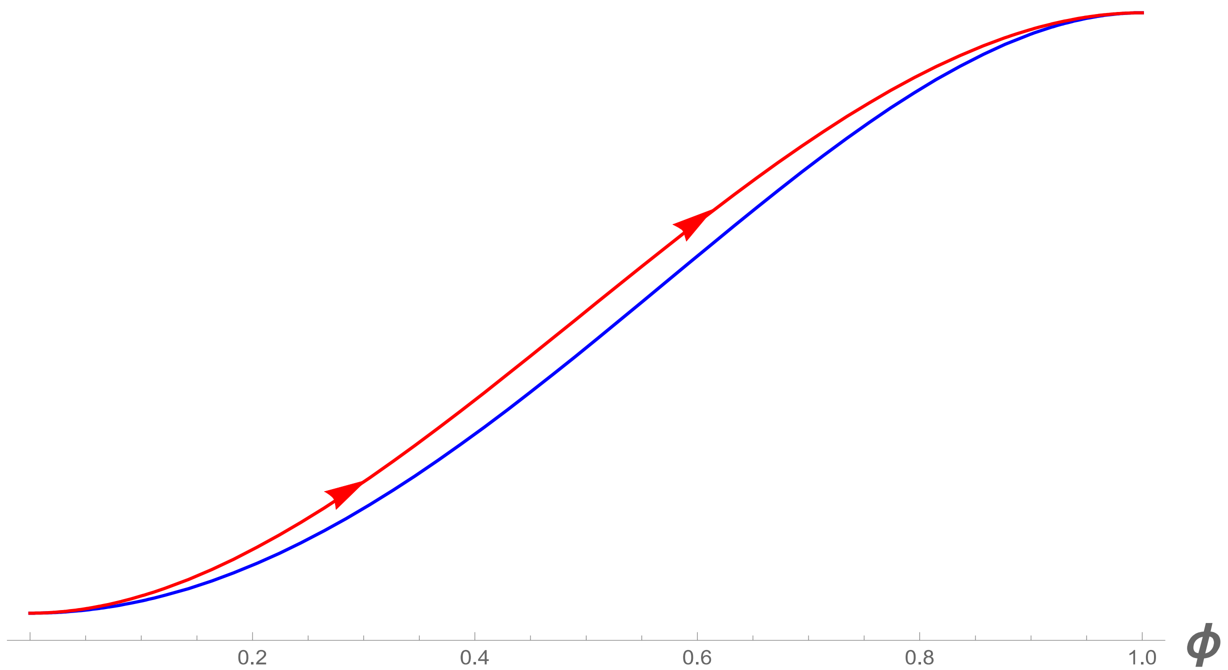
Mexican Hat potential

$$V(\phi) = -\frac{d(d-1)}{\ell^2} - \frac{m^2}{2}\phi^2 + \lambda\phi^4$$



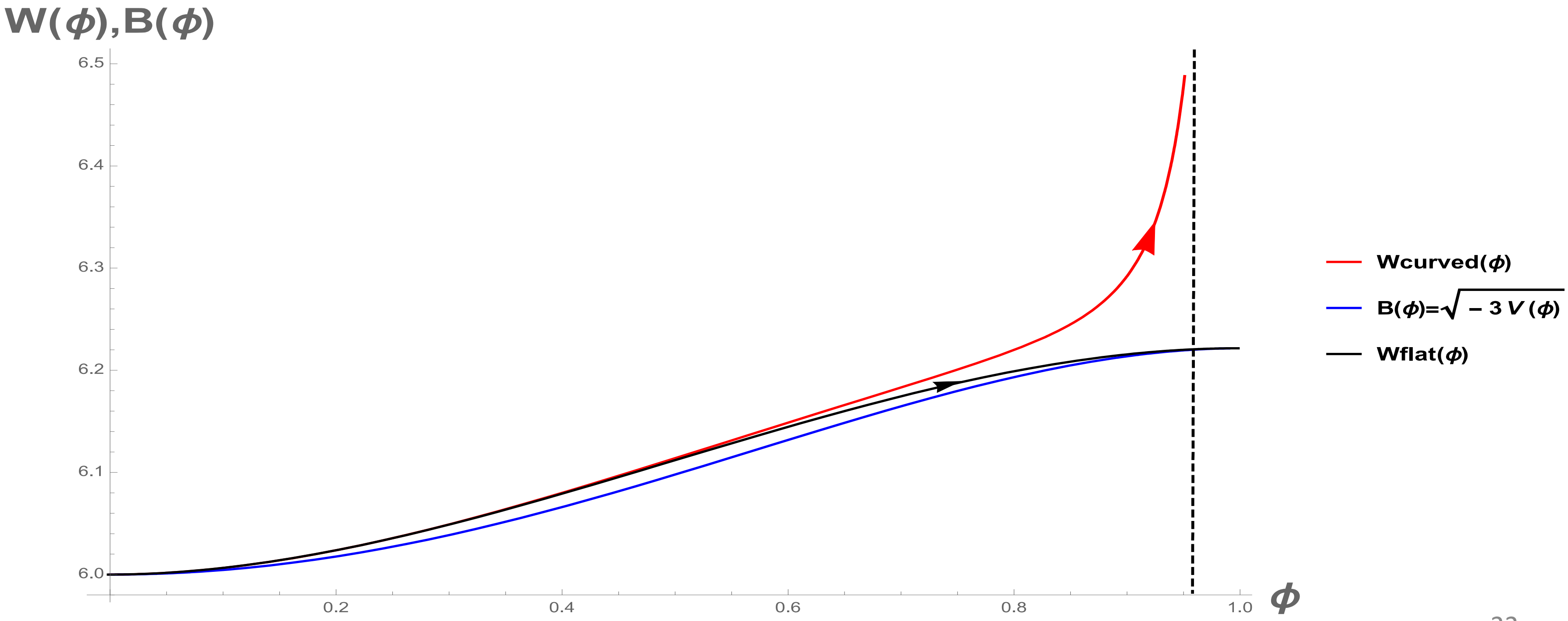
Flat case solution

- Flow starts from the maximum and ends in the minimum of the potential
- Maximum $\longrightarrow$ UV fixed point, minimum $\longrightarrow$ IR fixed point



Curved case (W)

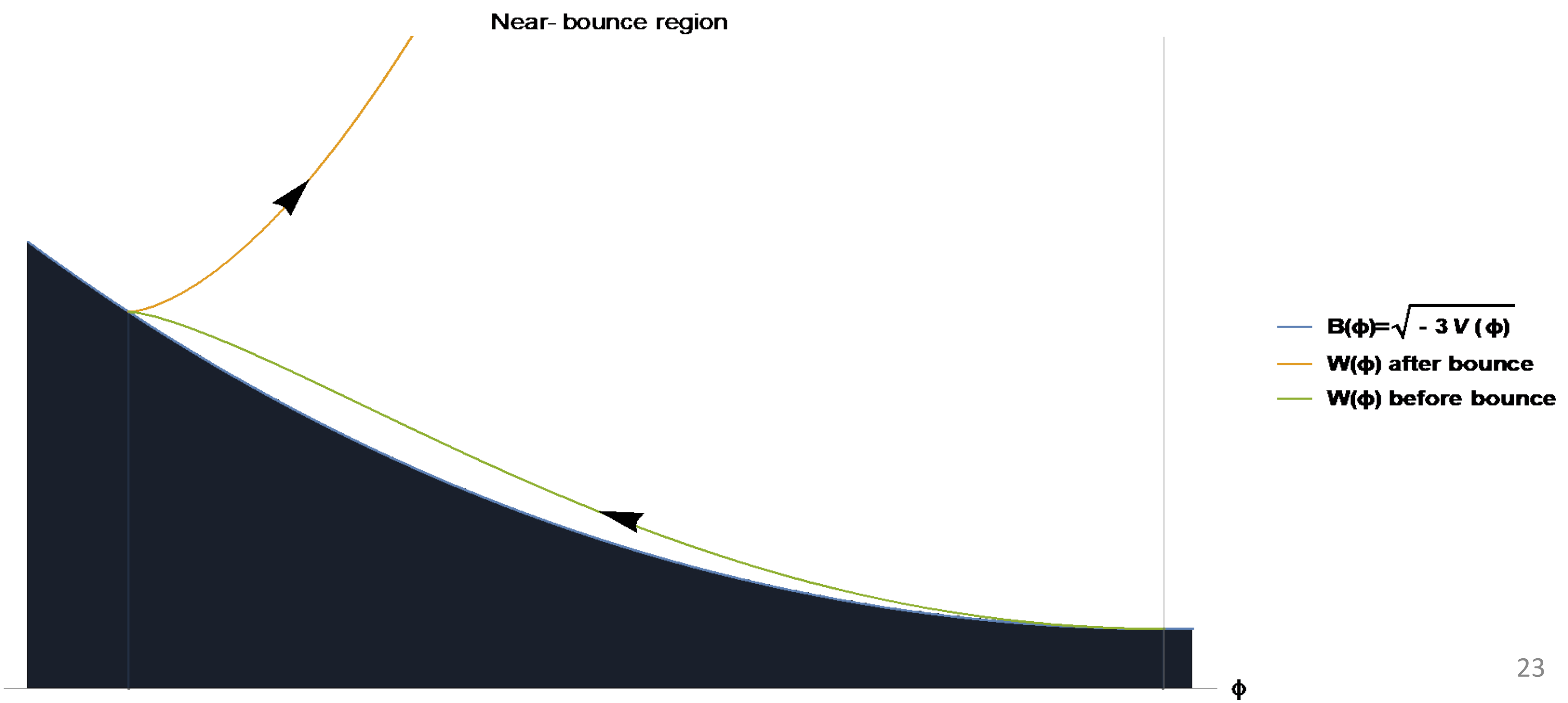
- Minimum cannot be reached for curved manifold
- W diverges at the IR
- The more we increase the curvature, the closer the IR point comes to the UV



Bounce Solution (Flat Case)

Near bounce: $W(\phi) \sim (\phi - \phi_B)^{3/2}$

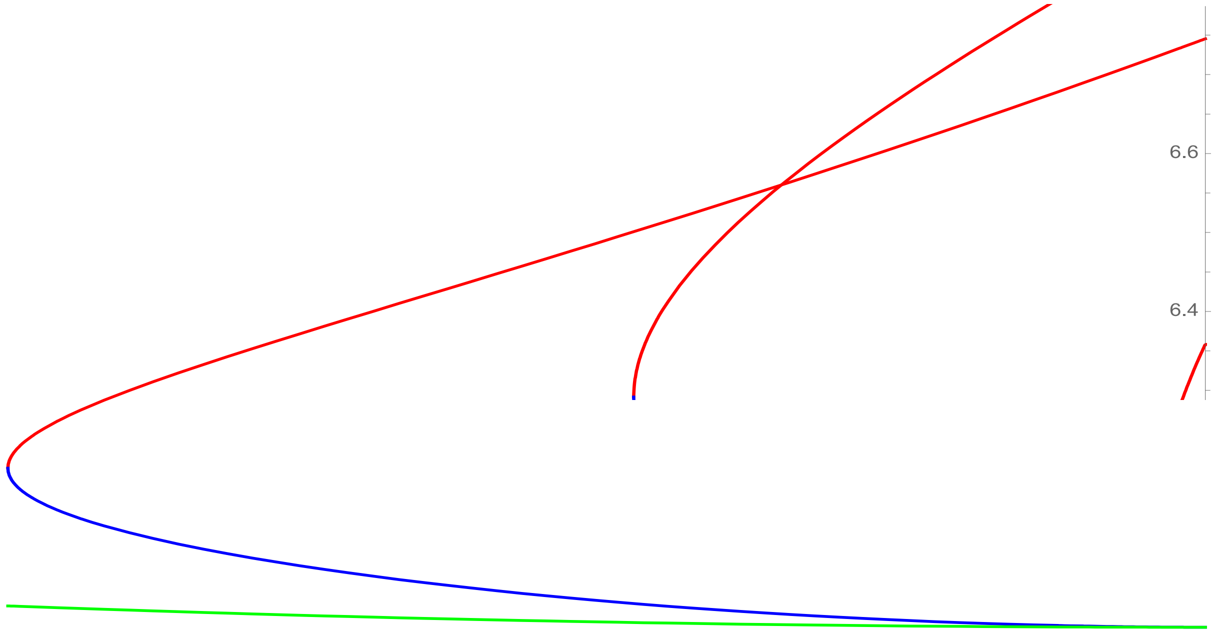
Bounce happens on the critical curve $B(\phi) = \sqrt{-3V(\phi)}$



Bounce Solution (Curved case)

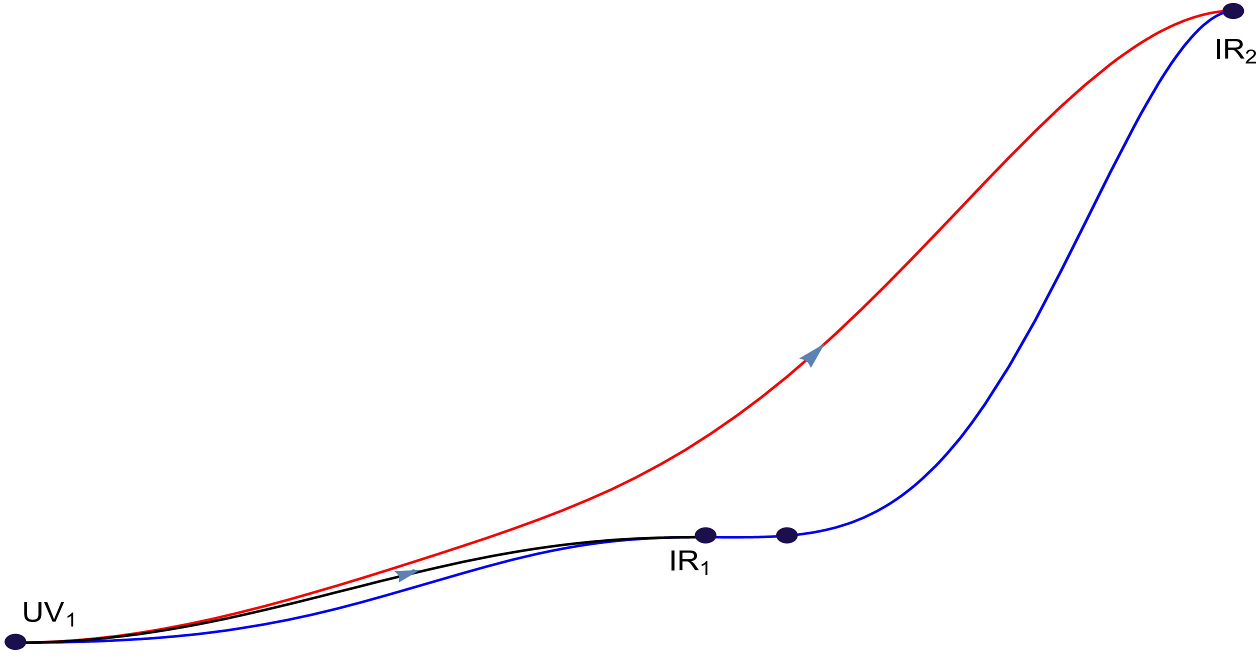
Near bounce: $W(\phi) \sim (\phi - \phi_B)^{1/2}$

Bounce cannot happen on the critical curve $B(\phi) = \sqrt{-3V(\phi)}$



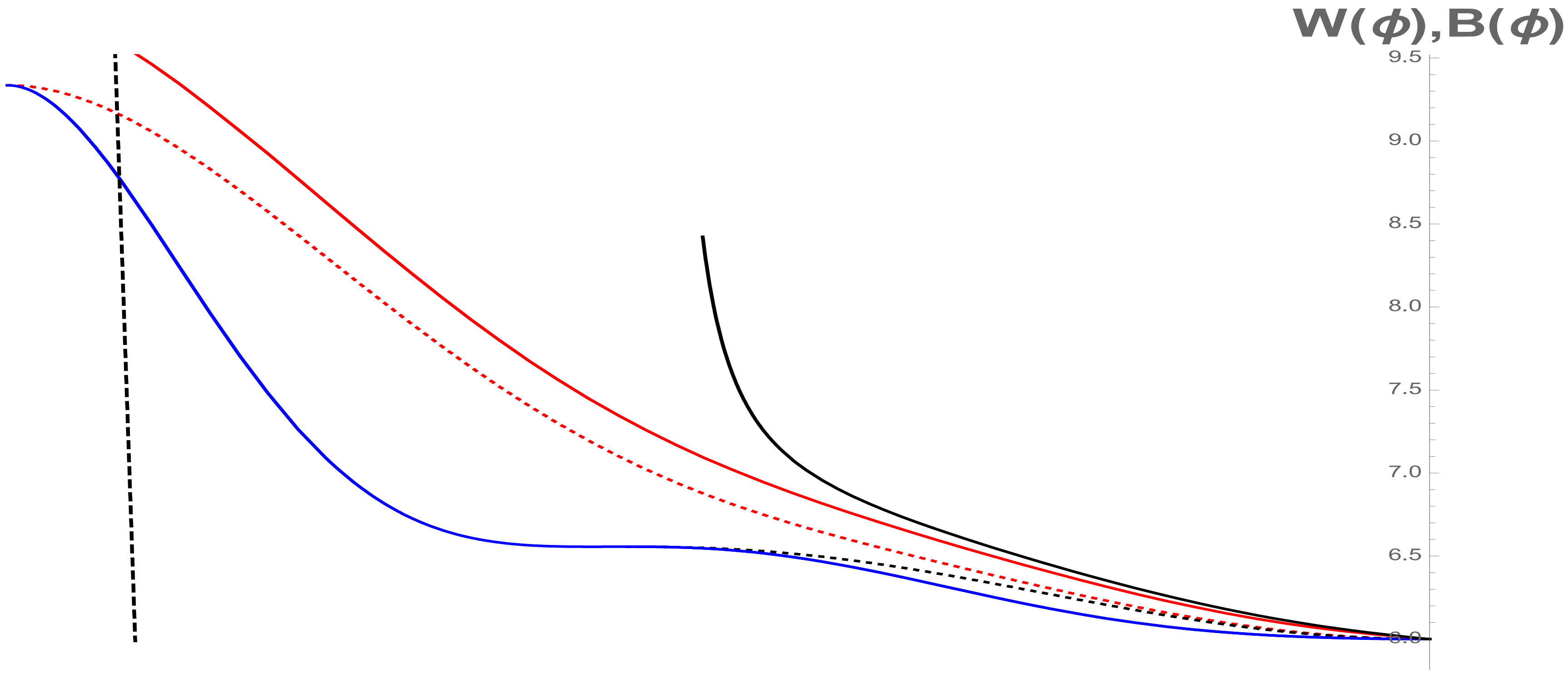
Skipping Solutions(Flat case)

- Potential is polynomial of order 12
- Several extrema: UV1, UV2, IR1, IR2
- Two solutions from UV1(skipping and non-skipping), differ by vev



Skip and non-skip flows
(Curved case)

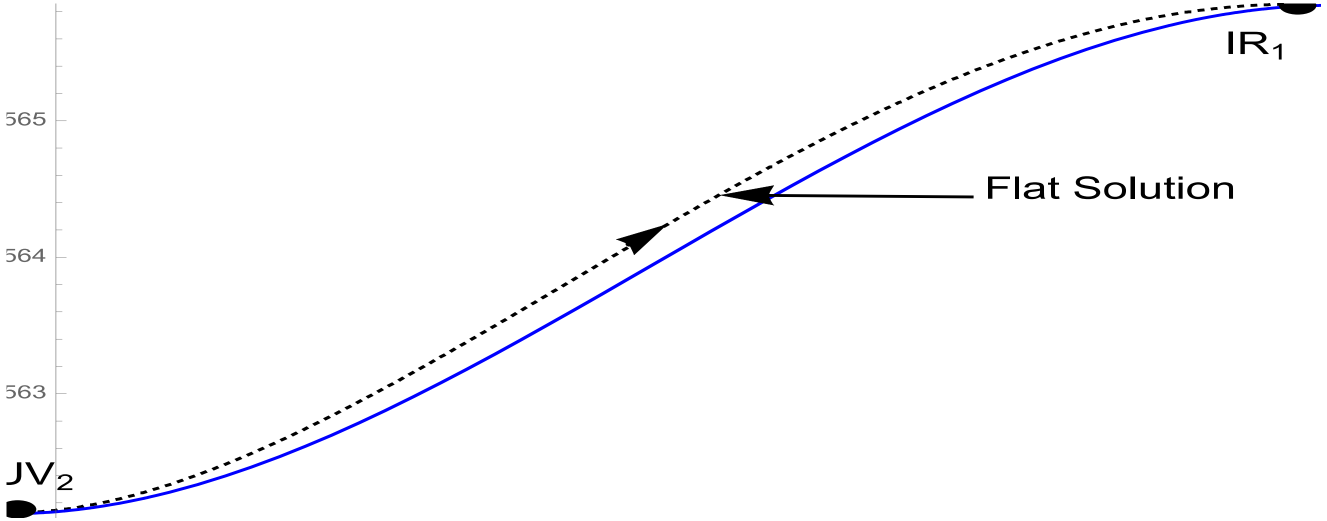
- For small curvature 3 solutions: 2 skipping and 1 non-skipping
- Increasing curvature moves the flow end-points
- Above a certain curvature skipping flows vanish, only the non-skipping flow remains



Direct and Bounce flows

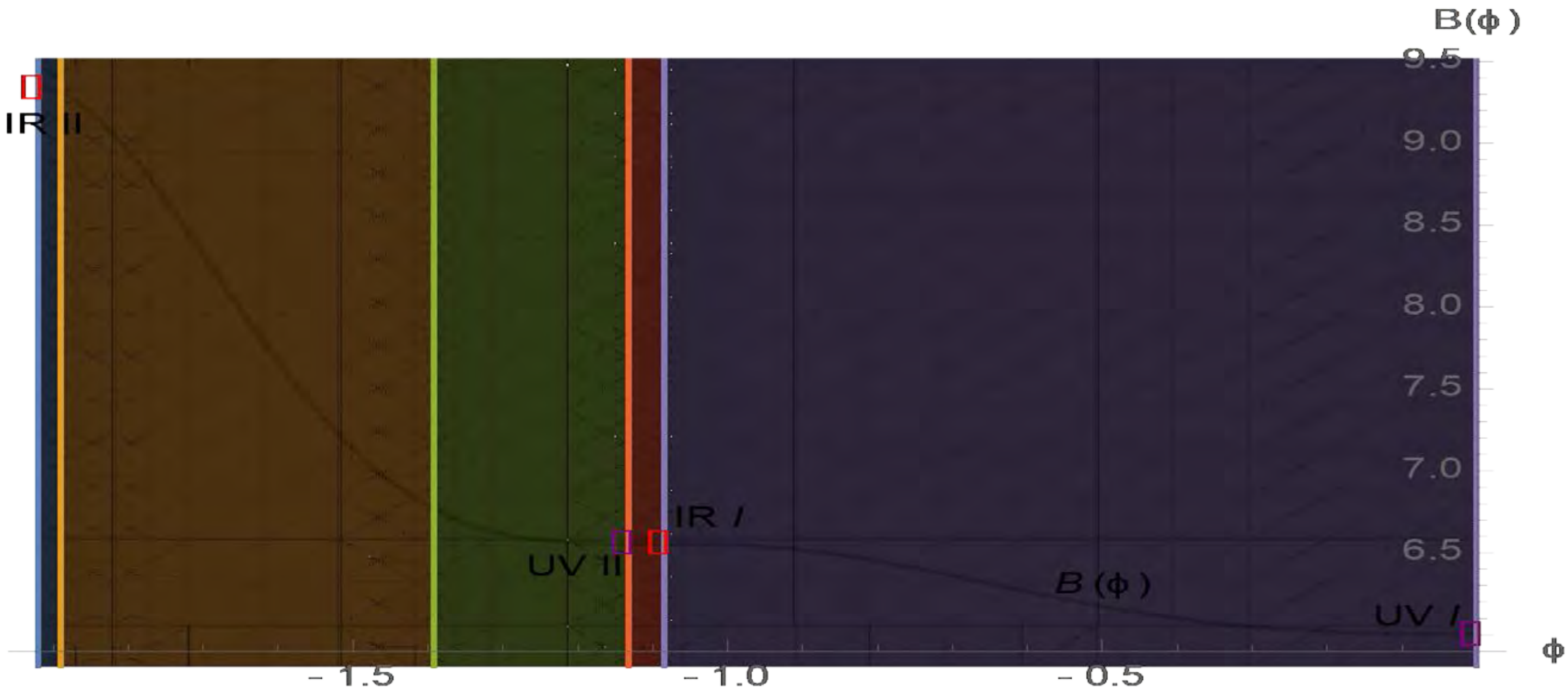
- There is a regular flow from UV2 to IR1 for flat case.
- Increasing curvature splits this solution into two:

  1) A **non-bouncing** flow, and 2) a **bouncing** solution
- Increasing curvature moves end-points closer to UV2 for both



Different flows

- Blue: IR region for skipping flows from UV1, negative source
- Yellow: IR region for bouncing flows from UV2, positive source
- Green: IR region for non-bouncing flows from UV2, negative source
- Red: IR region for non-bouncing flows from UV2, positive source
- Purple: IR region for non-skipping flows from UV1, negative source



On-Shell Action

- Action: $S = S_{\text{bulk}} + S_{\text{GHY}}$

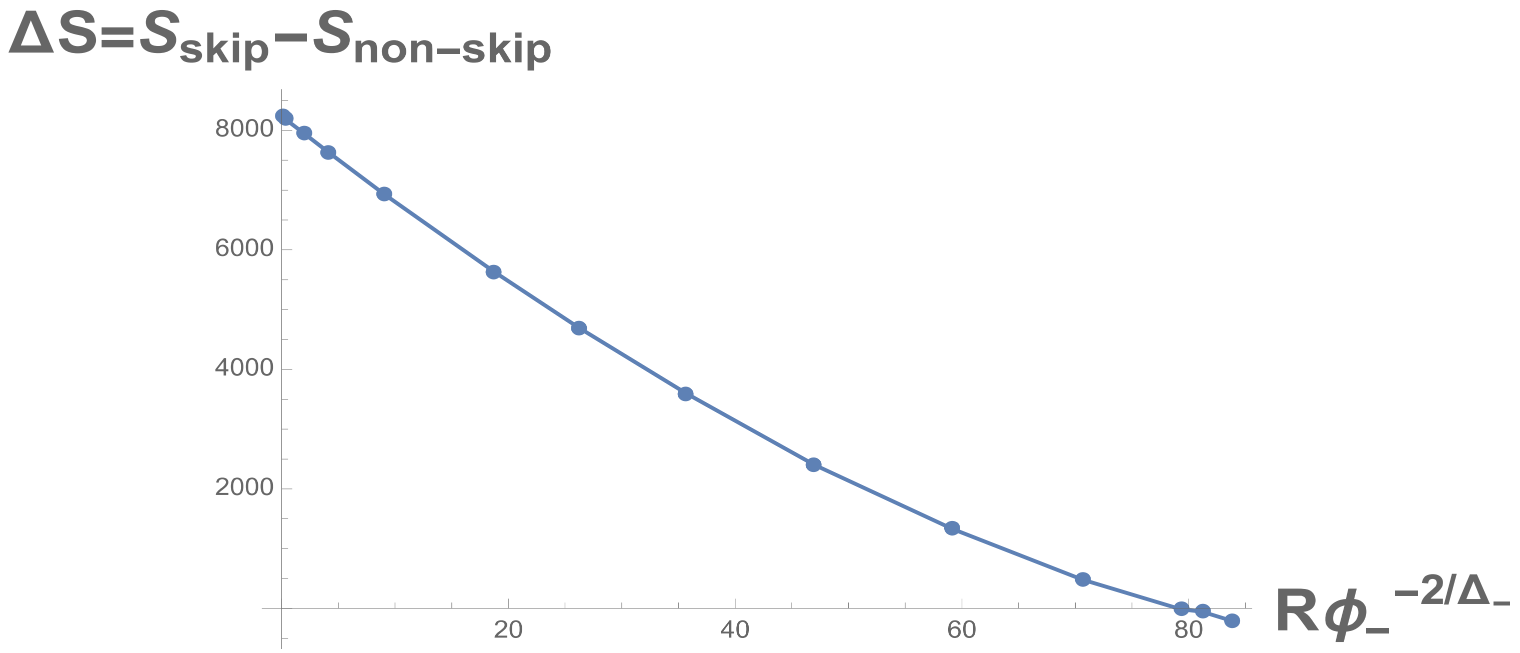
$$S_{\text{bulk}} = \int d^d x du \sqrt{-g} \left[R^{(g)} - \frac{1}{2} S^2 - V(\phi) \right]$$

$$S_{\text{GHY}} = 2 \int_{\text{boundary}} d^d x \sqrt{-\gamma} K$$

- Using equations of motion and definitions of superpotentials

$$S_{\text{on-shell}} = \int d^d x \sqrt{-\zeta} \left(-2(d-1) \left[e^{dA} \dot{A} \right]_{UV} + \frac{2R^{d/2}}{d} \int_{\phi_{UV}}^{\phi_{IR}} d\phi \frac{1}{S T^{(d-2)/2}} \right)$$

On-shell action: skipping vs non-skipping



- At low curvature, skipping flows have higher on-shell action
- After a certain curvature, non-skipping flows have higher on-shell action
- For Euclidean QFT, free energy is negative of on-shell action
- First order quantum phase transition

Conclusions and outlook

Conclusions

- QFT defined on a curved manifold with definite curvature, definite source, there are finite number of flows.
- Solutions differ by vev.
- Flow ends on a generic point which is not minimum of the potential.
- Exotic solutions exist (Bouncing and Skipping Flows)
- First order quantum phase transition been found between skipping and non-skipping flows

Outlook

- Explore F-theorem in this holographic setting.
- Reinstate brane with curved world-volume and study self-tuning.

Thank You

On-Shell Action

- Various terms are divergent. Need counter-terms.

$$S_{ct}^{(0)} = -\int_{UV} d^d x \sqrt{-\gamma} W_{0,ct}(\phi)$$

$$S_{ct}^{(1)} = \int_{UV} d^d x \sqrt{-\gamma} R^{\gamma} U_{ct}(\phi)$$

$$S_{ct}^{(2)} = -\int_{UV} d^d x \sqrt{-\gamma} (R^{\gamma})^2 \frac{\ell^3}{48\Delta_-} \log \phi_{UV}$$

- Functions satisfy

$$\frac{d}{4(d-1)} W_{0,ct}^2(\phi) - \frac{1}{2}(W_{0,ct}'(\phi))^2 = -V(\phi)$$

$$W_{0,ct}'(\phi) U_{ct}'(\phi) - \frac{d-2}{2(d-1)} W_{0,ct}(\phi) U_{ct}(\phi) = 1$$