Weyl Anomalies and Cosmology

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References

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Anomalous dependence on scale factor

- Logarithmic scaling violations due to renormalization, viewed as Weyl anomalies, lead to an anomalous dependence on the scale factor of the metric. Since the scale factor undergoes several e-foldings as the universe expands, large logarithms are involved.
- *Can we compute these effects systematically?*
- *Can they lead to interesting effects in cosmology?*

*The leading large logarithms can be re-summed using **local renormalization group** to obtain a **nonlocal** quantum effective action.*

Anomalous dependence on scale factor

- I'll discuss two examples where these effects are calculable and lead to interesting effects.
- *Two-dimensional Gravity: dilution of vacuum energy*
- *Four-dimensional massless electrodynamics: of possible interest for primordial magnetogenesis in early universe.*

A two-dimensional model

- Einstein gravity with positive cosmological constant

$$\frac{1}{2\kappa^2}\int d^dx\sqrt{-g}\,[R-2\Lambda]$$

- Reduces to *timelike* Liouville theory for $d = 2+\epsilon$

$$\frac{1}{4\pi\beta^2}\int d^2x\sqrt{-h}\left(\frac{R_h}{\epsilon}+\underbrace{(\nabla\Omega)^2+R_h\,\Omega-4\pi\mu\,e^{2\Omega}}\right)$$

$$\kappa^2 = 2\pi\beta^2\epsilon \qquad g_{\mu\nu} = e^{2\Omega}h_{\mu\nu} \qquad \mu = \frac{\Lambda}{\kappa^2}$$

- Alternatively, Polyakov action for supercritical matter.
- Classically, ***de Sitter*** is a solution.

Weyl Anomaly of the Cosmological Term

- The cosmological constant operator $e^{2\Omega}$ has anomalous dimension $\gamma = 2\beta^2$.
- It is essentially like a tachyon vertex operator which is an exponential of a free field.
- This implies that the trace of the energy-momentum tensor receives a quantum correction:

$$g^{\mu\nu}T_{\mu\nu} = -2(1-\beta^2)e^{-2\beta^2\Omega}$$

- There is a unique covariantly conserved quantum momentum tensor with this modified trace*: 3 equations for 3 functions.* But it must be *nonlocal.*

Nonlocal Quantum Momentum Tensor

$$T_{\mu\nu}(x) = -\mu\,(1-\beta^2)\,\eta_{\mu\nu}\,e^{-2(1-\beta^2)\Omega} + 2\,\mu\,\beta^2\,S_{\mu\nu}$$

$$S_{\mu\nu} = \int dy\,\left(\nabla_\mu\nabla_\nu - \frac{1}{2}\,g_{\mu\nu}\,\nabla^2\right) G_{xy}\,e^{-2\Omega(y)} + \int dy\,dz\,\left(\nabla_{(\mu}G_{yx}\nabla_{\nu)}G_{xz} - \frac{1}{2}\,g_{\mu\nu}\,g^{\alpha\beta}\,\nabla_\alpha\,G_{yx}\,\nabla_\beta\,G_{xz}\right)\,e^{-2\Omega(y)}\,R_g(z)$$

- Follows from a nonlocal quantum effective action:

$$I_{eff}[g] = \frac{1}{2\kappa^2}\int d^2x\,\sqrt{-g}\,\left(R_g - 2\,\Lambda\,\mathbf{e}^{-2\beta^2\Omega}\right)$$

- The scale factor is a nonlocal functional of the metric:

$$R_g = e^{-2\Omega}\,\left(R_h - 2\,\nabla_h^2\,\Omega\right) \qquad \Omega(x) = \frac{1}{2}\int d^2y\,\sqrt{-g}\,G_g(x,y)\,R_g(y)$$

Quantum effects modify barotropic index

- Homogeneous and isotropic universe:

$$g_{\mu\nu} = a^2(\tau)\,\eta_{\mu\nu} \qquad a(\tau) = a_* e^{2\Omega}$$

- Perfect fluid with a quantum corrected barotropic index :

$$T_{\mu\nu} = \begin{pmatrix} \rho_\Lambda & 0 \\ 0 & p_\Lambda \end{pmatrix} \qquad \frac{p_\Lambda}{\rho_\Lambda} = \boxed{w_\Lambda = -1 + 2\beta^2}$$

- The nonlocal momentum tensor becomes effectively ***local*** for Robertson-Walker metrics.
- The net effect of quantum corrections is to modify the barotropic index slightly.

Quantum *dilution* of vacuum energy

- Quasi-de Sitter power-law expansion:

$$\boxed{a(t) = a_*(1+\beta^2 H_* t)^{\frac{1}{\beta^2}}} \longrightarrow \quad a(t) = a_* e^{H_* t}$$

- Dilution of vacuum energy:

$$\boxed{\rho_\Lambda(t) = \rho_*(\frac{a}{a_*})^{-2\beta^2}} \longrightarrow \quad \rho(t) = \rho_*(1 - 2\beta^2 \log\frac{a(t)}{a_*} + \ldots)$$

Large logarithms have been re-summed.

No particle production. Nonlinear dilution of vacuum energy.

Generalization to four dimensions

- Can one expect a similar story in four dimensions?.

$$I_{eff}[g] = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R_g e^{-\Gamma_K(\Omega)} - 2\,\Lambda\, \mathbf{e}^{-\mathbf{\Gamma}_{\mathbf{\Lambda}}(\mathbf{\Omega}(\mathbf{x}))} \right)$$

- In general, the anomalous dimensions will depend on scale and we have *integrated* anomalous dimensions appearing the action.
- More generally, all standard model fields have nontrivial beta functions and hence a Weyl anomaly and an anomalous dependence on the scale factor.
- Can we compute these systematically?

Quantum effective action at one-loop

- The 1PI action for the classical background fields is given in terms of the trace of the heat kernel in curved spacetime with a covariant regulator.

$$I_{eff} = I + \frac{1}{2}\int_{\epsilon}^{\infty} \frac{d\tau}{\tau} \mathrm{Tr}(\mathrm{K}(\tau))$$

- *A **well-posed** problem but **intractable** in general.*
- For the nonlocal 1PI action we need the heat kernel for ***all*** values of proper time. The Schwinger-deWit expansion is valid for short proper time and gives a Wilsonian action in derivative expansion.

Barvinsky-Vilkovisky curvature expansion

- *A **curvature** expansion and not a derivative expansion*

$$\Box\mathcal{R} \gg \mathcal{R}^2$$

$$\mathrm{TrK}(\tau) = \frac{1}{(4\pi\tau)^{\mathrm{d}/2}} \int \mathrm{d^dx}\sqrt{\mathrm{g}}\ \mathrm{Tr}\Bigg\{ \mathbf{1} + \tau\left(\frac{\mathbf{R}}{6} - \mathbf{E}\right)$$

$$+ \tau^2 \sum_{i,j} \mathcal{R}_i f_{ij}(-\tau\Box_j)\mathcal{R}_j + +\tau^3 \sum_{i=1}^{11} \mathcal{F}_i(-\tau\Box_1, -\tau\Box_2, -\tau\Box_3)\mathcal{R}_1\mathcal{R}_2\mathcal{R}_3(i) +$$

$$+ \tau^4 \sum_{i=12}^{25} \mathcal{F}_i(-\tau\Box_1, -\tau\Box_2, -\tau\Box_3)\mathcal{R}_1\mathcal{R}_2\mathcal{R}_3(i) + \ldots$$

Here $\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3$ are various curvature tensors, $\mathbf{E}$ is the potential. And $\mathcal{F}_i$ are *nonlocal* form factors.

Nonlocal quantum effective action

- The form factors are functions of the type

$$f(x) \equiv \int_0^1 e^{-u(1-u)x} du$$

- Lead to *complicated nonlocal* expressions but give better insight into anomalies. For example, in four dimensions, one can derive the Riegert action (the analog of the Polyakov action) *including extra terms independent of the scale factor*.
- Unfortunately not in the regime of interest in cosmology where every thing is of Hubble scale.

Local renormalization group

(Drummond, Shore, Osborn…)

$$I = I_0 - \int d^d x \sqrt{-g} \sum_i \lambda_i \mathcal{O}_i$$

- There is no reason why physicists in Andromeda galaxy will use the same renormalization scale as us.

$$g^{\mu\nu} T_{\mu\nu}(x) = \frac{\delta I_{eff}[\Omega]}{\delta \Omega} = \beta_i [\mathcal{O}_i]_g(x) \qquad g_{\mu\nu} = e^{2\Omega} \eta_{\mu\nu}$$

- The beta functions are determined by the ***local*** short-time Schwinger-deWit expansion.
- For *Robertson-Walker* background the entire classical dynamics is contained in the scale factor.

Nonlocal action from Weyl Anomalies

- The local RG equation gives the functional derivative for the action in terms of the beta functions that the capture the Weyl anomaly away from a fixed point.

Functionally integrating this equation completely determines the dependence on the scale factor.

- Additional terms with logarithms of the Laplacian that depend on the scale factor.

Terms independent of the scale factor follow from the flat space limit.

Weyl anomalies completely determine the action.

Nonlocal action for quantum electrodynamics

- Integrate out massless charged fermions (as in the early universe) to obtain the nonlocal action:

$$I_{eff} = -\int \frac{1}{4} F^{\mu\nu} \left[\frac{1}{e^2(\mu)} - 2b_i\Omega + b_i \log\left(\frac{-\partial^2}{\mu^2} \right) \right] F_{\mu\nu} \, dx$$

- The logarithms have a spectral representation

$$\log\left(\frac{-\partial^2}{\mu^2} \right) \equiv \int_0^\infty dm^2 \left(\frac{1}{m^2 - \partial^2} - \frac{1}{-m^2 + \mu^2} \right)$$

- We have computed similarly the quantum effective action for all standard model fields.

Comparison with Barvinsky-Vilkovisky

$$I_{eff} = -\frac{1}{4}\int d^4x\sqrt{-g}\left\{\frac{1}{e^2}F_{\mu\nu}F^{\mu\nu} - b_i\left[F^2\frac{1}{\nabla^2}R + F^{\mu\nu}\log\left(\frac{-\nabla^2}{\mu^2}\right)F_{\mu\nu} + \right.\right.$$
$$+ 4R^{\mu\nu}\frac{1}{\nabla^2}\left(\log\left(\frac{-\nabla^2}{\mu^2}\right)T^{cl}_{\mu\nu} - F^{\mu\sigma}\log\left(\frac{-\nabla^2}{\mu^2}\right)F^{\sigma}_{nu} +\right.$$
$$\left.\left. + \ \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}\log\left(\frac{-\nabla^2}{\mu^2}\right)F_{\alpha\beta}\right) - \frac{1}{3}RF^{\mu\nu}\frac{1}{\nabla^2}F_{\mu\nu} + W^{\alpha}{}_{\beta\mu\nu}F_{\alpha}{}^{\beta}\frac{1}{\nabla^2}F^{\mu\nu}\right] +$$
$$\left. + \ 4n_iF^{\mu\nu}F_{\alpha}{}^{\beta}\frac{1}{\nabla^2}W^{\alpha}{}_{\beta\mu\nu}\right\} + \mathcal{O}(\mathcal{R}^4)$$

- All but the first two terms vanish for Weyl-flat metric in agreement with our result to this order.

Results from local RG rely only on local expansion and hence can have larger domain of validity.

Primordial Magnetogenesis

- ***HESS observatory*** observes TeV photons. Scattering with starlight produces electron-positron pairs which will up-scatter CMB radiation to GeV photons.
- *Non-observation* of GeV photons in ***Fermi-LAT*** leads to a ***lower*** bound on the intergalactic magnetic field.

$$B_0(k) \geq 10^{-15}\, Gauss \quad for \quad \frac{a_0}{k} \geq Mpc$$

- Such large scale magnetic field cannot be generated by local processes and must be primordial.

Weyl Anomalies & Primordial magnetogenesis

- Classical maxwell field is Weyl invariant and hence insensitive to the expansion of the universe. With the anomalous dependence on the scale factor, this is no longer true. *The quantum fluctuations of the maxwell field can get amplified during inflation.*
- The answer is dependent on a few parameters like Hubble scale at inflation, scale factor at the exit etc.
- Since the anomalies are universal (determined by the microscopic charge content), these can provide interesting constraints.

(*with Andre Benevides and Takeshi Kobayashi*)

Summary

- The two-dimensional model exhibits an interesting mechanism for dilution of vacuum energy that slows down de Sitter expansion to power law expansion.
- In four dimensions, the nonlocal modification due to these Weyl anomalies can be computed using the local RG for Robertson-Walker metrics. A new and powerful method which needs to be developed.
- These nonlocal quantum effects can have interesting implications, e. g. primordial magnetogenesis and possibly more.