Regional Meeting

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Hebrew University

Jerusalem, Israel

Geometry and Quantum Noise

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a panorama of previous work...

Peres (1984)

Deutsch (1991)

Srednicki (1994)

Maldacena (2001)

Dyson, Kleban & Susskind (2002)

Birmingham, Sachs & Solodukhin (2003)

Barbon & Rabinovici (2003)

Kleban, Porrati & Rabadan (2004)

Festuccia & Liu (2007)

RECENTLY...

Marolf & Polchinski

Shenker & Stanford

Susskind

Balasubramanian, Berkooz, Ross & Simon

Barbon & Rabinovici Fortschr. Phys

- **Introduction- What Does(does not) Geometry capture?**
- **Quantum Noise I - QFT, BH Information**
- **VERY(!) long time correlations. VERY small.**
- **Quantum Noise II- מועד ב-Firewalls?**
- **Geometry and Noise II**
- **Discussion**

BB

N=4 describes also a theory of a string moving in a background a AdS_5 X S^5 And a black hole in AdS_5 X S^5

The AdS/CFT Correspondence

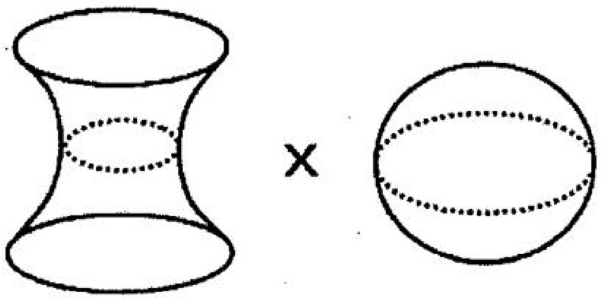
[Maldacena '97]

D=4, N=4 , SUSY Y.M. SU(N)

't Hooft coupling: $\lambda = N g_{\rm YM}^2$ 1/color number: $\frac{1}{N}$ theta angle: $\theta_{\rm YM}$

$\mathcal{N} = 4$ SYM was conjectured to be dual to a string theory:

IIB Superstrings on $AdS_5 \times S^5$



tension: $\frac{R^2}{\alpha'} = \sqrt{\lambda}$ coupling: $g_s = \frac{\lambda}{4\pi N}$ axion: $\langle C \rangle = \theta_{\rm YM}$

- AdS_5 metric

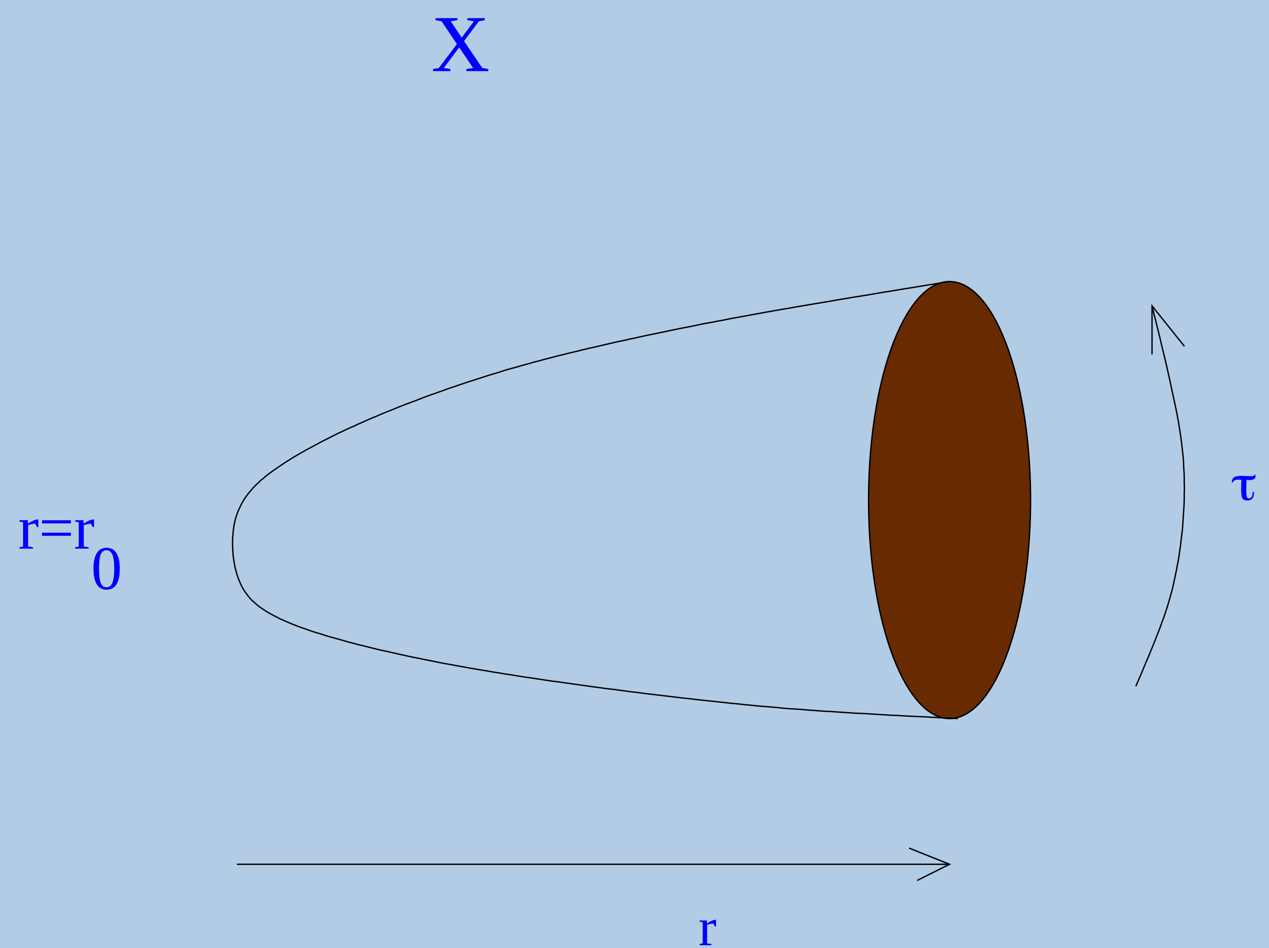
$$ds^2 = -\left(1+\frac{r^2}{R^2}\right)dt^2 + \left(1+\frac{r^2}{R^2}\right)^{-1} dr^2 + r^2\, d\Omega_3^2 + R^2\, d\Omega_5^2$$

- Effective temperature

$$T(r) = \frac{T(0)}{\sqrt{1+r^2/R^2}}$$

- Black Hole in AdS_5 metric

$$\begin{aligned} ds^2 &= -\left(1+\frac{r^2}{R^2}-\frac{M}{Cr^2}\right)dt^2 + \left(1+\frac{r^2}{R^2}-\frac{M}{Cr^2}\right)^{-1} dr^2 \\ &+ r^2\, d\Omega_3^2 + R^2\, d\Omega_5^2 \end{aligned}$$


X
r=r
0
τ
r

- For $T < 1/R$
  Only thermal AdS

- For $T \gtrsim 1/R$
  Thermal AdS plus BH in AdS,
  (actually two Black Holes)

- For $T > 1/R$
  BH dominates

Black Hole Information Paradoxes

- **BH formation paradox**
- **Eternal BH paradox** (Maldacena)

  Tool for CFT $\Longrightarrow$ AdS

- **In Principle**

$$
\begin{array}{ccc}
\text{Initial bulk state} & \Longrightarrow & \text{Initial CFT state} \\
. & & \Downarrow \\
\text{Final bulk state} & \Longleftarrow & \text{Final CFT state}
\end{array}
$$

Instead consider slight deviation from thermal equilibrium on the field theory side

Consider

$$G(t) = Tr\left[\rho A(t) A(0)\right]$$

For very large time scale

$$C(t) = \sum_{mn} \rho_m \, B_{mn} \, B_{nm} \, e^{i(E_m - E_n)t} \ ,$$

Aspects of Long Time Scales in Field Theory

Classical		**Quantum**
Compact Phase Space	$\Longleftrightarrow$	Discrete Spectrum
Volume Conservation	$\Longleftrightarrow$	Unitarity

Then, If

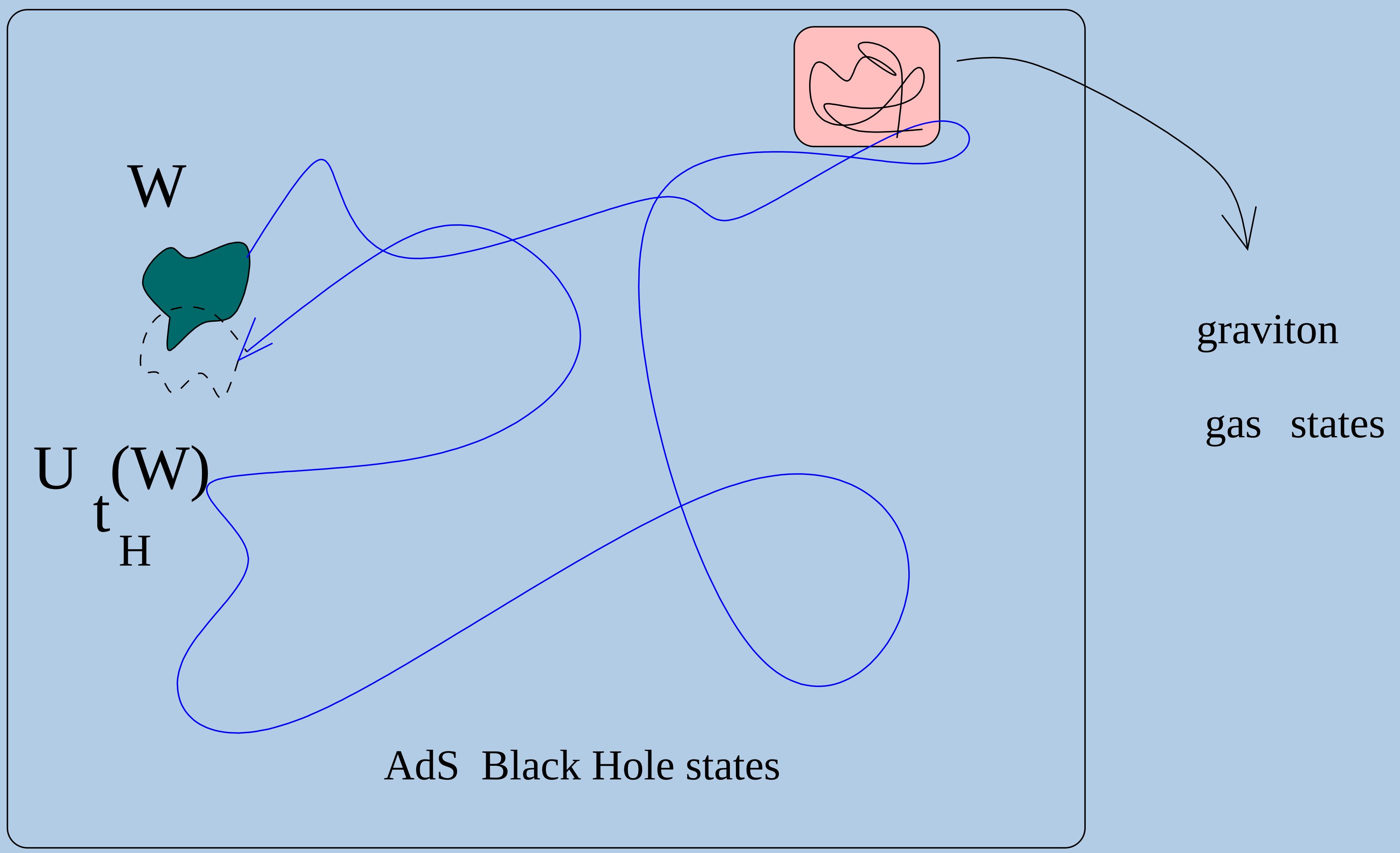
$$G(t_0) =< \theta_1(t_0, x_1)|\theta_2(0, x_2) >$$

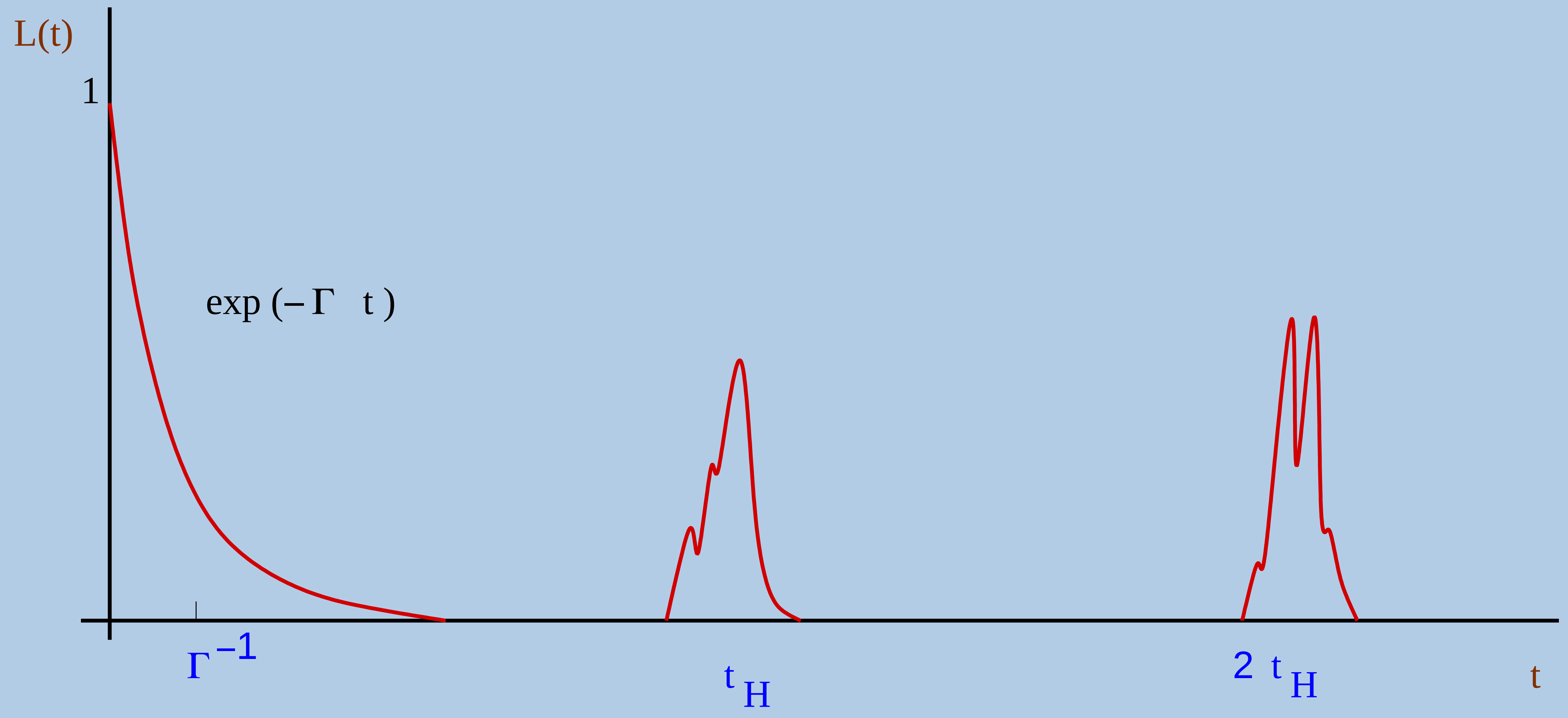
for any ϵ there is a $t^P(\epsilon)$ such that

$$|G(t^P(\epsilon)) - G(t^0)| < \epsilon$$

You See It All!

Energy >> 1/R (P , Q)





L(t)
1
exp (−Γ t)
Γ −1
t H
2 t H
t

$$\overline{C(t)} \equiv \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau dt\, C(t) = \sum_m \rho_m |B_{mm}|^2 \; .$$

An estimate gives a normalisation Exp(-S) times

a number. So the decay must stop, the discrete

nature of the spectrum felt and the magnitude

is Exp(-S) *

Γ is not universal

$$t_H = \frac{1}{< w >} \qquad < w > = < E_i - E_j >$$

$$< w > \sim \frac{\Gamma}{\Delta n_\Gamma},$$

Δn_Γ is the number of states in a band of width Γ.

$$t_H \sim \frac{1}{\Gamma} \exp(S(\beta))$$

$$t^P(\epsilon) \sim \exp(f(\epsilon) \exp S) \qquad |G(t^P(\epsilon)) - G(0)| < \epsilon$$

$$C(t) = \sum_{mn} \rho_m \, B_{mn} \, B_{nm} \, e^{i(E_m - E_n)t} \ ,$$

Estimate of Poincare Time

Consider “clocks” Exp(iEt)

t=1/v   v = (Δα/2π)Neff

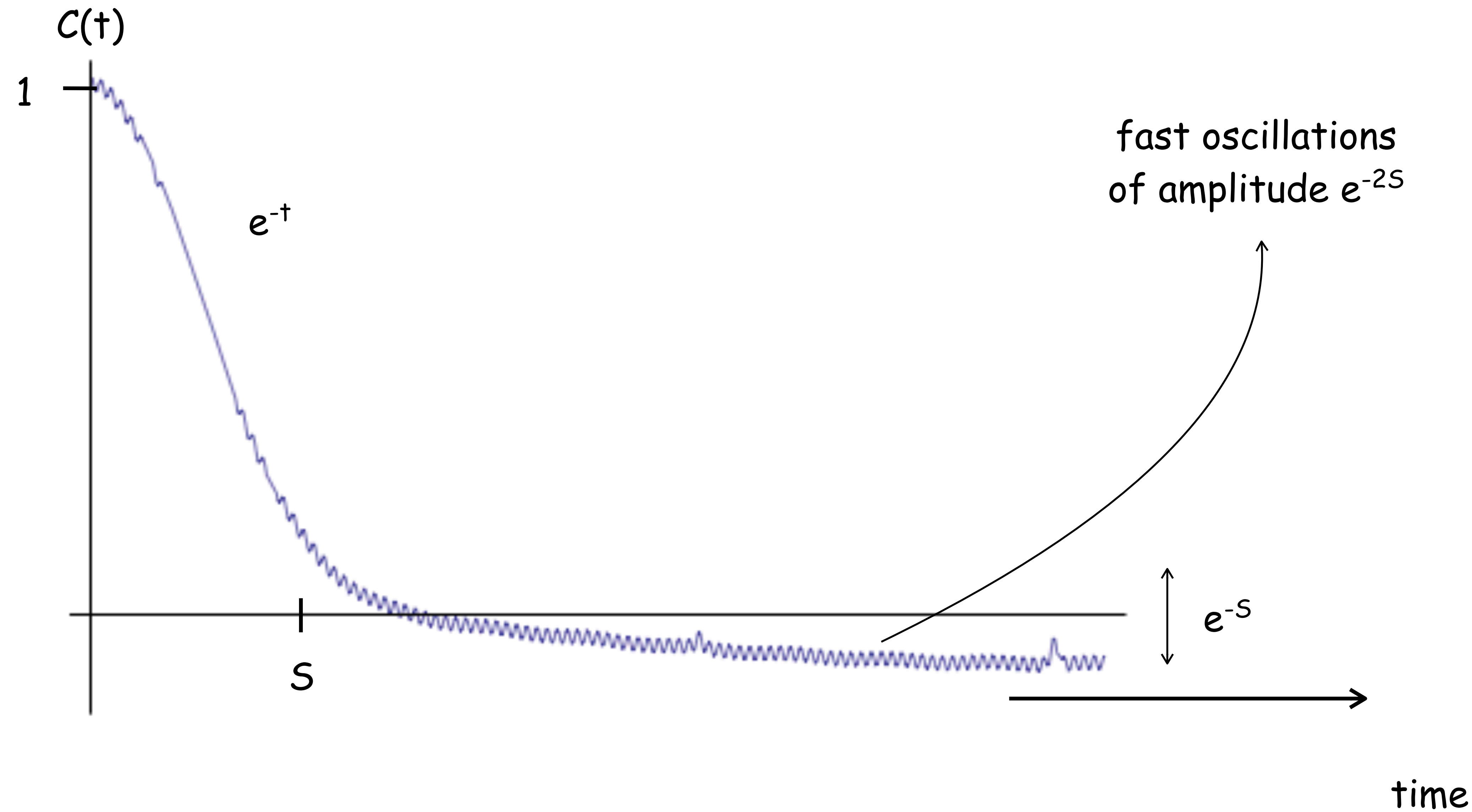
Et ~exp (Neff log(2π/Δα))~ exp(exp(S*log(2π/Δα)))

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



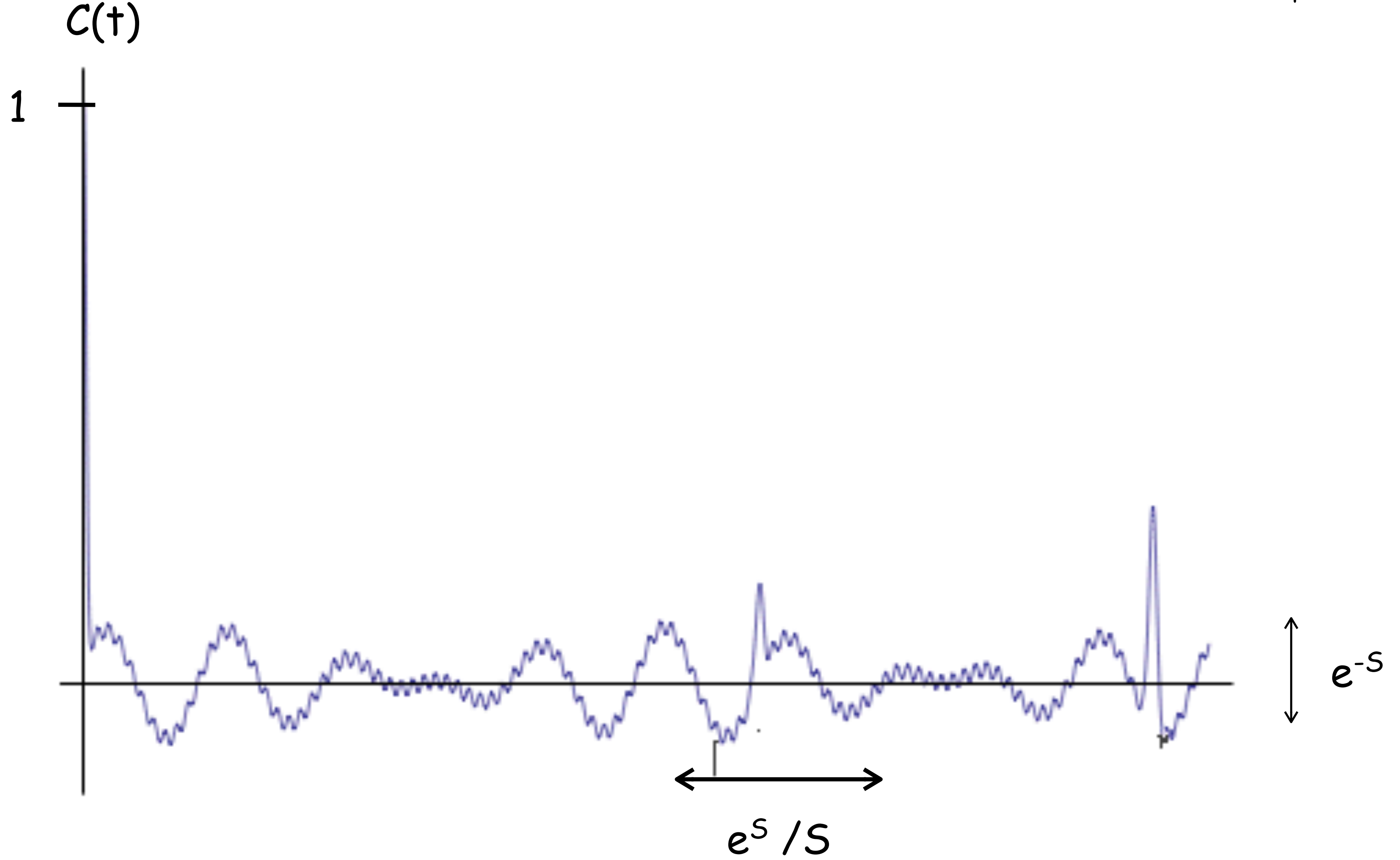
Page time scale

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



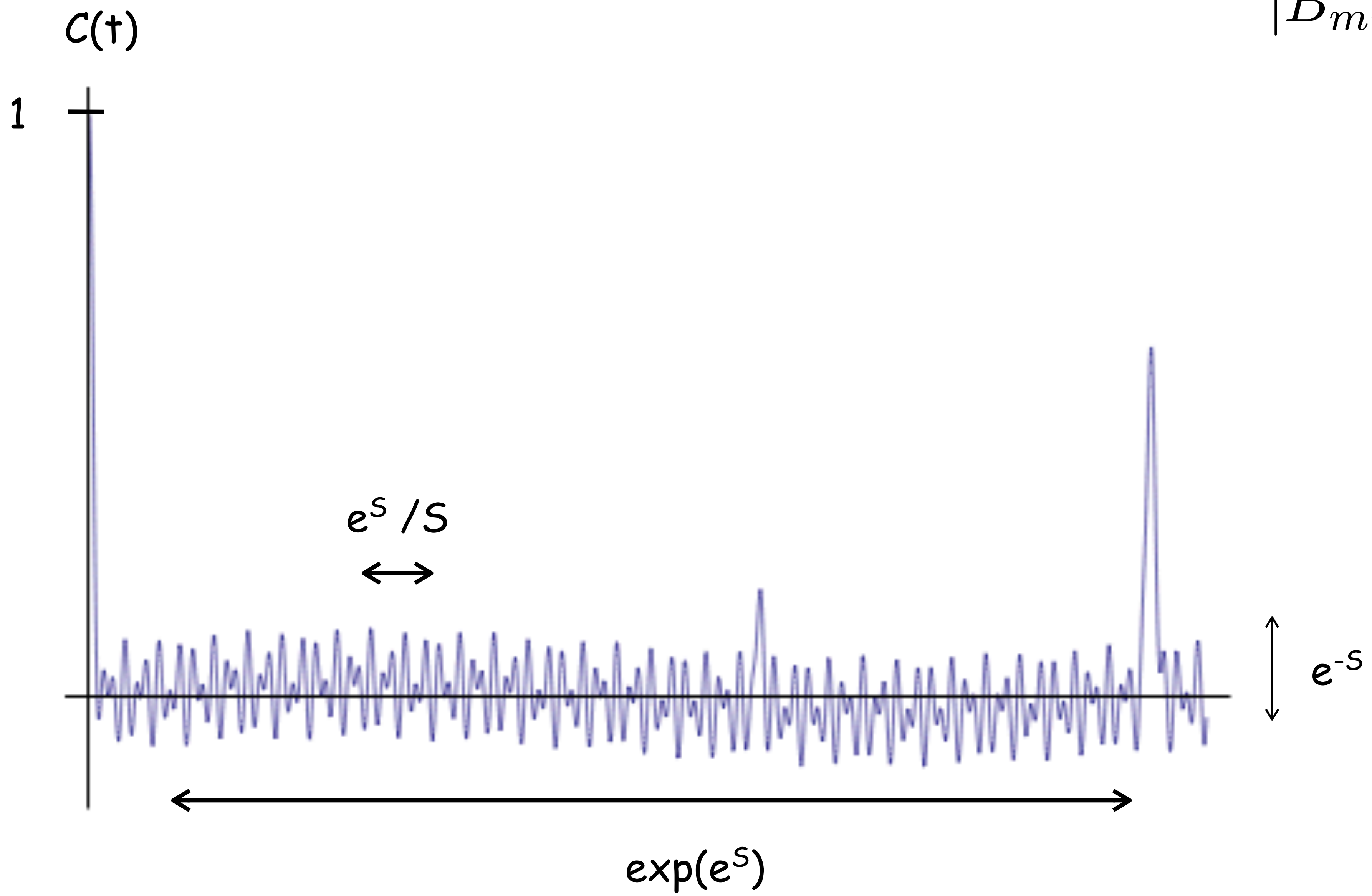
Heisenberg time scale

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



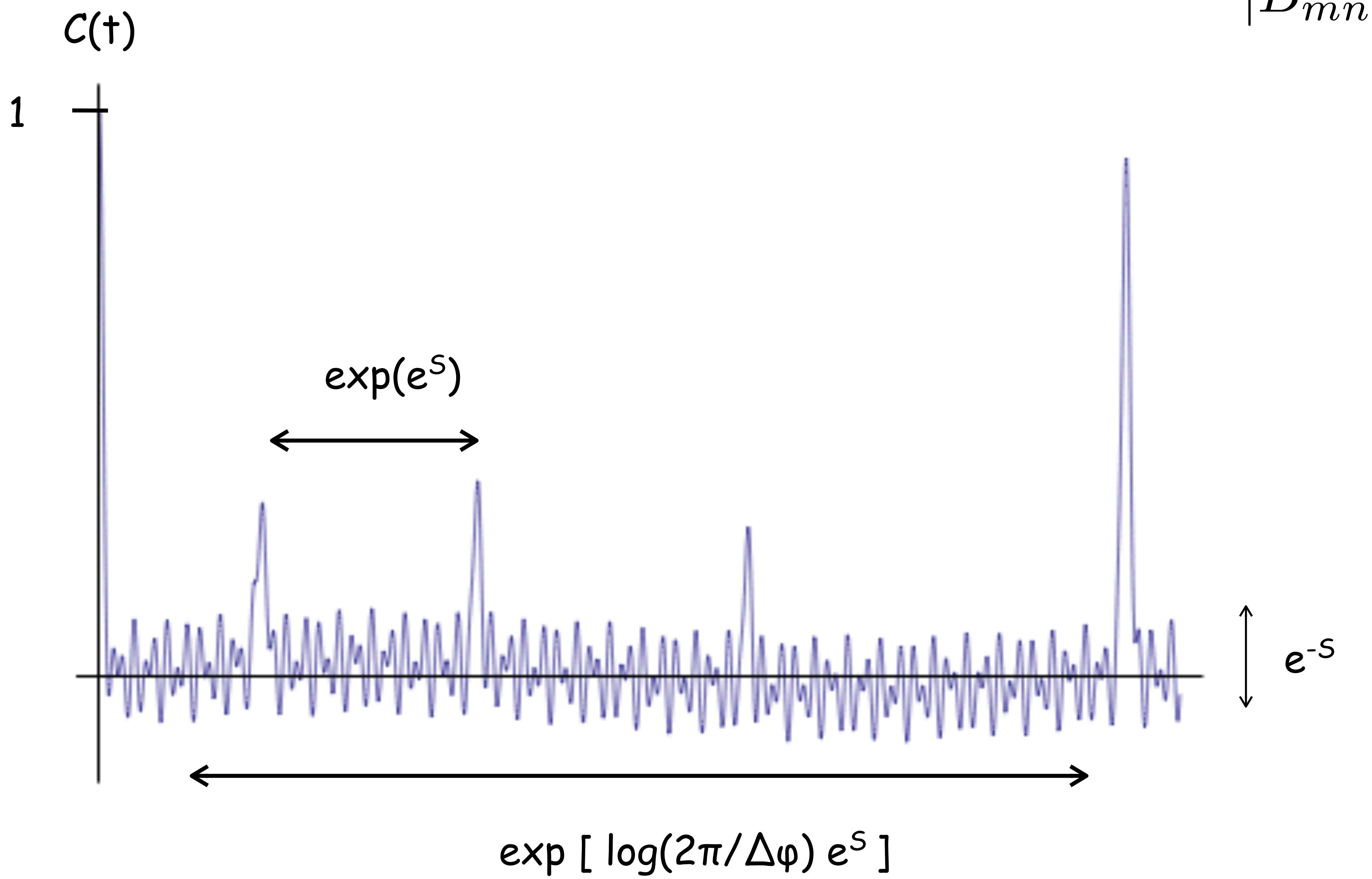
Poincaré time scale

$$C(t) = \sum_{mn}^{e^{2S}} \rho_m \, |B_{mn}|^2 \, e^{i(E_m - E_n)t}$$

$$C(0) = 1$$

$$\rho_m \sim e^{-S}$$

$$|B_{mn}| \sim e^{-S/2}$$



detailed Poincaré time scale

Some Proportion -Page Times S

In units of the Universe's Life UL

Page time for a BH the size of a proton 10^10 ULs

Page time of a BH in 10^9 sm Quasar 3 km 10^87 ULs

This is just S!!!

One reaches for Poincare 10 to the 10 five times…

Summary:

Time Scales related by Log

Log S - Scrambling time BH, 1/S boundary(UP,T)

S-Page time, end of decay

Exp(S)- Heisnberg time

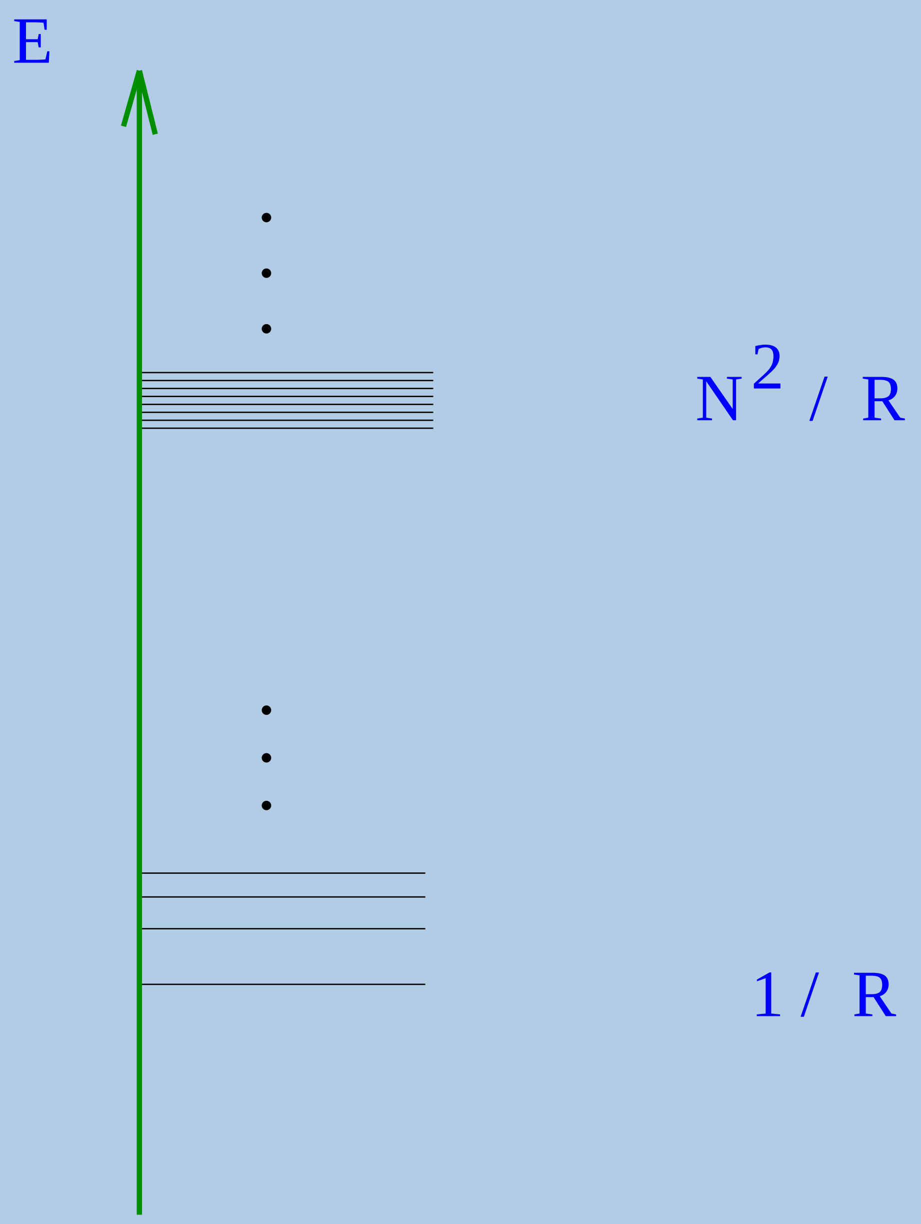
Exp(Exp(#S))- Poincare time

Consider

$$L(t) = \left| \frac{G(t)}{G(0)} \right|^2$$

$$\bar{L} = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt\, L(t)$$

The CFT is unitary and has a Gap


E
N^2 / R
$1 / R$

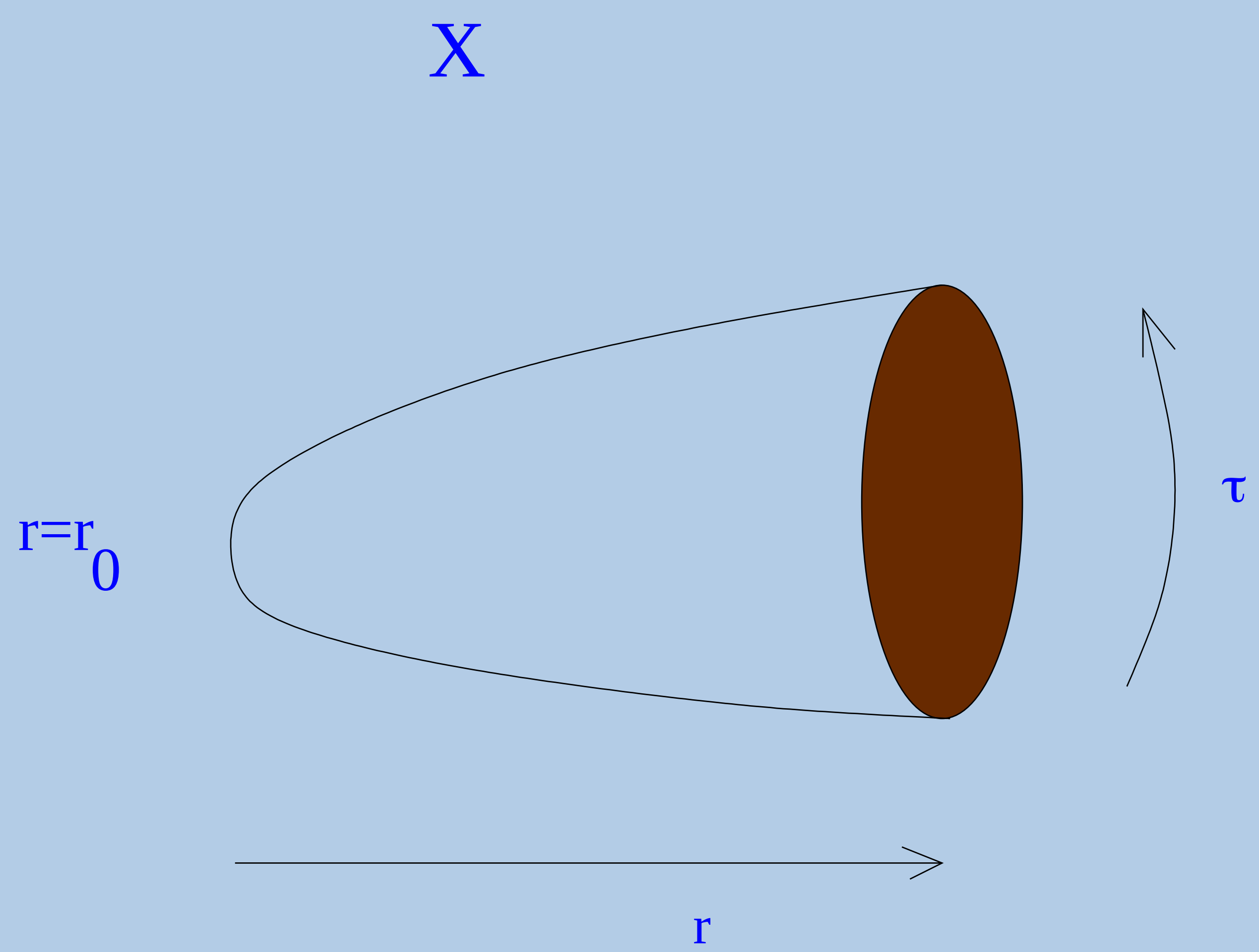
$$\bar{L} \sim \frac{\Delta L}{\Gamma t_H} \sim \exp(-S(\beta))$$

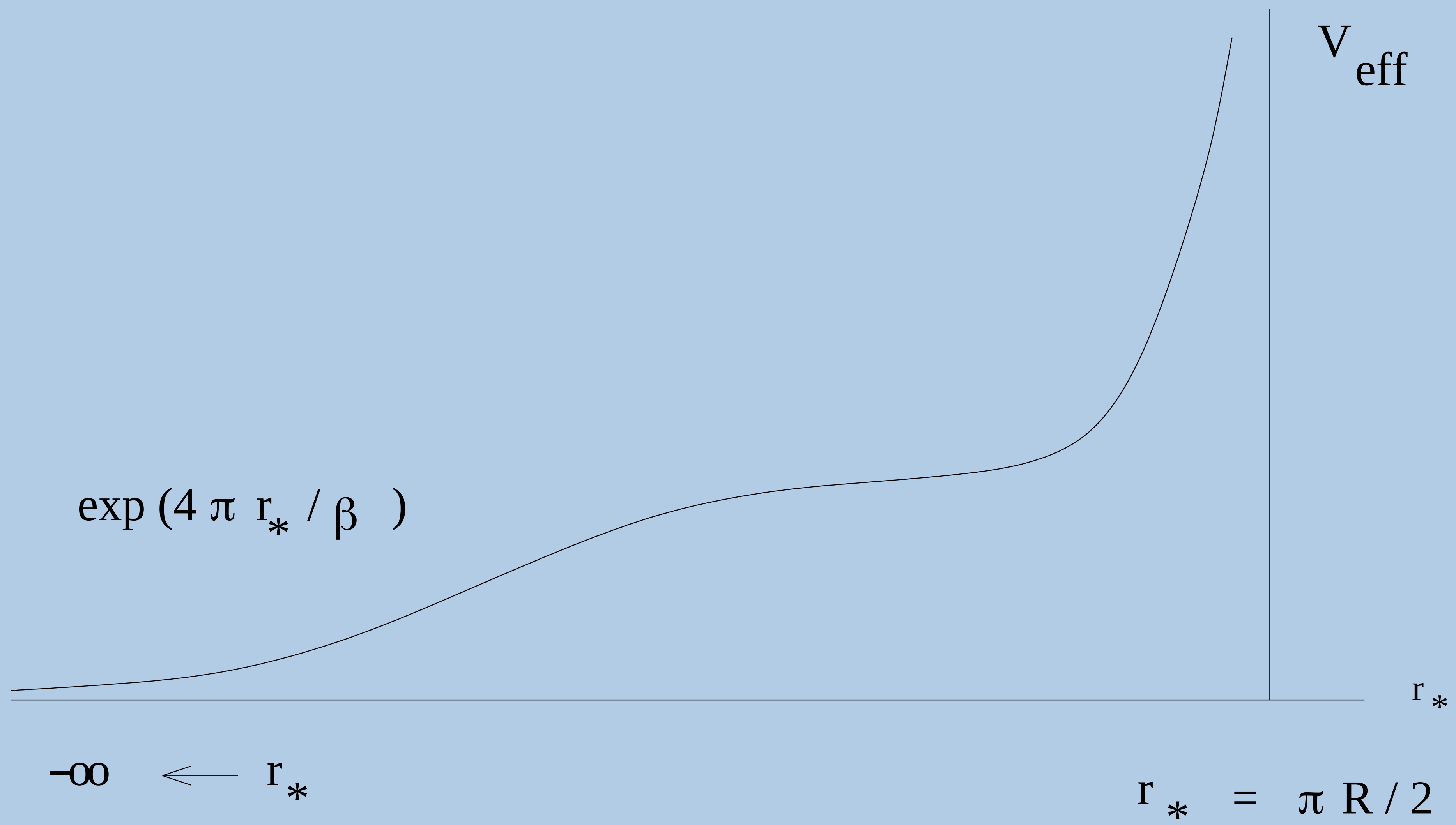
,

$$\bar{L} \sim \exp(-N^2 \dots) \sim \exp\left(-\frac{1}{G_N} \dots\right)$$

Non Perturbative from Gravity Point of View

For BH background $\bar{L} \to 0$, Reason:
No Gap in the presence of a BH.


X
r=r0
τ
r

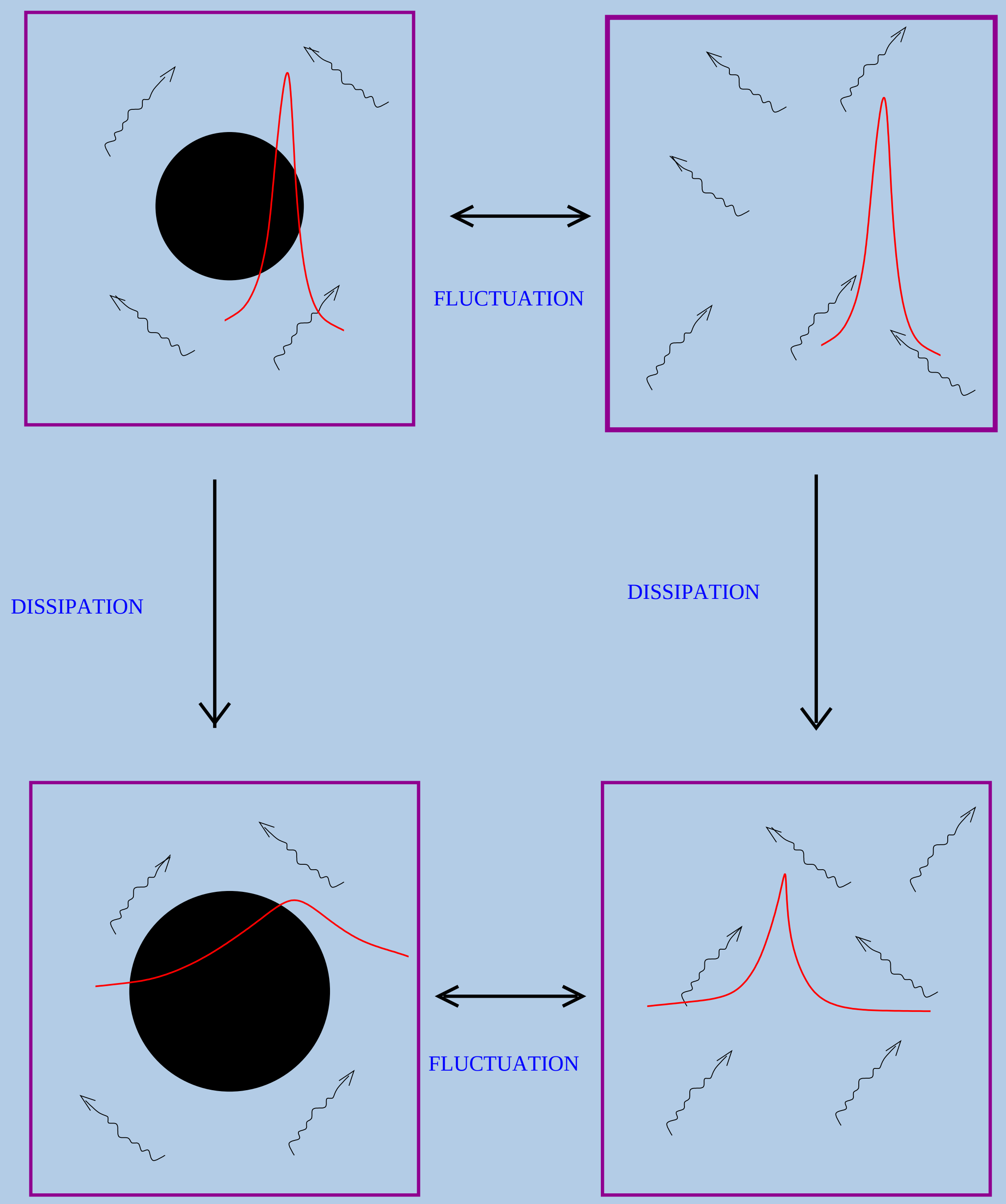

V_{eff}
$\exp(4\pi \; r_* / \beta)$
r_*
$-\infty \leftarrow r_*$
$r_* = \pi R / 2$

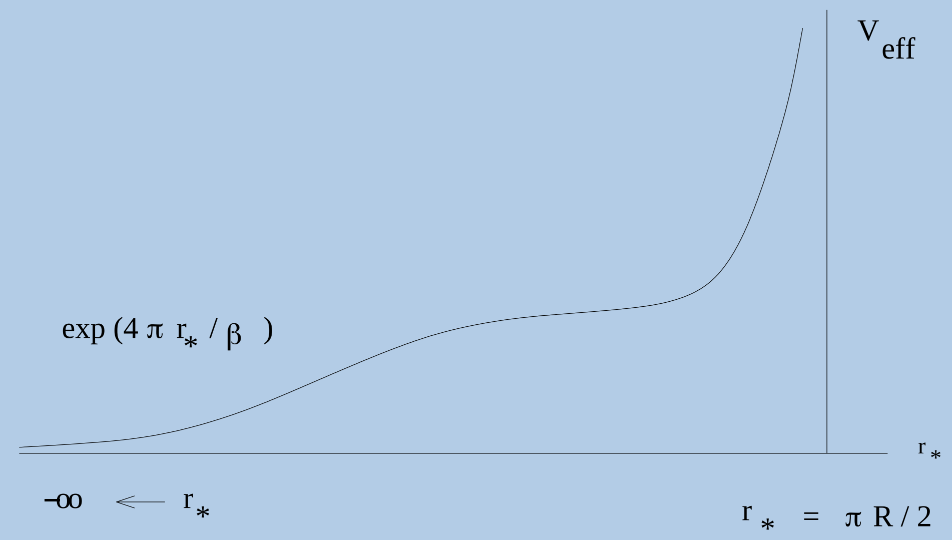
$$\bar{L}_{CFT} = \exp(-S) \Rightarrow \bar{L} = \exp(-S)$$

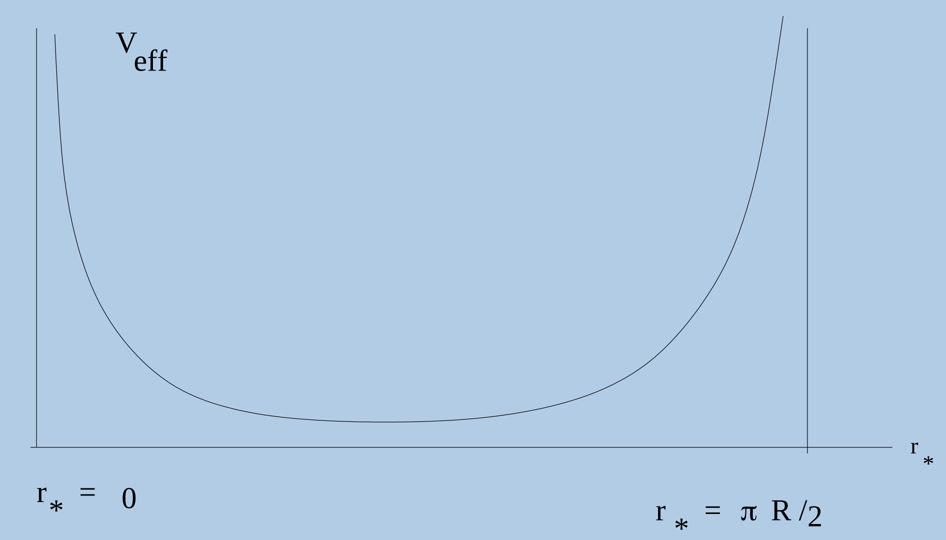
But it seems $\bar{L}_{Bulk} = 0$

Contradiction ?

Poincarè Recurrences and Topological Diversity

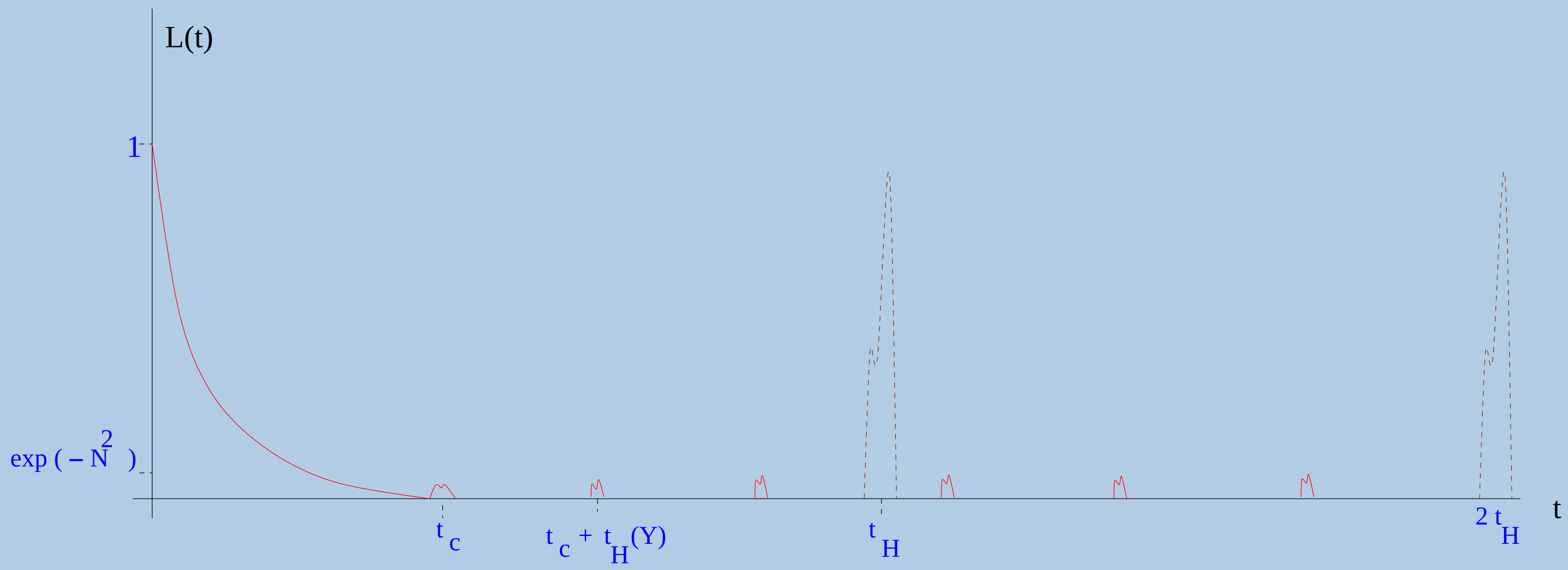

FLUCTUATION
DISSIPATION
DISSIPATION
FLUCTUATION





In a Thermal AdS Background a gap is formed and now

$$\bar{L}_{Bulk} \approx \exp(-S) > 0$$



L(t)
1
exp (− N 2)
t
t c
t c + t H (Y)
t H
2 t H

- $\bar{L}$ **reasonable course grained**
- $L(t)$ **not reproduced**
- **Stretched horizon, Brick Wall?**

Conclusions

- **The Burden of Proof That a Well Defined Information Paradox Exists Shifts to Claimer**
- **Topological Diversity is Required**
- **String theory is Quite a Formidable Bastion of Consistency**

Geometry Reproduces Average Result

Geometry May Well Miss some Exp(-S)

Features.

Firewall

ER= EPR

???

NOISE

The Noise is defined by

$$|\text{noise}| \equiv \left[\overline{|C(t)|^2}\right]^{1/2}$$

$$\overline{|C(t)|^2} = \sum_{mnrs} \rho_m \rho_r |B_{mn}|^2 \, |B_{rs}|^2 \, \overline{e^{i(E_m - E_n + E_s - E_r)t}} \, .$$

B has no diagonal elements so

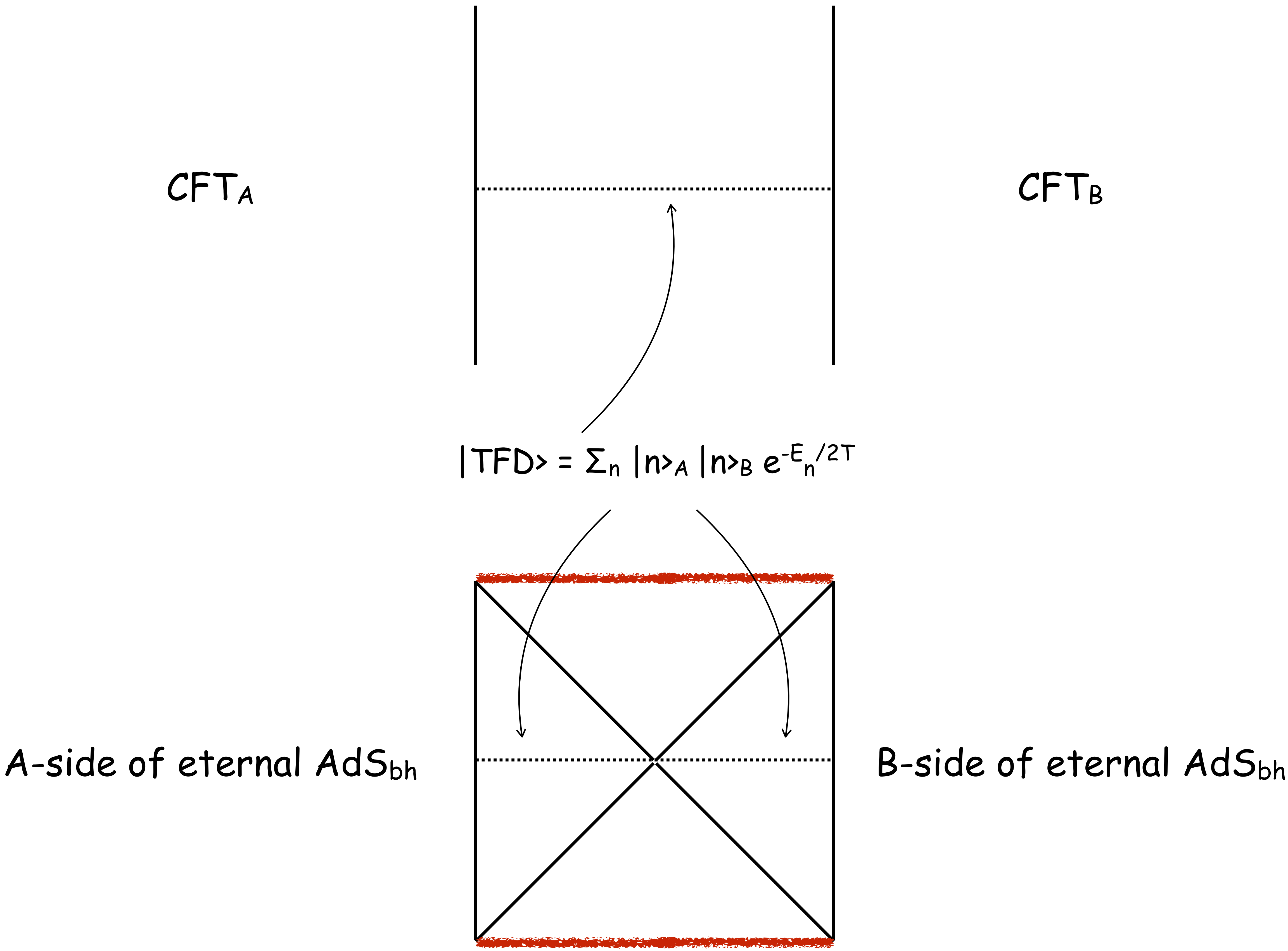
$$E_m = E_r \text{ and } E_n = E_s$$

$$|\text{noise}| = \left[\sum_{mn} \rho_m^2 \, |B_{mn}|^4\right]^{1/2}$$

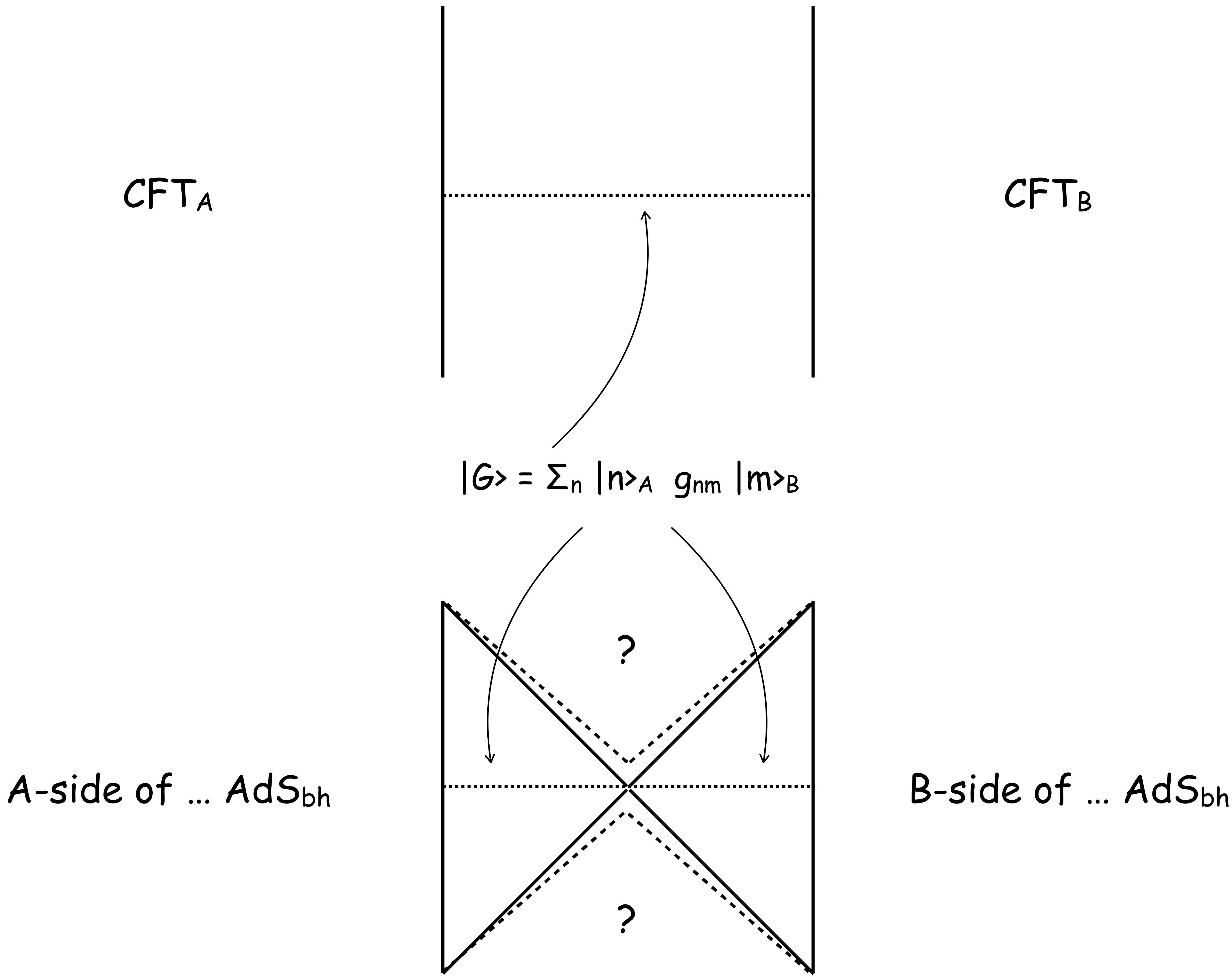
$$|\text{peak}| \sim |C(t)|_{\max} = \sum_{mn} \rho_m \, |B_{mn}|^2$$

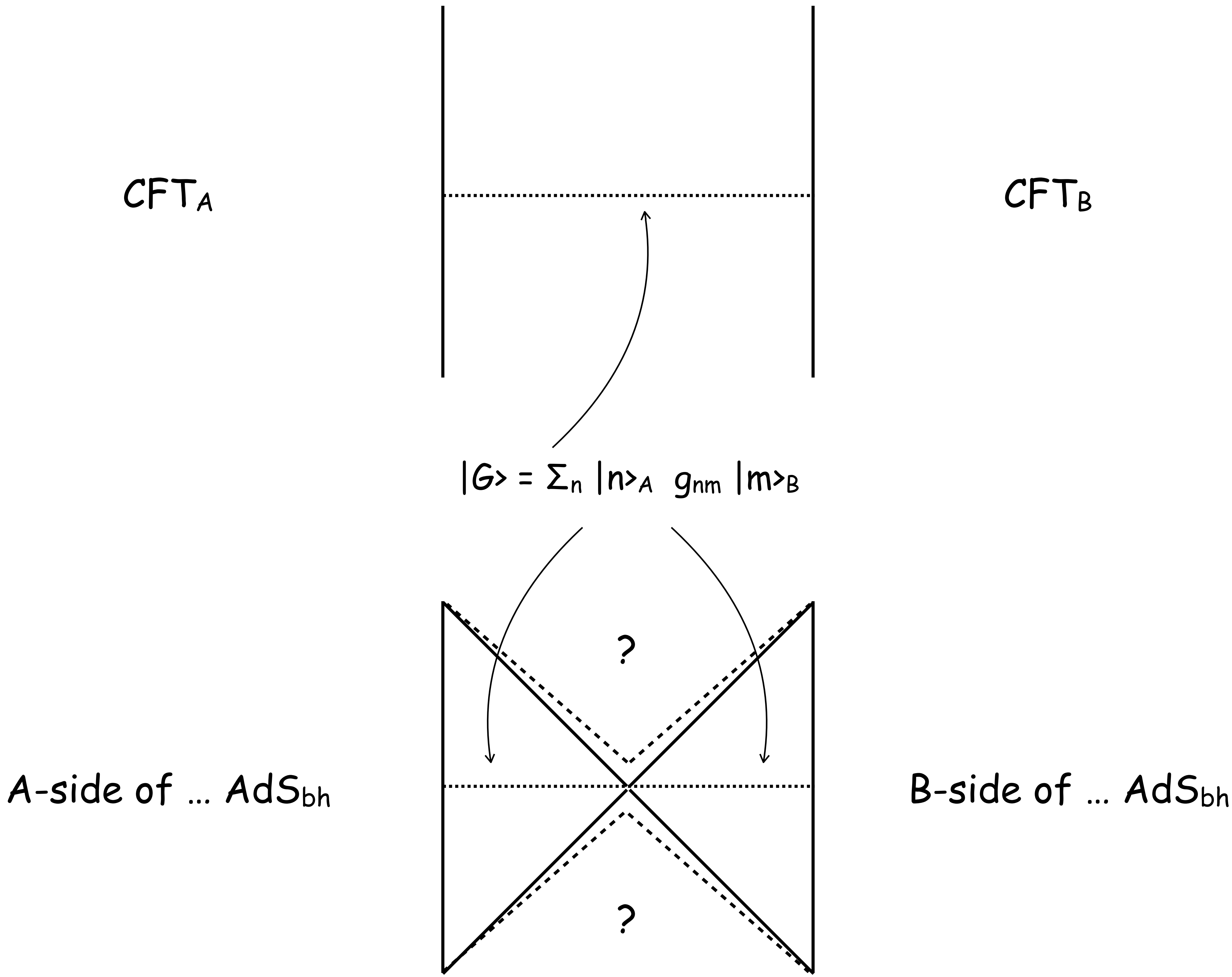
$$\frac{|\text{noise}|}{|\text{peak}|} = \left[\frac{\sum_{mn} \rho_m^2 \, |B_{mn}|^4}{\left(\sum_{mn} \rho_m \, |B_{mn}|^2\right)^2}\right]^{1/2} .$$

= Exp(-S) SQRT{Exp(2S)/Exp(4S)}



CFT_A
CFT_B
|TFD> = Σ_n |n>_A |n>_B e^{-E_n/2T}
A-side of eternal AdS_bh
B-side of eternal AdS_bh


CFT_A
CFT_B
|G> = Σn |n>A gnm |m>B
?
?
A-side of ... AdSbh
B-side of ... AdSbh


CFT_A
CFT_B
$|G\rangle = \Sigma_n \, |n\rangle_A \;\; g_{nm} \, |m\rangle_B$
?
?
A-side of … AdS_{bh}
B-side of … AdS_{bh}

$C(t=0) \sim 1$

- Noise~ Exp(-S)

More General Case

- **Two different operators.**
- **If the density matrix is diagonal in the energy basis-there is only dependence on time differences.**
- **Otherwise there is a dependence on both times.**
- **Even for a diagonal density matrix there is no generic peak at t=0**

$$G_{BB'}(t,t') = \mathrm{Tr}\left[\rho\, B(t)\, B'(t')\right] = \sum_{mnr} \rho_{mn}\, B_{nr}\, B'_{rm}\, e^{i(E_n-E_r)(t-t')}\, e^{-i(E_m-E_n)t'}$$

For enough entanglement one can construct the Alice surrogate

$$\langle\, A(t_A)\, B(t_B)\,\rangle_G = \mathrm{Tr}\,\left[\rho\, B(t_B)\, B_{A(t_A)}\right]\ ,$$

$$\left(B_{A(t_A)}\right)_{\alpha\beta} = \sqrt{\rho_\alpha}\,\left(\Omega_A^\dagger\, A(t_A)\,\Omega_A\right)_{\beta\alpha}\,\frac{1}{\sqrt{\rho_\beta}}$$

$$\langle A(t_A)B(t_B)\rangle_{\mathrm{TFD}} = \mathrm{Tr}\,\left[\rho_T\,\widetilde{A}(t_A - i\beta/2)B(t_B)\right]$$

Representative Dynamics and Observables

- **Dynamics- “nearly” Integrable.**
- **Operators- Fields of Quasi Particles- Sparse- Gravitons in Thermal AdS**

Representative Dynamics and Observables

- **Dynamics- Chaotic.**
- **Operators Bs- They do not commute with the Hamiltonian, H, moreover their eigenfunctions are uncorrelated with those of H.**
- **U is "Pseudo Random"- Black Hole**

$$B_{mn} = (U\, b\, U^\dagger)_{mn} = \sum_\alpha b_\alpha\, U_{m\alpha}\, (U_{n\alpha})^*$$

ETH Observable

$$B_{mn} = \bar{B}(\bar{E})\delta_{mn} + b(\bar{E},\omega)\, e^{-S(\bar{E})/2}\, R_{mn}$$

$$\bar{E} = \tfrac{1}{2}(E_m + E_n)\ , \quad \omega = E_m - E_n$$

$$B_{mn} = \bar{B}(\bar{E})\delta_{mn} + b(\bar{E},\omega)\, e^{-S(\bar{E})/2}\, R_{mn}$$

$$\sum_\alpha |U_{\alpha n}|^2 = 1$$

$$B_{mn} = (U\, b\, U^\dagger)_{mn} = \sum_\alpha b_\alpha\, U_{m\alpha}\, (U_{n\alpha})^*$$

U elements are of order Exp(-S/2).

Off diagonal elements are Exp(-S/2), random walk.

Noise Estimates

- Bob’s Noise(one sided)
- EPR Noise(two sides)
- Several Narrow bands Noise.
- Thermal Gas Noise

Bob’s Noise

ETH, one “narrow” band with thermal width T

Constant functions in the band.

ETH*ETH=ETH

$$(R^{B})_{mn}\,(R^{B'})_{rs} = (\mathcal{D}_{BB'})_{mn}\,\delta_{ms}\,\delta_{nr} + (\text{erratic})_{mnrs}$$

First term is “smooth” in m,n

Second term gives the leading answers

Noise from the Smooth Part.

$$G^{(s)}_{BB'}(t) \sim |b\,b'|\,e^{-S}\sum_{\alpha}\rho_{\alpha}\,\sum_{mn}(\Omega^{\dagger}_{B})_{\alpha m}\,e^{i(E_m - E_n)t}\,(\Omega_B)_{m\alpha}$$

$$|\text{noise}^{(s)}|_{\text{pure diag}} \sim |b\, b'|\, e^{-S/2}$$

$$|\text{noise}^{(s)}|_{\text{pure non-diag}} \sim |b\, b'|\, e^{-S}$$

$$|\text{noise}^{(s)}|_{\text{mixed diag}} \sim |b\, b'|\, e^{-S}$$

$$|\text{noise}^{(s)}|_{\text{mixed non-diag}} \sim |b\, b'|\, e^{-3S/2}$$

Noise From the Erratic Component-Dominates.

$$G^{(e)}_{BB'}(t) \sim |b\, b'|\, e^{-S/2} \sum_{\alpha} \rho_{\alpha} \left(\Omega^{\dagger}_{B}\, R_{BB'}\, \Omega_{B} \right)_{\alpha\alpha} \sim |b\, b'|\, e^{-S/2} \sum_{\alpha} \rho_{\alpha}\, \left(R_{\Omega^{\dagger} BB' \Omega} \right)_{\alpha\alpha}$$

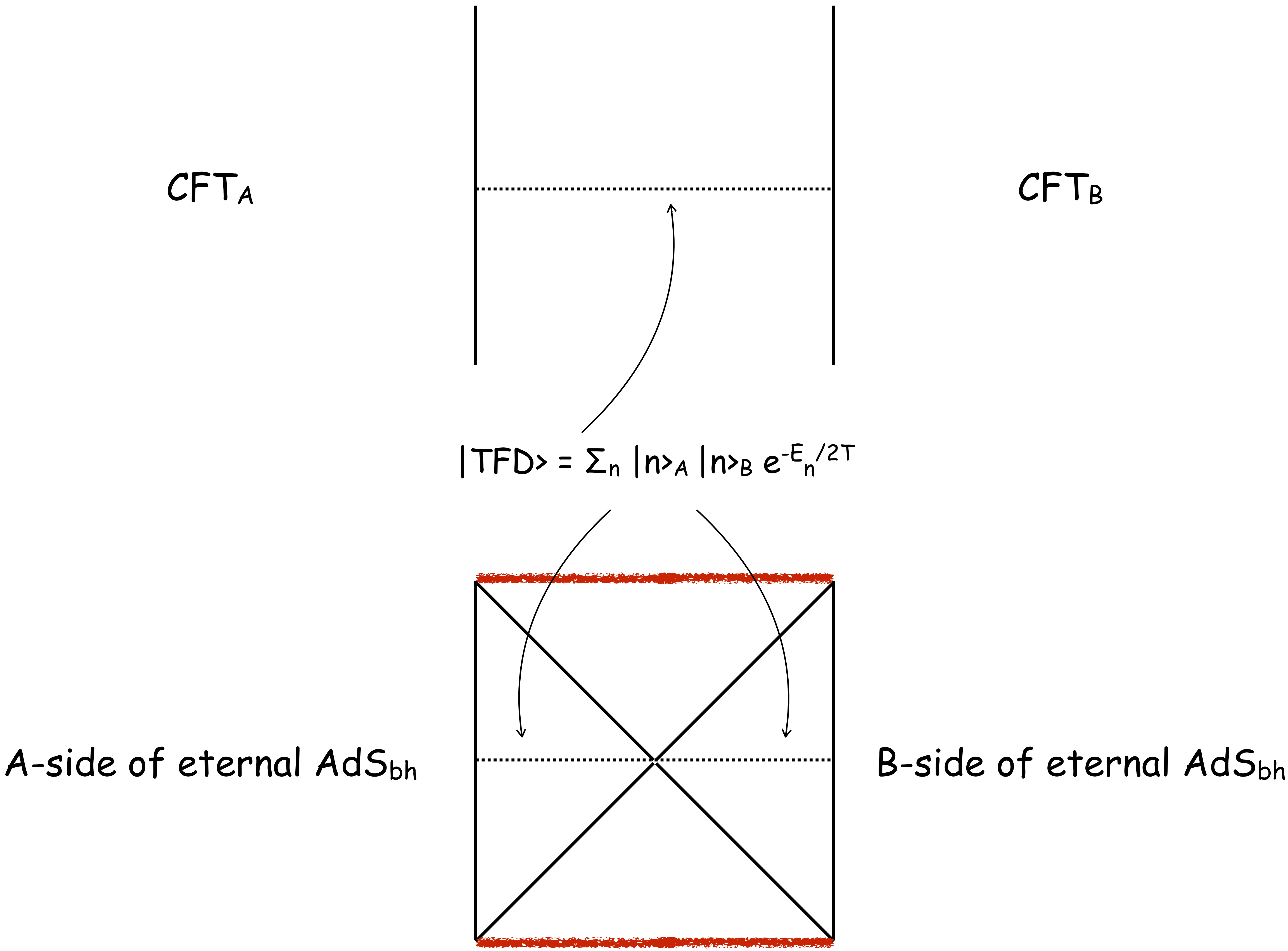
$$|\mathrm{noise}|_{\mathrm{mixed}} \sim |b\, b'|\, e^{-S} \ , \qquad |\mathrm{noise}|_{\mathrm{pure}} \sim |b\, b'|\, e^{-S/2}$$

Does NOT depend on the alignment of B!

EPR Noise

$$G_{AB}(t) = \sum_{\alpha\beta} \sqrt{\rho_\alpha \rho_\beta} \left(\Omega_A^\dagger \, A(t) \, \Omega_A \right)_{\alpha\beta} \left(\Omega_B^\dagger \, B(0) \, \Omega_B \right)_{\alpha\beta}$$

EPR Noise
For the Diagonal Term



CFT_A
CFT_B
$|TFD\rangle = \Sigma_n |n\rangle_A |n\rangle_B e^{-E_n/2T}$
A-side of eternal AdS_{bh}
B-side of eternal AdS_{bh}

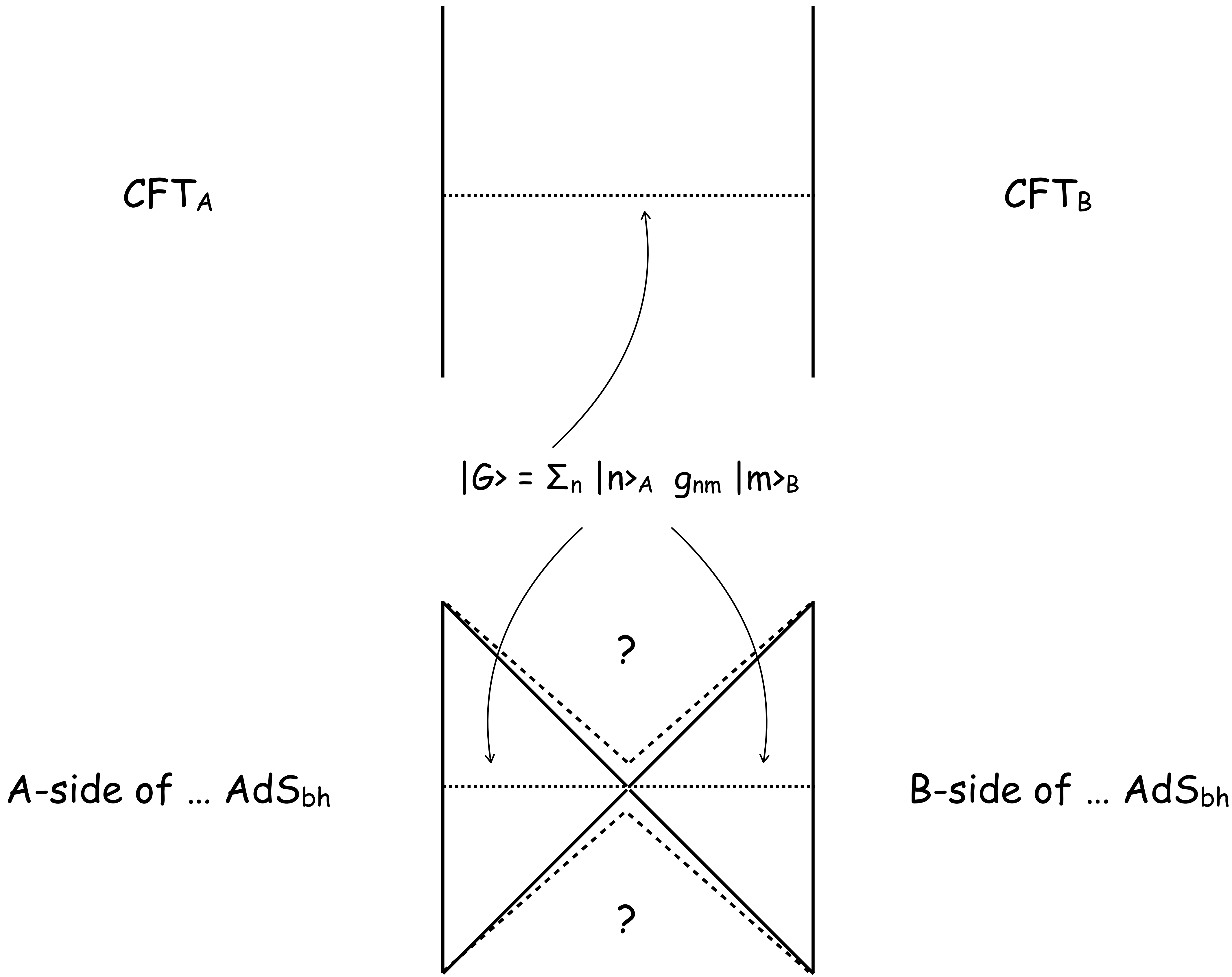
EPR Noise

For the Diagonal State

Value of the Correlator at t=o
O(1) Peak as a “Geometry”

The Noise is

abExp(-S)


CFT_A
CFT_B
$|G\rangle = \Sigma_n \, |n\rangle_A \; g_{nm} \, |m\rangle_B$
?
?
A-side of … AdS_{bh}
B-side of … AdS_{bh}

EPR Noise

For a non diagonal state

Value of the correlator at t=o
O(abExp(-S)) peak "NOT" as a "Geometry"

The noise is HOWEVER again

abExp(-S)

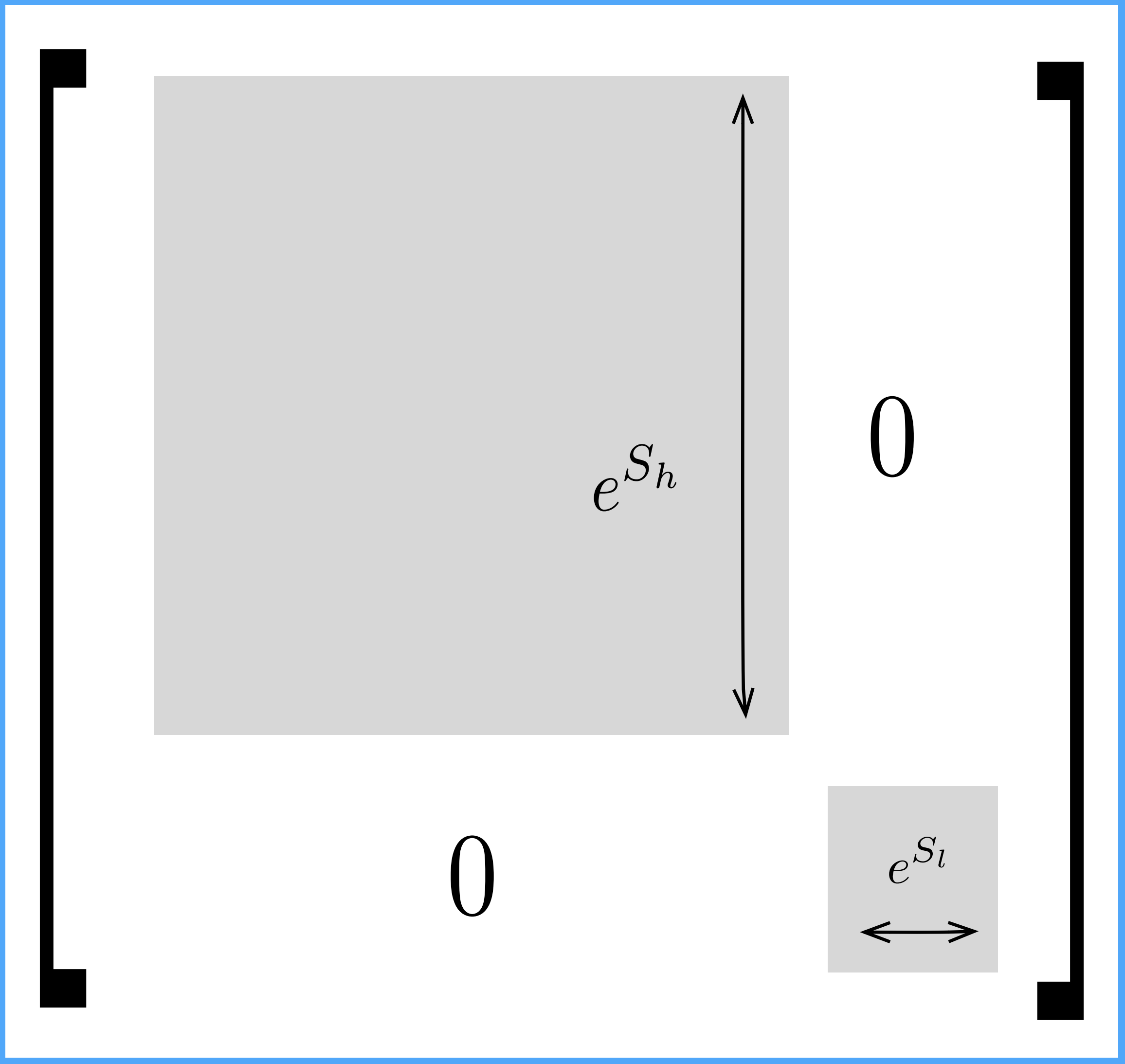
This does NOT depend on the amount of

Entanglement.

To ensure O(1) amplitude for the noise at time, t
One needs a state dependent condition.

$$(R^A)_{mn}\,(R^B)_{rs} \sim (\Omega_A\,\Omega_B^T)_{mr}\,(\Omega_A^*\,\Omega_B^\dagger)_{ns} + (\mathrm{erratic})_{mnrs}$$

Several Bands
Large gap, large difference in entropy



The Lower(est) Band Dominates.

$$\rho = \sum_b p_b \, \rho_b = \sum_b p_b \, e^{-S_b} \mathbf{1}_b \; , \qquad \sum_b p_b = 1$$

$$C(t) \approx \sum_b p_b \, C_b(t) \qquad C_b(t) \sim |\mathrm{noise}|_b \, f_b(t)$$

$$C(t) \sim \sum_b |bb'|_b \, p_b \, e^{-S_b} \, f_b(t)$$

$$p_b \Big|_{\mathrm{canonical}} = \frac{e^{-I_b(\beta)}}{Z(\beta)}$$

The Lowest Energy Band Dominates

$$Z(\beta) \equiv \sum_b e^{-I_b(\beta)} \text{ and } I_b(\beta) = \beta E_b - S_b$$

$$e^{-I_b(\beta)} e^{-S_b}$$

$$\exp(-\beta E_b)$$

*** For bands which are all quasi integrable, the noise is determined by the thermodynamical dominant.**

Thermal Gas Noise

$$B_1 = \frac{1}{L^{\frac{d-1}{2}}} \sum_s \left(b_s \, a_s + b_s^* \, a_s^\dagger \right)$$

$$\langle B_1(t) B_1(0) \rangle_{\text{gas}} = \frac{1}{L^{d-1}} \sum_s \left[(1 + f(\omega_s)) |b_s|^2 \, e^{-i\omega_s t} + f(\omega_s) |b_s|^2 \, e^{i\omega_s t} \right] \; + \text{inter}$$

$$f(\omega_s) = (e^{\beta \omega_s} - 1)^{-1}$$

Thermal Gas Noise-Large

$$\overline{|\langle B_1(t)B_1(0)\rangle_{\text{gas}}|^2} \sim \frac{1}{L^{2d-2}} \sum_{\omega_s<T} \left(1+2f(\omega_s)+2f(\omega_s)^2\right)|b_s|^4 \sim L^{2-2d}\,(LT)^{d-2}$$

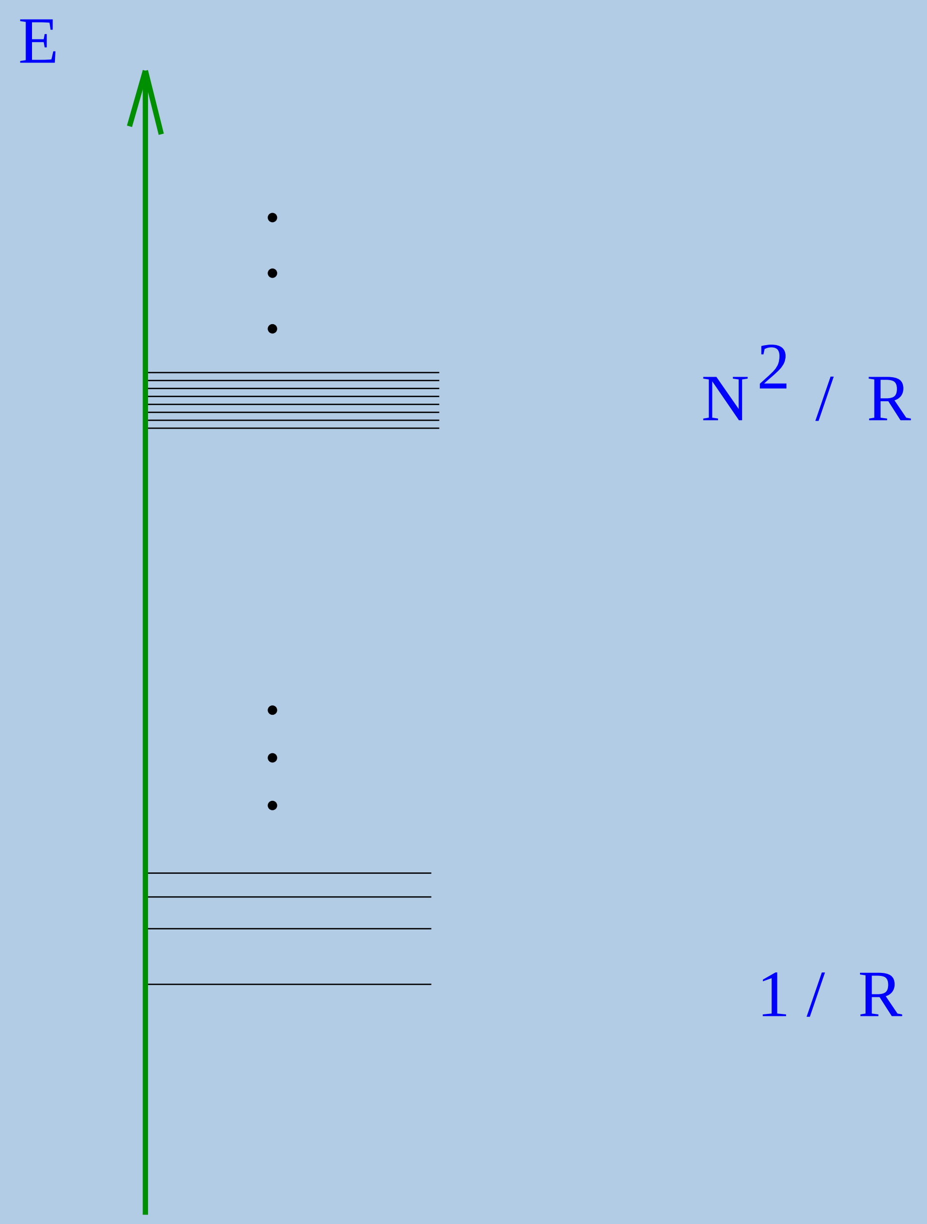
$$\frac{|\text{noise}|}{|\text{peak}|} \sim \frac{1}{(LT)^{d/2}} \sim \frac{1}{\sqrt{S_{\text{gas}}}}$$

The peak scales as $\langle B_1^2\rangle \sim L^{1-d}(LT)^{d-1}$.

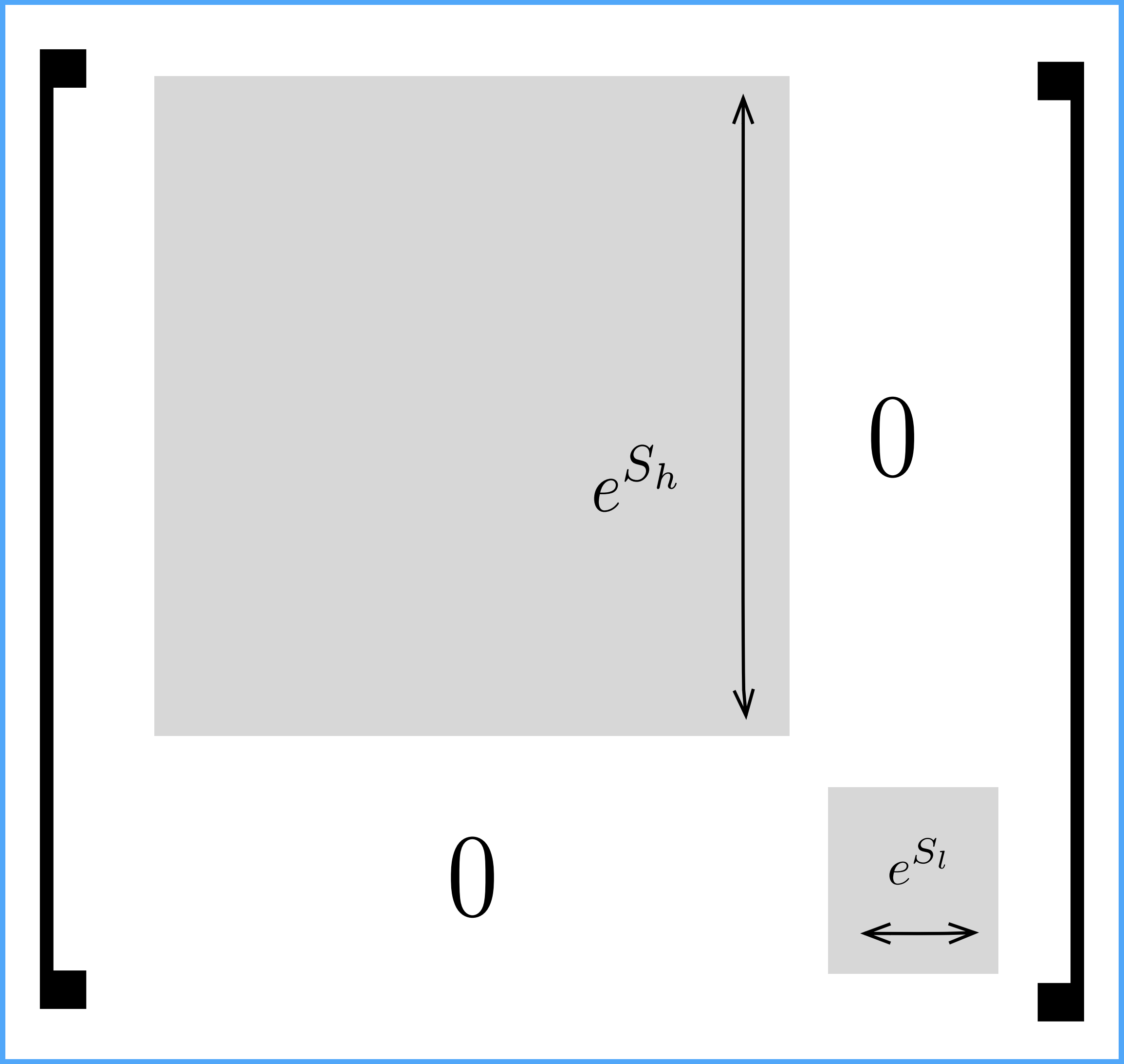
- **Geometry reproduces correctly the average property.**
- **Geometry reproduced a non perturbative result.**
- **Geometry does not reproduce even finer details of the non perturbative behaviour of the time dependent correlations.**

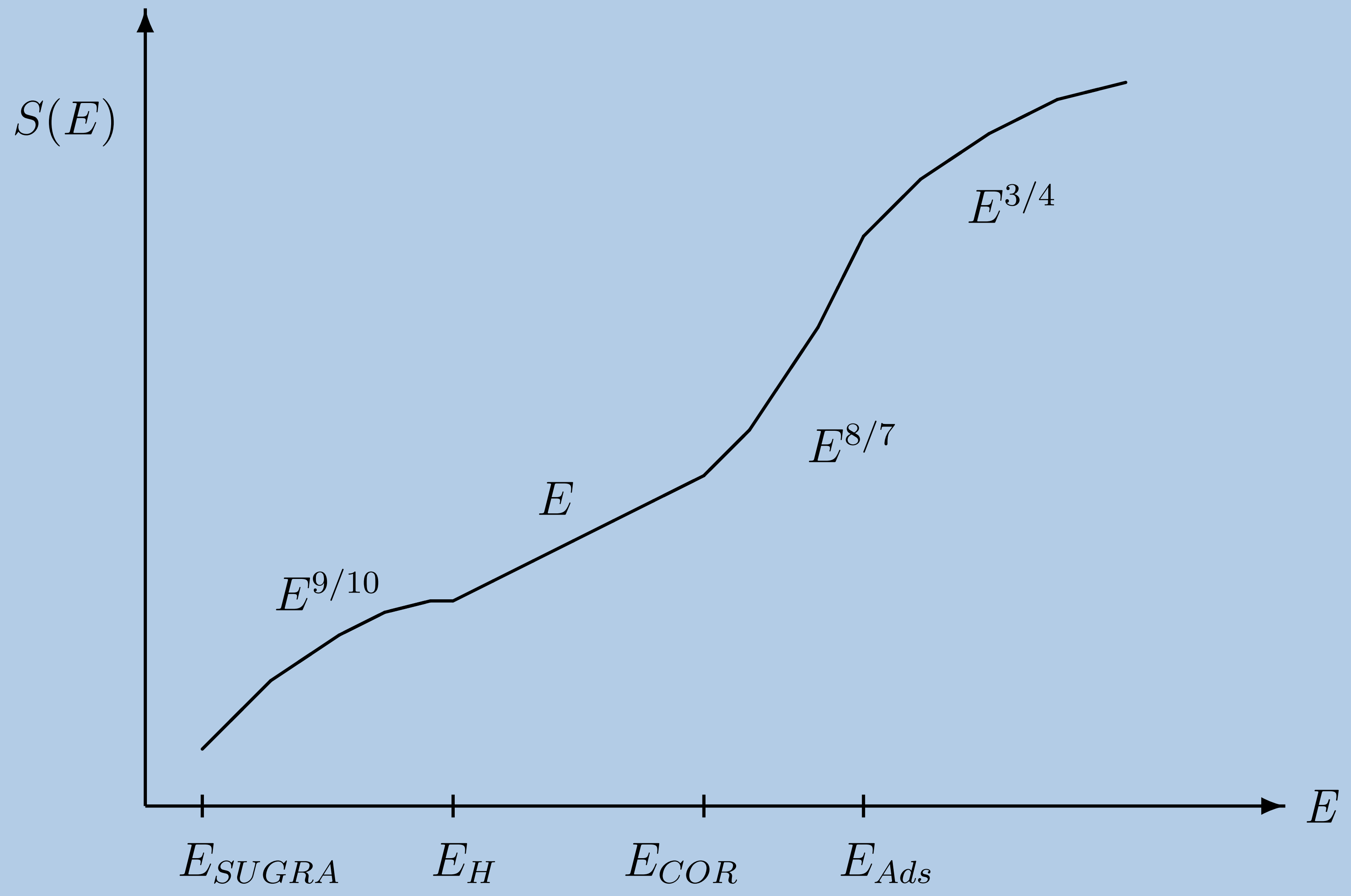
This will have consequences in AdS CFT

Listen to the AdS Noise


E
N2 / R
1 / R

Several Bands
Large gap, large difference in entropy




$S(E)$
$E^{3/4}$
$E^{8/7}$
E
$E^{9/10}$
E
E_{SUGRA}
E_H
E_{COR}
E_{Ads}

The Lowest Energy Band Dominates

$$Z(\beta) \equiv \sum_b e^{-I_b(\beta)} \text{ and } I_b(\beta) = \beta E_b - S_b$$

$$e^{-I_b(\beta)} e^{-S_b}$$

$$\exp(-\beta E_b)$$

ETH For BHs and Strings.

For the Gas:

$$B \sim \frac{1}{N} \mathrm{Tr}\ F^{\,n}$$

For T small relative to the critical T S and I are O(1) in

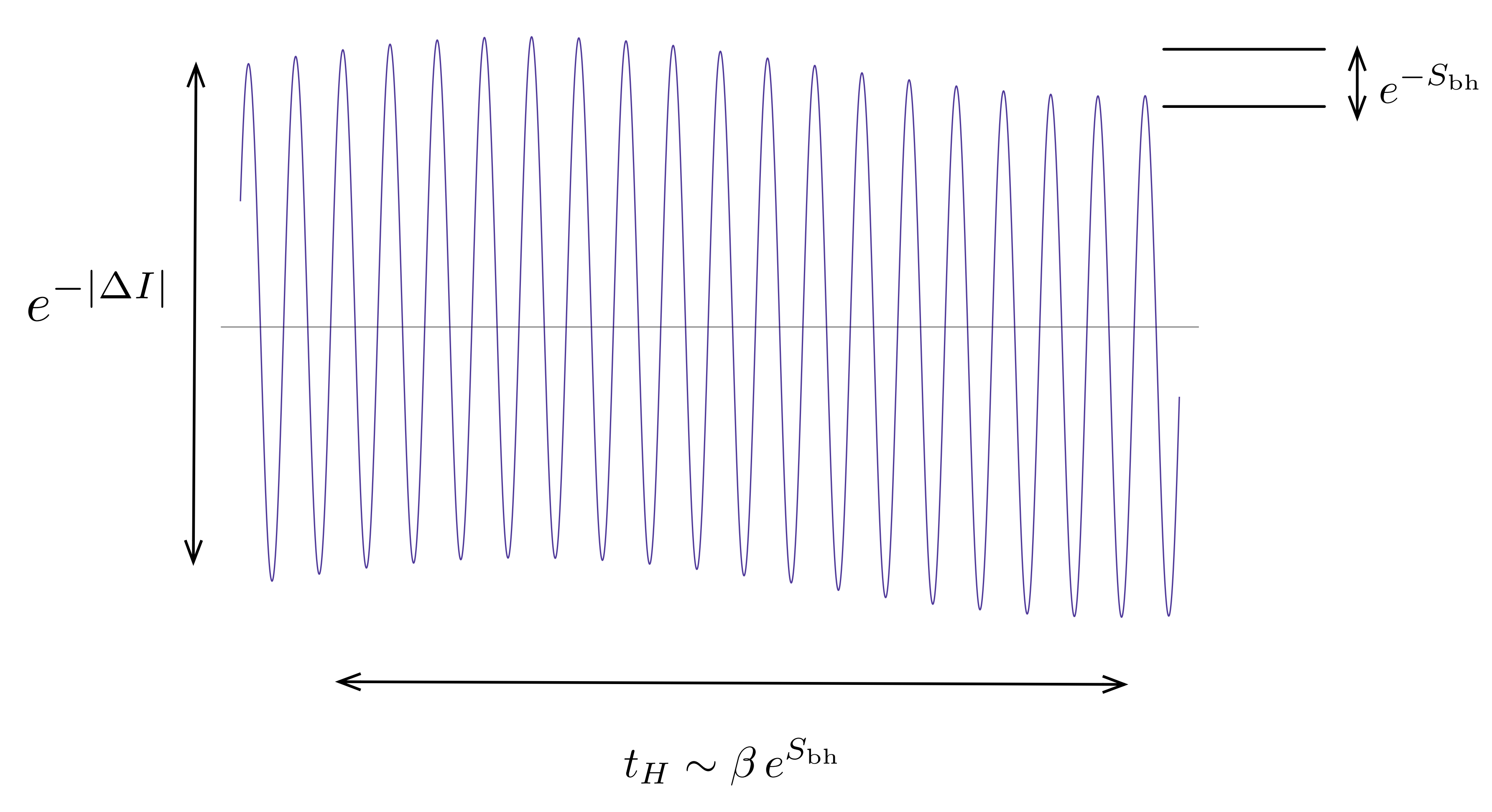
$$|\mathrm{noise}|_{\mathrm{canonical}} \sim |b|^2 \, \frac{e^{-|\Delta I|}}{\sqrt{S_{\mathrm{gas}}}}$$

For T>>T critical

$$|\mathrm{noise}|_{T\gg T_c} \sim |b|^2\, e^{I_{\mathrm{bh}}} \left[\frac{1}{(RT)^{9/2}} + O\left(e^{-c_{\mathrm{Hag}}\,\lambda^{5/2}}\right) \right.$$

$$\left. + O\left(e^{-c_{\mathrm{sh}}\,N^2/\lambda^{7/4}}\right) + O\left(e^{-c_{\mathrm{bh}}\,N^2}\right) \right]$$

The Noise is determined by the lowest band , the fast O(1) variations are determined by it as well. But the hight and the long time variations are determined by the thermodynamical dominant configuration.



Slogans:

1. Diversity Counts.

2. Geometry can capture non perturbative average observables.

3. Geometry may well miss some parts.