BH S-Matrix and the YM Meron Wormhole

Nava Gaddam

July 14, 2017

9th Crete Regional Meeting on String Theory, Kolymbari 2017

Based on:

- JHEP 1611 (2016) 131 with P. Betzios and O. Papadoulaki
- On going work with P. Betzios, O. Papadoulaki and G. 't Hooft

Schwarzschild vs In-falling

Often claimed:

Information paradox for the Schwarzschild observer is solved!

- AdS/CFT? Unitary boundary theory.

Therefore, all the interesting puzzles and paradoxes concern in-falling observers. (Firewalls, Fuzzballs, Mirror Operators, Non-locality, etc.)

Schwarzschild vs In-falling

Often claimed:

Information paradox for the Schwarzschild observer is solved!

- AdS/CFT? Unitary boundary theory.

Therefore, all the interesting puzzles and paradoxes concern in-falling observers. (Firewalls, Fuzzballs, Mirror Operators, Non-locality, etc.)

In this talk, I will say (almost) nothing about the in-falling observer!

Black Hole S-Matrix 1

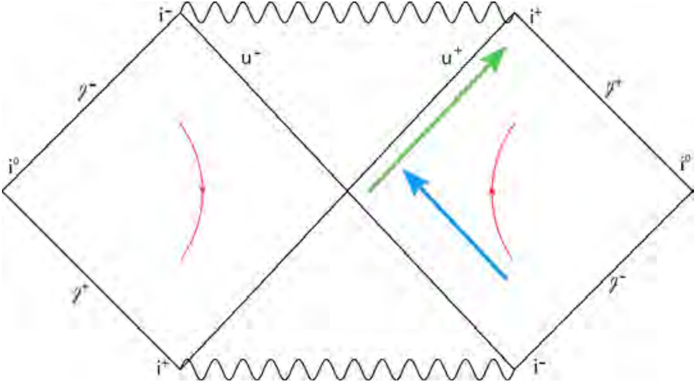
Let's say I throw a (massless) particle (with position u^+_{in} and momentum p^-_{in}) into a black hole.

Black Hole S-Matrix 1

Let's say I throw a (massless) particle (with position u^+_{in} and momentum p^-_{in}) into a black hole.

Question

How is the information about this particle retrieved by the Schwarzschild observer?

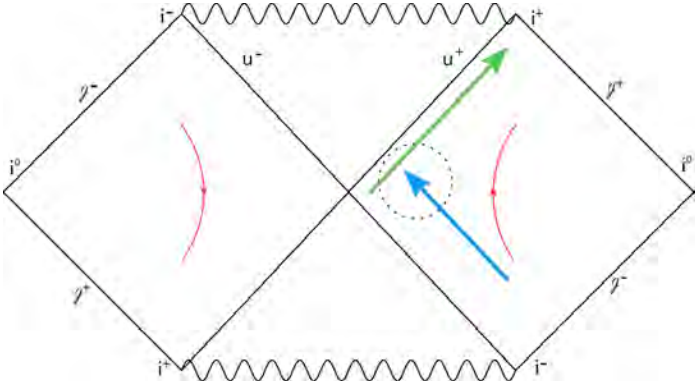


Black Hole S-Matrix 1

Let's say I throw a (massless) particle (with position u^+_{in} and momentum p^-_{in}) into a black hole.

Question

How is the information about this particle retrieved by the Schwarzschild observer?



Plan for the talk

Black Hole S-Matrix

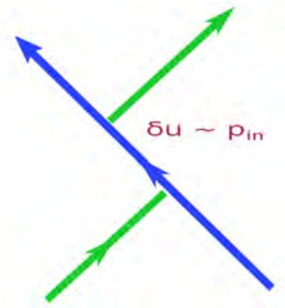
Inverted Harmonic Oscillators

Yang-Mills Meron-Wormhole — a gravitational instanton

Conclusions

Gravitational back-reaction

[Aichelburg-Sexl; Dray-'t Hooft]



$$u_{\mathrm{out}}^{-}(\Omega) \;=\; 8\pi G R^2 \int d^{d-2}\Omega' \, \tilde{f}\left(\Omega,\Omega'\right)\, p_{\mathrm{in}}^{-}\left(\Omega'\right)$$

The effect is quantum gravitational, albeit semi-classical. No out-going modes without quantum mechanics. No back-reaction without gravity. So, observed effect is zero when $G \to 0$ or $\hbar \to 0$.

The scattering algebra

['t Hooft]

Relation between in and out states can be expanded in partial waves:

$$\hat{u}_{lm}^{\pm} = \mp \frac{8\pi G}{R^2\left(l^2+l+1\right)}\hat{p}_{lm}^{\pm} =: \mp \lambda \hat{p}_{lm}^{\pm}$$

Out-going mode is a Fourier transform of the in-going mode!

The scattering algebra

['t Hooft]

Relation between in and out states can be expanded in partial waves:

$$\hat{u}^{\pm}_{lm} = \mp \frac{8\pi G}{R^2\left(l^2+l+1\right)}\hat{p}^{\pm}_{lm} =: \mp \lambda \hat{p}^{\pm}_{lm}$$

Out-going mode is a Fourier transform of the in-going mode!
Canonical commutation relations yield the scattering algebra:

$$\begin{aligned}
\left[\hat{u}^{\pm}_{lm}, \hat{p}^{\mp}_{l'm'}\right] &= i\delta_{ll'}\delta_{mm'} \\
\left[\hat{u}^{+}_{lm}, \hat{u}^{-}_{l'm'}\right] &= i\,\lambda\,\delta_{ll'}\delta_{mm'} \\
\left[\hat{p}^{+}_{lm}, \hat{p}^{-}_{l'm'}\right] &= -\frac{i}{\lambda}\,\delta_{ll'}\delta_{mm'}
\end{aligned}$$

Black Hole S-Matrix 2

Algebra leads to an inner product on the Hilbert-space. Massage the Fourier transform to find:

$$\begin{pmatrix} \phi^{\text{out}}(+,\rho^-) \\ \phi^{\text{out}}(-,\rho^-) \end{pmatrix} = \int_{-\infty}^{\infty} dx \begin{pmatrix} A(+,+,x) & A(+,-,x) \\ A(-,+,x) & A(-,-,x) \end{pmatrix} \begin{pmatrix} \phi^{\text{in}}(+,x+\log\lambda-\rho^-) \\ \phi^{\text{in}}(-,x+\log\lambda-\rho^-) \end{pmatrix}$$

Black Hole S-Matrix 2

Algebra leads to an inner product on the Hilbert-space. Massage the Fourier transform to find:

$$\begin{pmatrix} \phi^{\mathrm{out}}(+,\rho^-) \\ \phi^{\mathrm{out}}(-,\rho^-) \end{pmatrix} = \int_{-\infty}^{\infty} dx \begin{pmatrix} A(+,+,x) & A(+,-,x) \\ A(-,+,x) & A(-,-,x) \end{pmatrix} \begin{pmatrix} \phi^{\mathrm{in}}(+,x+\log\lambda-\rho^-) \\ \phi^{\mathrm{in}}(-,x+\log\lambda-\rho^-) \end{pmatrix}$$

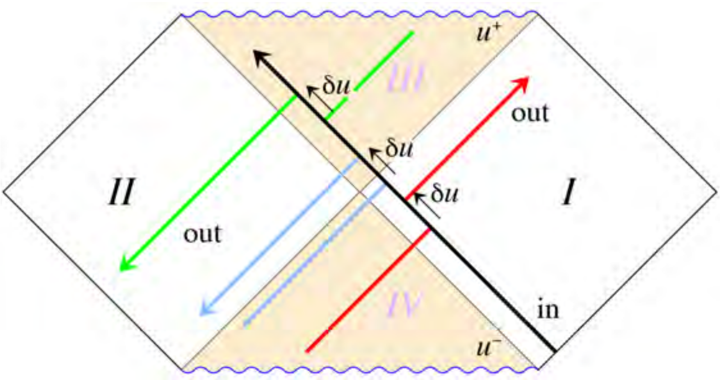
This defines an S-matrix mapping the in and out states:

$$\begin{aligned} S(k_l,\lambda_l) &= e^{-ik_l\log\lambda_l} \begin{pmatrix} A(+,+,k_l) & A(+,-,k_l) \\ A(-,+,k_l) & A(-,-,k_l) \end{pmatrix} \\ &= \frac{1}{\sqrt{2\pi}}\Gamma\left(\frac{1}{2}-ik_l\right) e^{-ik_l\log\lambda_l} \begin{pmatrix} e^{-i\frac{\pi}{4}}\,e^{-k_l\frac{\pi}{2}} & e^{i\frac{\pi}{4}}\,e^{k_l\frac{\pi}{2}} \\ e^{i\frac{\pi}{4}}\,e^{k_l\frac{\pi}{2}} & e^{-i\frac{\pi}{4}}\,e^{-k_l\frac{\pi}{2}} \end{pmatrix} \end{aligned}$$

Can check for unitarity, explicitly.

Antipodal entanglement

Drawing stolen from 't Hooft!



Radiation from antipodal points is entangled!

Plan for the talk

Black Hole S-Matrix

Inverted Harmonic Oscillators

Yang-Mills Meron-Wormhole — a gravitational instanton

Conclusions

Rearranging degrees of freedom

[Betzios-N.G-Papadoulaki]

Question 1

What kind of a quantum mechanical potential allows for only scattering states?

Rearranging degrees of freedom

[Betzios-N.G-Papadoulaki]

Question 1

What kind of a quantum mechanical potential allows for only scattering states?

- An unstable one

Rearranging degrees of freedom

[Betzios-N.G-Papadoulaki]

Question 1

What kind of a quantum mechanical potential allows for only scattering states?

- An unstable one

Question 2

And what potential allows for energy eigenstates such as those of Rindler space?

Rearranging degrees of freedom

[Betzios-N.G-Papadoulaki]

Question 1

What kind of a quantum mechanical potential allows for only scattering states?

- An unstable one

Question 2

And what potential allows for energy eigenstates such as those of Rindler space?

- Ubiquity of Rindler space suggests a quadratic potential

Inverted Harmonic Oscillators

A collection of inverted harmonic oscillators:

$$H_{tot} = \sum_{lm} \frac{1}{2}\left(p_{lm}^2 - x_{lm}^2\right)$$
$$= \sum_{lm} \frac{1}{2}\left(\tilde{u}_{lm}^{+}\tilde{u}_{lm}^{-} + \tilde{u}_{lm}^{-}\tilde{u}_{lm}^{+}\right)$$

S-Matrix of each oscillator = that of corresponding partial wave!

The algebra revisted

Remember the λ-dependent algebra! Different time coordinate for each oscillator?

$$
\begin{aligned}
\left[\hat{u}^{\pm}_{lm}, \hat{p}^{\mp}_{l'm'}\right] &= i\delta_{ll'}\delta_{mm'} \\
\left[\hat{u}^{+}_{lm}, \hat{u}^{-}_{l'm'}\right] &= i\,\lambda\,\delta_{ll'}\delta_{mm'} \\
\left[\hat{p}^{+}_{lm}, \hat{p}^{-}_{l'm'}\right] &= -\frac{i}{\lambda}\,\delta_{ll'}\delta_{mm'}
\end{aligned}
$$

The algebra revisted

Remember the λ-dependent algebra! Different time coordinate for each oscillator?

$$\begin{aligned} \left[\hat{u}^{\pm}_{lm}, \hat{p}^{\mp}_{l'm'}\right] &= i\delta_{ll'}\delta_{mm'} \\ \left[\hat{u}^{+}_{lm}, \hat{u}^{-}_{l'm'}\right] &= i\,\lambda\,\delta_{ll'}\delta_{mm'} \\ \left[\hat{p}^{+}_{lm}, \hat{p}^{-}_{l'm'}\right] &= -\frac{i}{\lambda}\,\delta_{ll'}\delta_{mm'} \end{aligned}$$

Can make the algebra λ-independent. Price we pay: chemical potential!

The partial waves

- How do the partial waves differ from each other?

The partial waves

- How do the partial waves differ from each other? Via a chemical potential; the energies are given by

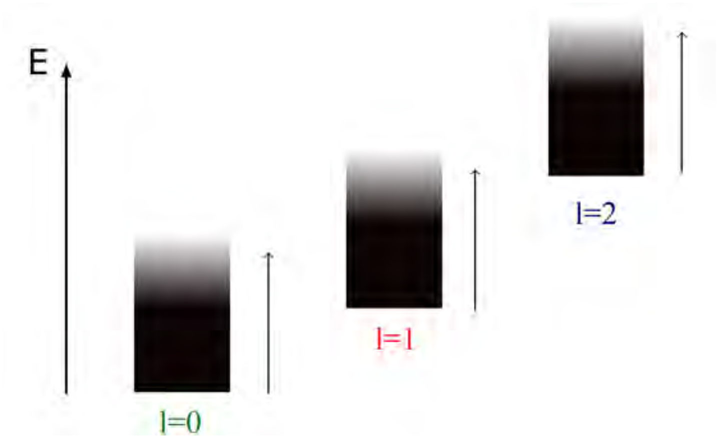
$$k_l = \omega_l + \frac{l^2 + l + 1}{c}, \qquad \text{and} \qquad E_{\rm tot}^{\rm Rindler} = \sum_l k_l \,.$$

- Remember

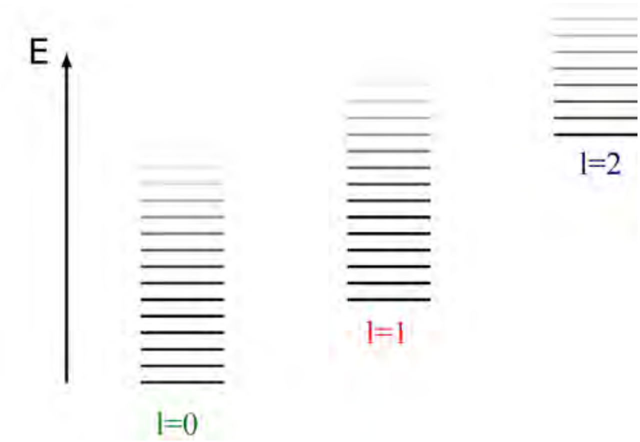
$$\frac{\ell^2 + \ell + 1}{c} = \frac{1}{\lambda} = \frac{R^2 \left(\ell^2 + \ell + 1\right)}{8\pi G}$$

The algebra is now λ-independent and all the ℓ dependence is in the chemical potential. Unique time-coordinate!

The spectrum



The spectrum



Relation to 2d Quantum Gravity

IHOs have been studied for ever. Also in context of 2d QG and black holes. [Lage-N refs.]

Time delays

Singlet sector Matrix model calculations — intermediate state doesn't live long enough to represent a 2d black hole.

Singlet sector $\sim$ One oscillator

Relation to 2d Quantum Gravity

IHOs have been studied for ever. Also in context of 2d QG and black holes. [Lage-N refs.]

Time delays

Singlet sector Matrix model calculations — intermediate state doesn't live long enough to represent a 2d black hole.

Singlet sector $\sim$ One oscillator

Here, many spherical waves. NOT in the singlet sector.

[See talks by Betzios & Papadoulaki]

Degeneracy of states

Given an initial state of some total energy, can distribute it among finite no. of partial waves. Exponential degeneracy for large energies:

Hardy-Ramanujan

$$p(n) \sim \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

Degeneracy of states

Given an initial state of some total energy, can distribute it among finite no. of partial waves. Exponential degeneracy for large energies:

Hardy-Ramanujan

$$p(n) \sim \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

Can also study the number density. Approximate thermality?

$$N^{\text{out}}_{++}(k) = \frac{N^{\text{in}}_{++}(k)}{1+e^{2\pi k}} + \frac{N^{\text{in}}_{--}(k)}{1+e^{-2\pi k}}$$

$$N^{\text{out}}_{--}(k) = \frac{N^{\text{in}}_{--}(k)}{1+e^{2\pi k}} + \frac{N^{\text{in}}_{++}(k)}{1+e^{-2\pi k}}$$

Lessons learnt?

- Some features of BH horizons needn't be all that complicated!

Lessons learnt?

- Some features of BH horizons needn't be all that complicated!
- Antipodal entanglement = Entanglement between reflected and transmitted pieces of the wave-function!

Lessons learnt?

- Some features of BH horizons needn't be all that complicated!
- Antipodal entanglement = Entanglement between reflected and transmitted pieces of the wave-function!
- Bell-pairs here vs many-body entanglement in gauge/gravity duality? Each 'particle' is actually a partial wave. Need a more refined understanding of the Hilbert space to compare with gauge/gravity duality.

Lessons learnt?

- Some features of BH horizons needn't be all that complicated!
- Antipodal entanglement = Entanglement between reflected and transmitted pieces of the wave-function!
- Bell-pairs here vs many-body entanglement in gauge/gravity duality? Each 'particle' is actually a partial wave. Need a more refined understanding of the Hilbert space to compare with gauge/gravity duality.
- Cannot naively second quantise.

Lessons learnt?

- Some features of BH horizons needn't be all that complicated!
- Antipodal entanglement = Entanglement between reflected and transmitted pieces of the wave-function!
- Bell-pairs here vs many-body entanglement in gauge/gravity duality? Each 'particle' is actually a partial wave. Need a more refined understanding of the Hilbert space to compare with gauge/gravity duality.
- Cannot naively second quantise.
- Planckian physics: Incorporate transverse effects via interactions between oscillators.

Plan for the talk

Black Hole S-Matrix

Inverted Harmonic Oscillators

Yang-Mills Meron-Wormhole — a gravitational instanton

Conclusions

Black Hole — a Gravitational Instanton?

Euclidean wormhole

Large gravitational instanton with finite (possibly large) Euclidean size?

$$ds^2 = dr^2 + \left(r^2 + r_0^2\right) \underbrace{\left(d\psi^2 + \sin^2(\psi)\, d\theta^2 + \sin^2(\psi)\sin^2(\theta)\, d\phi^2\right)}_{d\Omega_3^2}$$

Black Hole — a Gravitational Instanton?

Euclidean wormhole

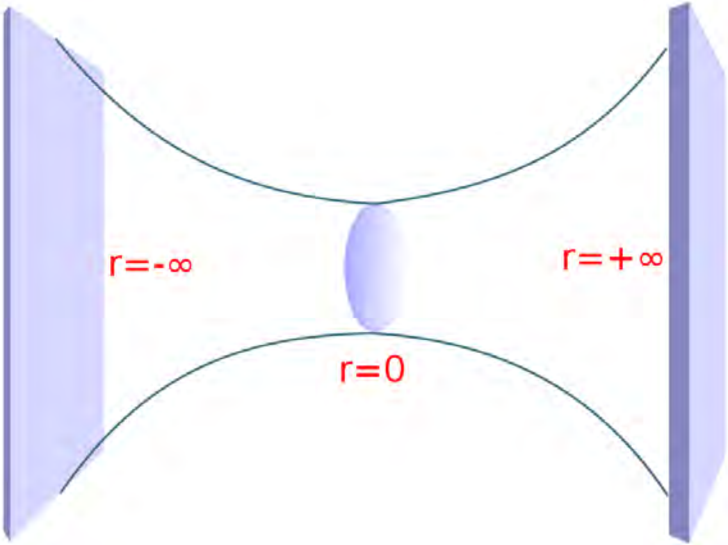
Large gravitational instanton with finite (possibly large) Euclidean size?

$$ds^2 = dr^2 + (r^2 + r_0^2)\underbrace{\left(d\psi^2 + \sin^2(\psi)\,d\theta^2 + \sin^2(\psi)\sin^2(\theta)\,d\phi^2\right)}_{d\Omega_3^2}$$

Supported by non-Abelian field strength with magnetic flux, carrying negative Euclidean energy density near the throat.

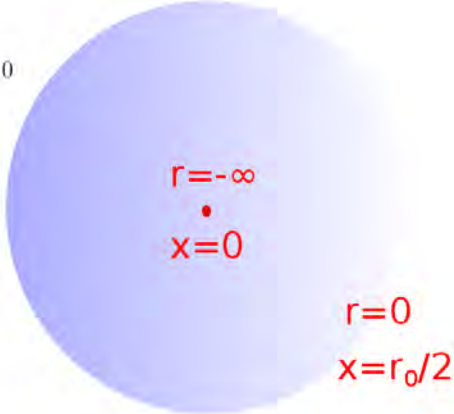
Merons: Studied in the late 70s, in flat space (singular; not solutions of EOMs!). [Alfaro-Fubini-Furlan;Callen-Dashen-Gross;Laughton]
Smooth but non-covariant merons [Hosoya-Ogura]

Yang-Mills Meron Wormhole



Yang-Mills Meron Wormhole

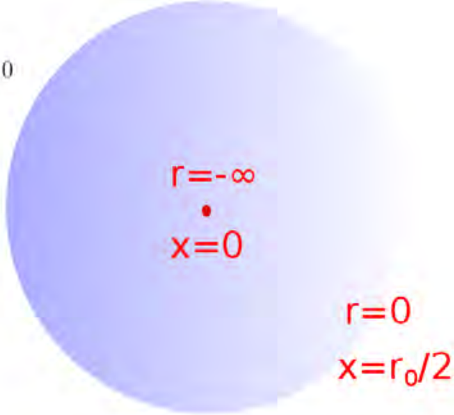
$$r = \mathbf{x} - \frac{r_0^2}{4\mathbf{x}} \quad \text{with} \quad \mathbf{x} = \sqrt{x_\mu x^\mu} \geq 0$$



$$ds^2 = \left(1 + \frac{r_0^2}{4\mathbf{x}^2}\right)^2 \delta_{\mu\nu} dx^\mu dx^\nu \quad \text{with} \quad r_0 = \sqrt{\frac{4\pi G_N}{g_{\rm YM}^2}}$$

Yang-Mills Meron Wormhole

$$r = \mathbf{x} - \frac{r_0^2}{4\mathbf{x}} \quad \text{with} \quad \mathbf{x} = \sqrt{x_\mu x^\mu} \geq 0$$



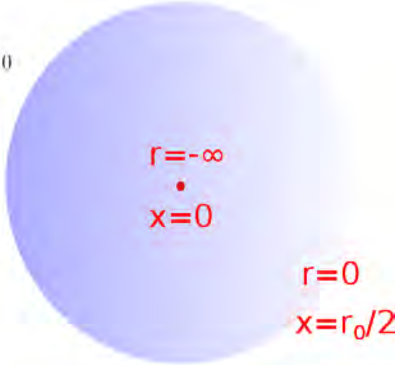
$$ds^2 = \left(1 + \frac{r_0^2}{4\mathbf{x}^2}\right)^2 \delta_{\mu\nu} dx^\mu dx^\nu \quad \text{with} \quad r_0 = \sqrt{\frac{4\pi G_N}{g_{\rm YM}^2}}$$

$$F^a_{\mu\rho} = \frac{1}{g_{\rm YM}}\left[\eta_{a\mu\rho}\frac{f_1(\mathbf{x})}{\mathbf{x}^2} + \left(x_\mu \eta_{a\rho\gamma} x_\gamma - x_\rho \eta_{a\mu\gamma} x_\gamma\right)\frac{f_2(\mathbf{x})}{\mathbf{x}^4}\right]$$

'The Vacuole'

$$r = \mathbf{x} - \frac{r_0^2}{4\mathbf{x}} \quad \text{with} \quad \mathbf{x} = \sqrt{x_\mu x^\mu} \geq 0$$

$$x^\mu \to \frac{r_0^2}{4\mathbf{x}} x^\mu$$



$$ds^2 = \left(1 + \frac{r_0^2}{4\mathbf{x}^2}\right)^2 \delta_{\mu\nu} dx^\mu dx^\nu \quad \text{with} \quad r_0 = \sqrt{\frac{4\pi G_N}{g_{\rm YM}^2}}$$

Antipodal mapping corresponds to Inversion.

Gaussian fluctuations

$$S^{(2)} = \int \left[A\, \mathcal{M}_A\, A + h\, \mathcal{M}_h\, h + h\, \mathcal{M}_{(h\, A)}\, A \right]$$

Gaussian fluctuations

$$S^{(2)} = \int \left[A\, \mathcal{M}_A\, A + h\, \mathcal{M}_h\, h + h\, \mathcal{M}_{(h\,A)}\, A \right]$$

$$A_\mu^{L,a} \left[\frac{1}{2}\left(\partial_{\mathbf{x}}^2 + \frac{3}{\mathbf{x}}\partial_{\mathbf{x}} - \frac{4L^2}{\mathbf{x}^2} \right) + \frac{4r_1^4}{\mathbf{x}^2\left(\mathbf{x}^2+r_1^2\right)^2} - \frac{2f\left(fT^2 + T\cdot L\right)}{\mathbf{x}^2} + \frac{2r_1^2\left(\mathbf{x}\partial_{\mathbf{x}} - 1\right)}{\mathbf{x}^2\left(\mathbf{x}^2+r_1^2\right)} \right.$$
$$\left. - \frac{4r_1^2}{\mathbf{x}^2\left(\mathbf{x}^2+r_1^2\right)^2}\left(5r_1^2 + 4fT\cdot S_1\left(\mathbf{x}^2+r_1^2\right)\right) \frac{2f_1\left(T\cdot S_1\right)}{\mathbf{x}^2} - \frac{6r_1^2}{\left(\mathbf{x}^2+r_1^2\right)^2} \right] A_\mu^{L,a}$$
$$+ A_\mu^{T,a} \left[\frac{1}{2}\left(\partial_{\mathbf{x}}^2 + \frac{3}{\mathbf{x}}\partial_{\mathbf{x}} - \frac{4L^2}{\mathbf{x}^2} \right) + \frac{4r_1^4}{\mathbf{x}^2\left(\mathbf{x}^2+r_1^2\right)^2} - \frac{2f^2T^2}{\mathbf{x}^2} - \frac{2fT\cdot L}{\mathbf{x}^2} \right.$$
$$\left. + \frac{2r_1^2\left(\mathbf{x}\partial_{\mathbf{x}} - 1\right)}{\mathbf{x}^2\left(\mathbf{x}^2+r_1^2\right)} - 2\frac{f_1}{\mathbf{x}^2}\left(S_1\cdot T\right) + \frac{2r_1^2}{\left(\mathbf{x}^2+r_1^2\right)^2} \right] A_\nu^{T,a}$$

Conclusions

Summary

- BH horizon has the dynamics of a a collection of Inverted Harmonic Oscillators.
- Euclidean Wormholes supported by a YM Meron—ideal toys for exciting physics (both for quantum gravity and QCD)

Conclusions

Summary

- BH horizon has the dynamics of a a collection of Inverted Harmonic Oscillators.
- Euclidean Wormholes supported by a YM Meron—ideal toys for exciting physics (both for quantum gravity and QCD)

Near-future directions

- Partition function for IHOs [Aharony-Marsano-Minwalla-Papadodimas-Van Raamsdonk]
- Find a Matrix Model for the oscillators (Non-singlet sectors?) [Hartnoll-Huijse-Mazenc; Anninos-Denef-Monten]
- Multi-vacuole (meron) solutions (dilute gas)? Consequences for QCD? [Callan-Dashen-Gross; Steele-Negele]
- Easy to embed in gauge/gravity duality → scope for better understanding.