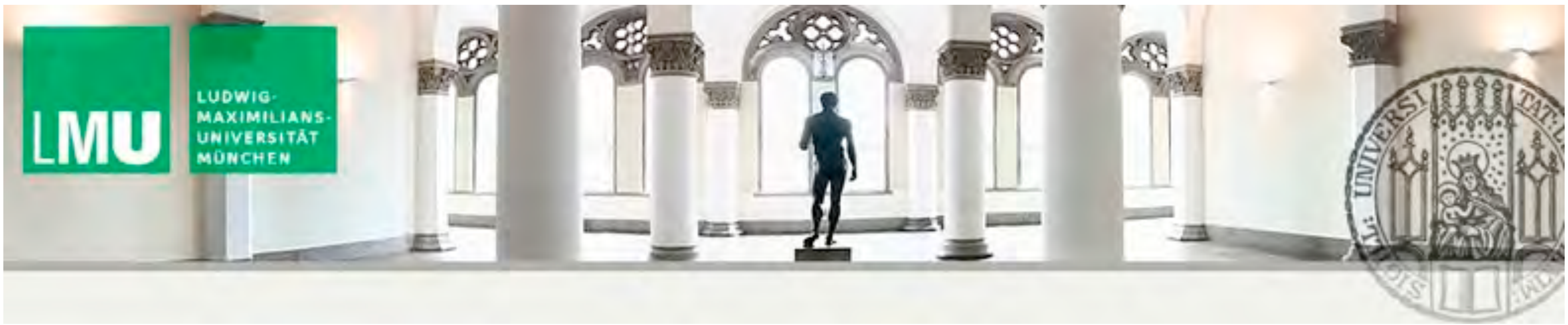
Large N Graviton Scattering and Black Hole Formation

DIETER LÜST (LMU-München, MPI)



8th. Crete Regional Meeting in String Theory, Nafplion, 8th. July 2015

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Work in collaboration with Gia Dvali, Cesar Gomez, Reinke Isermann and Stephan Stieberger, arXiv: 1409.7405

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Outline:

I) Unitarity in graviton scattering and black hole production

II) Large N graviton scattering amplitudes at high energies in field and string theory

III) Summary

I) Introduction

Quantum mechanics:

- Complementary picture:

Wave ⇔ N particles

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Wave $\Leftrightarrow$ N particles

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In quantum gravity and in string theory some of these statements have to be refined.

High energy string scattering:

[Amati, Ciafaloni, Veneziano (1987); Gross, Mende (1987)]

Refinement of Heisenberg relation:

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Refinement of Heisenberg relation:

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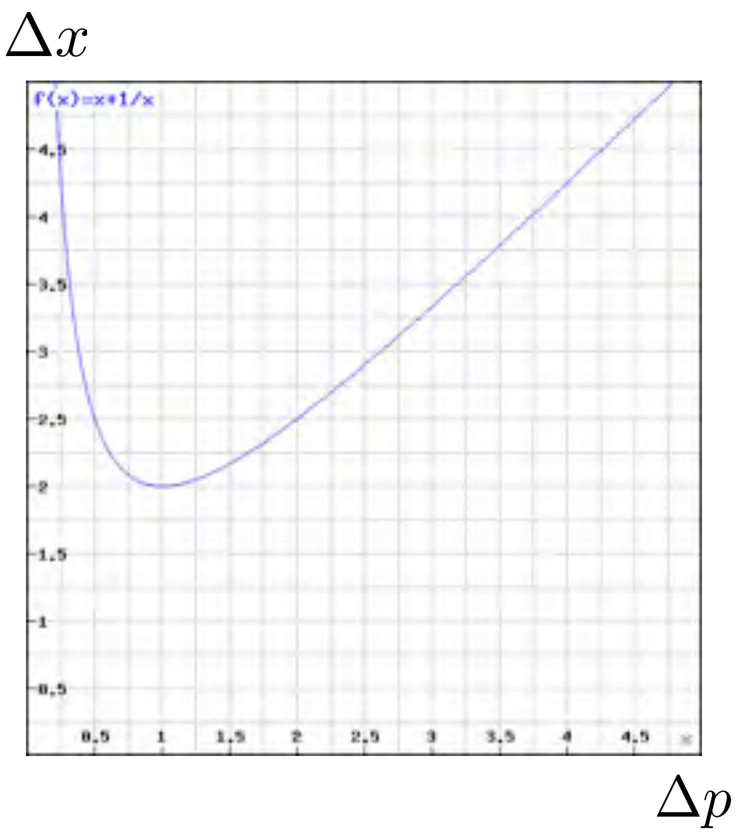
$\Rightarrow$ Smallest possible distance:

$$\Rightarrow \quad \Delta x \geq \sqrt{\hbar \alpha'} = \Delta x_{min}$$



In particular two questions and puzzles:

- What is the quantum nature of Black Holes ?

- What is the high energy behavior of graviton scattering amplitudes ?

Unitarity at tree level ?

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Solve these problems (partially) within Einstein gravity!

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$\Rightarrow$ Classicalization & the black hole N-portrait

- What is the high energy behavior of graviton scattering amplitudes ?

Unitarity at tree level ?

Classicalization & the black hole N-portrait:

[G. Dvali, C. Gomez, ...]

- Are described by IR physics,

.. where there is no need to modify gravity in the IR

Quantization of gravity in IR $\leftrightarrow$ semiclassical regime.

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- Are described by IR physics,

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Quantization of gravity in IR $\leftrightarrow$ semiclassical regime.

However there remain still some UV problems:

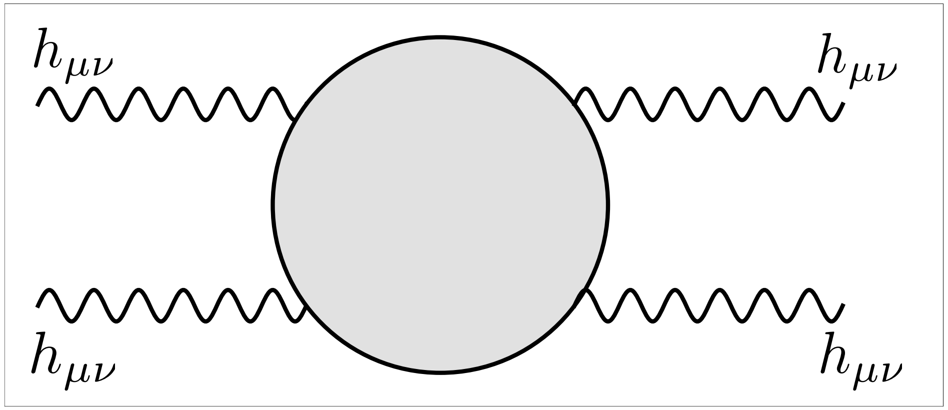
- Precise coefficient coefficient in black hole entropy:

$$S = \frac{1}{4} \frac{A}{L_P^2}$$

- Renormalization, UV finiteness of loop amplitudes

New UV degrees of freedom ➪ String theory !

Graviton scattering:



$\sim sL_P^2$

It is known that tree level graviton scattering amplitudes grow like s (center of mass energy).

$\Rightarrow$ Violation of unitarity at $s = M_P^2$

One possible solution: Wilsonian approach:

Amplitude is unitarized by integrating in new weakly coupled degrees of freedom of shorter and shorter wave lengths (at higher and higher energies).

However it is expected that black holes will be produced in particle scattering processes with high energies of the order

$$\sqrt{s} > R_s^{-1} \equiv (\sqrt{s} L_P^2)^{-1}$$

[´t Hooft (1987); Antoniadis, Arakani-Hamed, Dimopoulos, Dvali (1998); Banks, Fischler (1999); Dimopoulos, Landsberg (2001); Yoshino, Nambu (2002); Giddings, Thomas (2002); Eardley, Giddings (2002); Giddings, Rychkov (2004); ...]

Classicalization: Amplitudes get unitarized by classical black hole formation.

[G. Dvali, C. Gomez (2010); G. Dvali, G. Giudice, C. Gomez, A. Kehagias (2010)]

(Gravity protects itself at high energies by black hole formation.)

So we need a better understanding of how black holes are formed in graviton scattering amplitudes.

Black hole corpuscular N-portrait:

Quantum black hole = Bound state of N gravitons

(Bose-Einstein condensate)

[G. Dvali, C. Gomez (2011 - 2014); G. Dvali, C. Gomez, D.L. (2012)]

Black hole corpuscular N-portrait:

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Relevant properties:

[G. Dvali, C. Gomez (2011 - 2014); G. Dvali, C. Gomez, D.L. (2012)]

- N is large and the gravitons are soft.
- Interaction strength among individual gravitons is small:

$$\alpha = \frac{L_P^2}{R^2} << 1$$ (R ... graviton wave length)

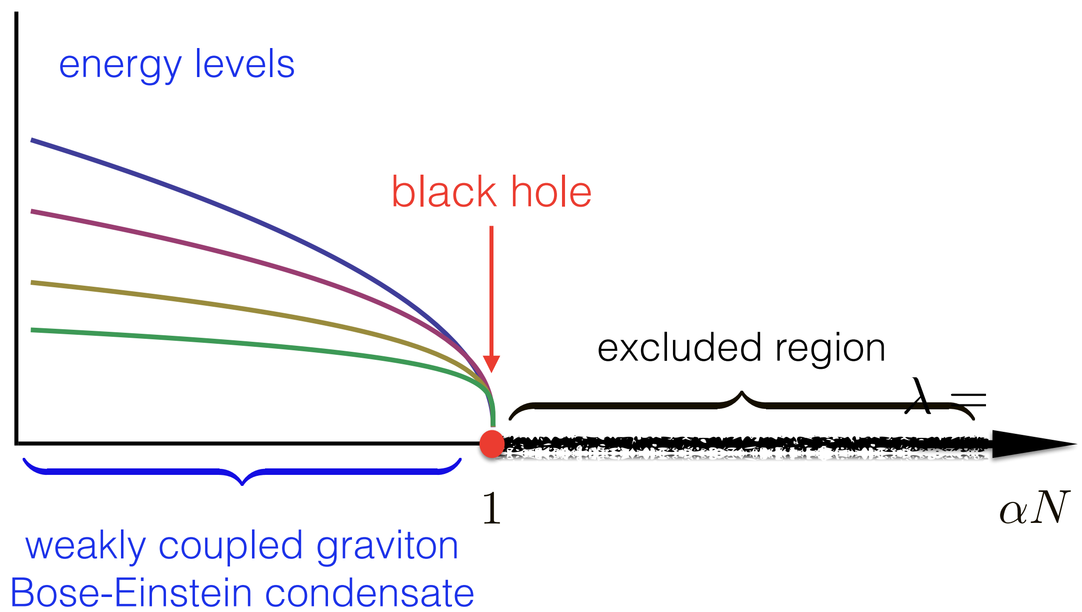
- Collective (`t Hooft like) coupling: $\lambda = \alpha N$
- Black holes are formed at the quantum critical point:

$$\lambda = 1 \qquad (R = \sqrt{N} L_P)$$

[G. Dvali, C. Gomez, arXiv:1207.4059; Flassig, Pritzel, Wintergerst, arXiv:1212.3344]

Black hole bound state (at $\lambda = 1$):

- Mass and size: $M_{BH} = \sqrt{N} M_P \,, \quad R_{BH} = \sqrt{N} L_P$
- Exponential degeneracy, entropy: $S \sim N$
- Semiclassical behavior: $N \to \infty$



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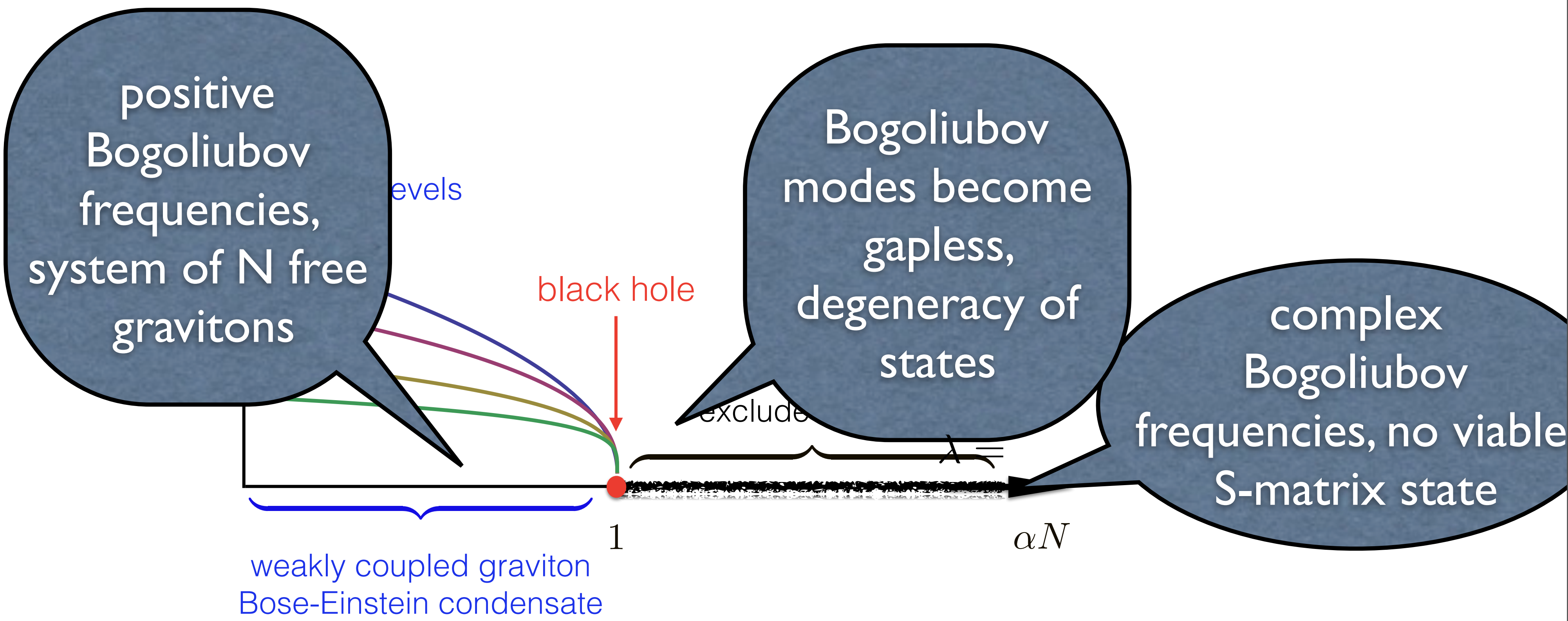

Can we reconcile this picture in graviton scattering processes (expressed in terms of N and λ)?

Is there a signal of non-perturbative black hole physics in perturbative graviton amplitudes?

So far: computation of graviton N-point amplitudes with small N (N=4).

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Our paper: new look at graviton scattering at trans-Planckian energy

- Explicit calculation of field theory and string amplitudes in a new kinematical large N regime, relevant for black hole production:

$$2 \longrightarrow N \quad \text{with} \quad N \to \infty$$

- We will argue that the perturbative $2 \longrightarrow N$ amplitude indeed contains relevant non-perturbative information supporting the picture of black hole production and classicalization.

Crossing the UV barrier:

The 2 $\to$ N string amplitude exhibits an interesting transition property:

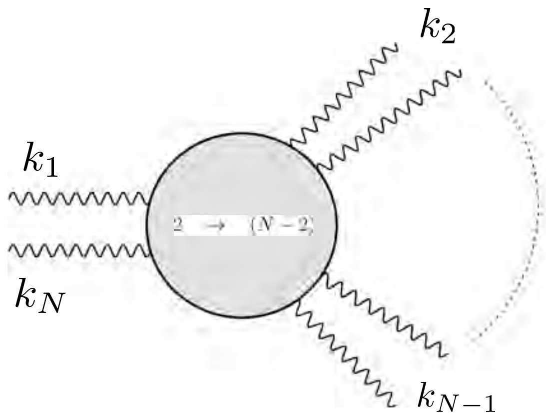
- Soft final gravitons: Unitarization by black holes.
- Hard final gravitons: Unitarization by string Regge states.

New trans-Planckian cross-over energy scale:

$$E_{\mathrm{IR/UV}} = N M_{\mathrm{string}}$$

II) Large N Graviton Scattering Amplitudes

$2 \longrightarrow N$ graviton amplitude with high center of mass s:



$$s_{ij} = (k_i + k_j)^2 \sim \begin{cases} s \,, & i,j \in \{1,N\} \,, \\ -\frac{s}{N-2} \,, & i \in \{1,N\} \,,\ j \notin \{1,N\} \,, \\ \frac{s}{(N-2)^2} \,, & i,j \notin \{1,N\} \,. \end{cases}$$

$p_{in} \sim \sqrt{s}$ and $p_{out} \sim \dfrac{\sqrt{s}}{N-2}$

Classicalization limit: soft gravitons in the final state.

$$s \to \infty \,, \qquad \epsilon = \frac{1}{N-2} \to 0 \quad \Longrightarrow \quad p_{out} < M_P$$

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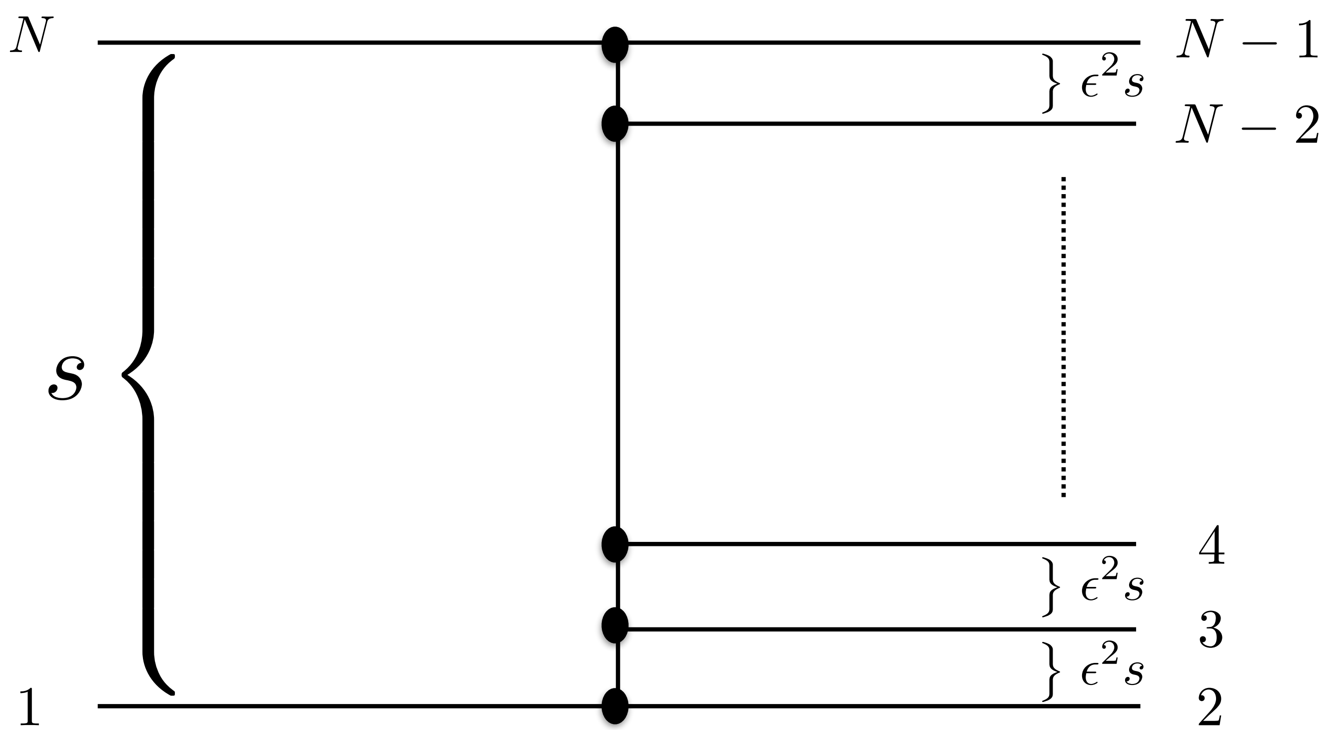

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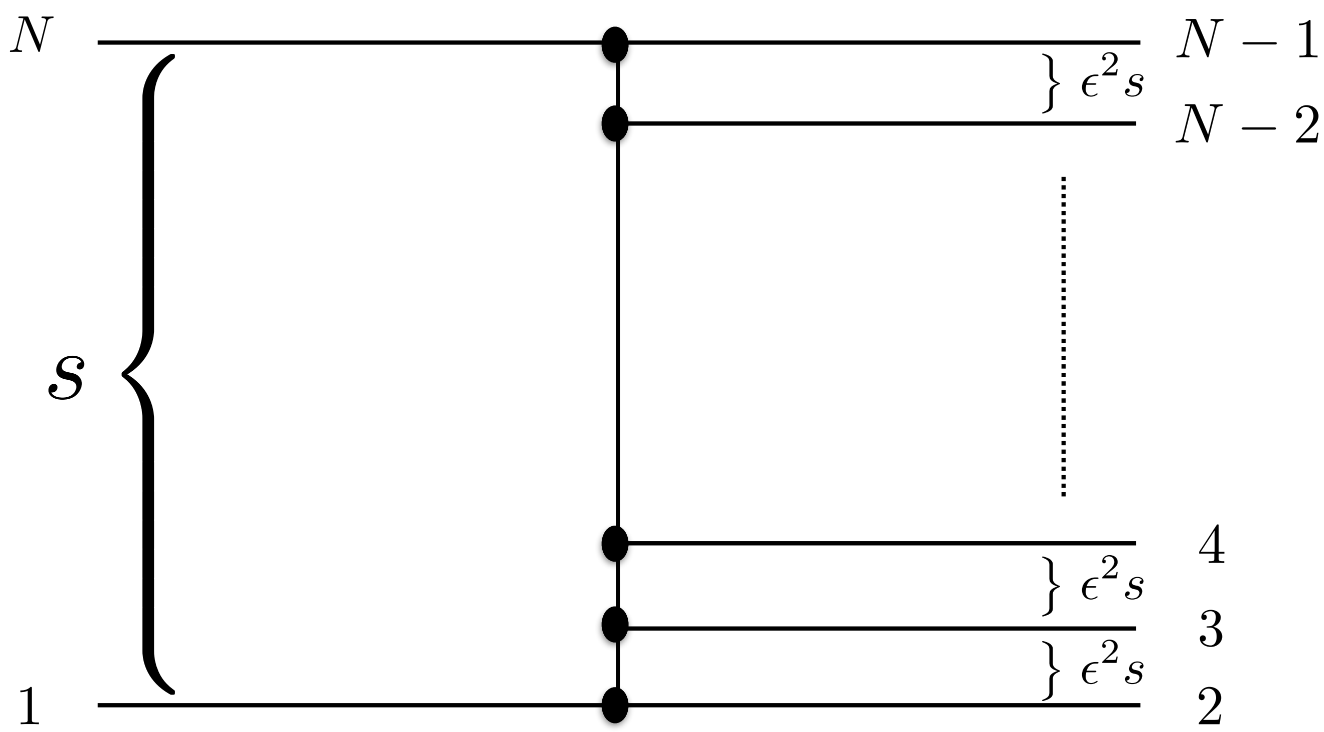

Double scaling limit: ($\Rightarrow$ small impact parameter)

$$N \to \infty\,, \quad s \to \infty\ (\sqrt{s} >> M_P) \quad \text{with} \quad \lambda = \frac{s}{M_P^2 N} \neq 0$$

(i) Field theory

To compute the graviton scattering amplitudes one can try on-shell methods and KLT techniques. [Kawai, Lewellen, Tye (1986)],

Problem: *KLT uses a double sum over (N-3)! squares of Yang-Mills amplitudes => in practice very hard to perform* $N \to \infty$ *limit*

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Instead we use the CHY formula for the N-graviton amplitude:

[Cachazo, He, Yuan (2013, 2014)]

$$M_N = \int \frac{d^N\sigma}{\text{Vol } SL(2,\mathbf{C})} \prod_{a=1}^{N}{}' \delta\left(\sum_{b\neq a} \frac{s_{ab}}{\sigma_a - \sigma_b}\right) E_N^2(\{k,\xi,\sigma\})$$

integral over N-punctered sphere

delta-function support on solutions of scattering equations

certain determinant (Pfaffian) encoding external momenta k and polarizations ξ

Scattering equations:

$$\sum_{b\neq a} \frac{s_{ab}}{\sigma_a - \sigma_b} = 0$$

$(N-3)!$
solutions

relate space of kinematic invariants of N gravitons to that of the positions of N points on a sphere

[cfr. with twistor approach by E. Witten (2003)]

Problem: for N > 5 the scattering equations are very hard to solve for generic momenta.

[See L. Dolan and P. Goddard (2013/2014),
C. Baadsgaard, N. Bjerrum-Bohr, J. Bourjaily, P. Damgaard, arXiv:1506.06137]

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Fortunately in the **classicalization limit**, i.e. the limit we are interested in **scattering equations** can be **solved explicitly**

classicalization limit can be parameterized as:

(in units of s/(N-2)^2)

$$s_{1,N} = \frac{1}{2}\,(N-3)\,(N-a-b)\,,$$

$$s_{N-1,N} = -\frac{1}{2}\,(N-3)\,(2-b)\,, \quad s_{1,N-1} = -\frac{1}{2}\,(N-3)\,(2-a)\,,$$

$$s_{1,i} = -\frac{1}{2}\,(N-2-b)\,, \quad s_{i,N} = -\frac{1}{2}\,(N-2-a)\,,$$

$$s_{N-1,i} = \frac{1}{2}\,(4-a-b)\,, \quad s_{ij} = 1 \quad , \quad i,j \in \{2,\ldots,N-2\}\,,$$

this gives rise to a two-parameter a,b solution, which is (N-3)!-fold degenerate

This parametrization can be mapped to a problem Kalousios (2013) has already studied:

Solutions of scattering equations are identified with the zeros of Jacobi polynomials.

$$
\begin{aligned}
M_N(1,\ldots,N) \quad &= \quad -\kappa^{N-2}\, 2^{8-N}\, \frac{s}{(N-2)^2}\, [(N-3)!!]^2\, \frac{\Gamma\left(\frac{a}{2}\right)\ \Gamma\left(\frac{3}{2}+\frac{b-N}{2}\right)\ \Gamma\left(\frac{1-N+a+b}{2}\right)}{\Gamma\left(1+\frac{a-N}{2}\right)\ \Gamma\left(\frac{b-1}{2}\right)\ \Gamma\left(\frac{a+b-3}{2}\right)} \\
&\times \quad \frac{\Gamma\left(\frac{3}{2}+\frac{a-N}{2}\right)\ \Gamma\left(\frac{b}{2}\right)\ \Gamma\left(\frac{a+b-2}{2}\right)}{\Gamma\left(1+\frac{b-N}{2}\right)\Gamma\left(\frac{a-1}{2}\right)\ \Gamma\left(\frac{a+b-N}{2}\right)}\ H_N(a,b)^2
\end{aligned}
$$

Exact in any real a,b and N

$$
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$$\times\,\frac{\Gamma\left(\frac{3}{2}+\frac{a-N}{2}\right)\,\Gamma\left(\frac{b}{2}\right)\,\Gamma\left(\frac{a+b-2}{2}\right)}{\Gamma\left(1+\frac{b-N}{2}\right)\Gamma\left(\frac{a-1}{2}\right)\,\Gamma\left(\frac{a+b-N}{2}\right)}\,H_N(a,b)^2$$

Field theory amplitude!

Exact in any real a,b and N

$$\xrightarrow{N\to\infty}\ \kappa^N\,\frac{s}{N^2}\,N!$$

Note: Incidentally the solutions to the scattering equations describe the saddle point contributions in the high-energy limit of open and closed string amplitudes (Gross, Mende) → see next part of the talk.

To obtain the physical probability, i.e. the S-matrix element, we have to consider phase space integral:

$$d|\langle 2|S|N-2\rangle|^2 = \frac{1}{(N-2)!}\prod_{i=2}^{N-1} dp_i^4 |M_N|^2 \delta^4(P_{total})$$

$$\left(\quad p_{in} \sim \sqrt{s} \ , \ p_{out} \sim \frac{\sqrt{s}}{N-2} \right)$$

Physical $2 \rightarrow \ N-2$ perturbative, scattering probability in classicalization regime:

$$|\langle 2|S|N-2\rangle|^2 = \left(\frac{L_P^2 s}{N^2}\right)^N N! = \left(\frac{\lambda}{N}\right)^N N! \sim e^{-N}\lambda^N$$

Collective coupling $\lambda \equiv \alpha N = s/M_P^2 N$

This perturbative scattering probability possesses a maximum at the following critical value for N:

$$N_{crit} = sL_P^2 \quad \Leftrightarrow \quad \lambda_{crit} = 1$$

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Remark: similar calculations can be done for scalar field theories, like $\lambda\phi^4$.

In this case the amplitudes show a different large N behavior:

$$A_N^2 \simeq \lambda^N N!$$

constant

Connection of the perturbative amplitude to the non-perturbative black hole bound state:

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The perturbative amplitude is suppressed by e^{-N} .

This is just the inverse of the degeneracy of states of a black hole with entropy $\mathcal{S} \sim N$.

Therefore this suppression factor is compensated at the critical point $\lambda = 1$ by e^{N} from the degeneracy of black hole states:

$$A_{BH} \sim \sum_j |\langle 2|S|N\rangle|_p^2 \, |\langle N|BH\rangle_j|_{np}^2 \sim \lambda^N e^{-N}|_p \times e^N|_{np}$$

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pert. amplitude

projection on black hole bound state

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So, black hole is exactly dominating at $\lambda = 1$.

In summary:

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$$|M_N^{pert.}|^2 \simeq \lambda^N e^{-N}, \quad \lambda = s/(M_P^2 N)$$

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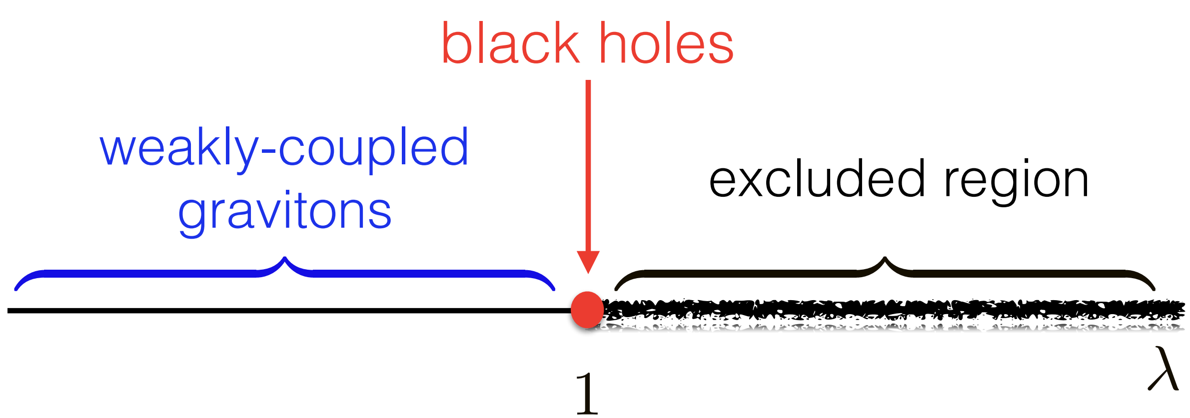
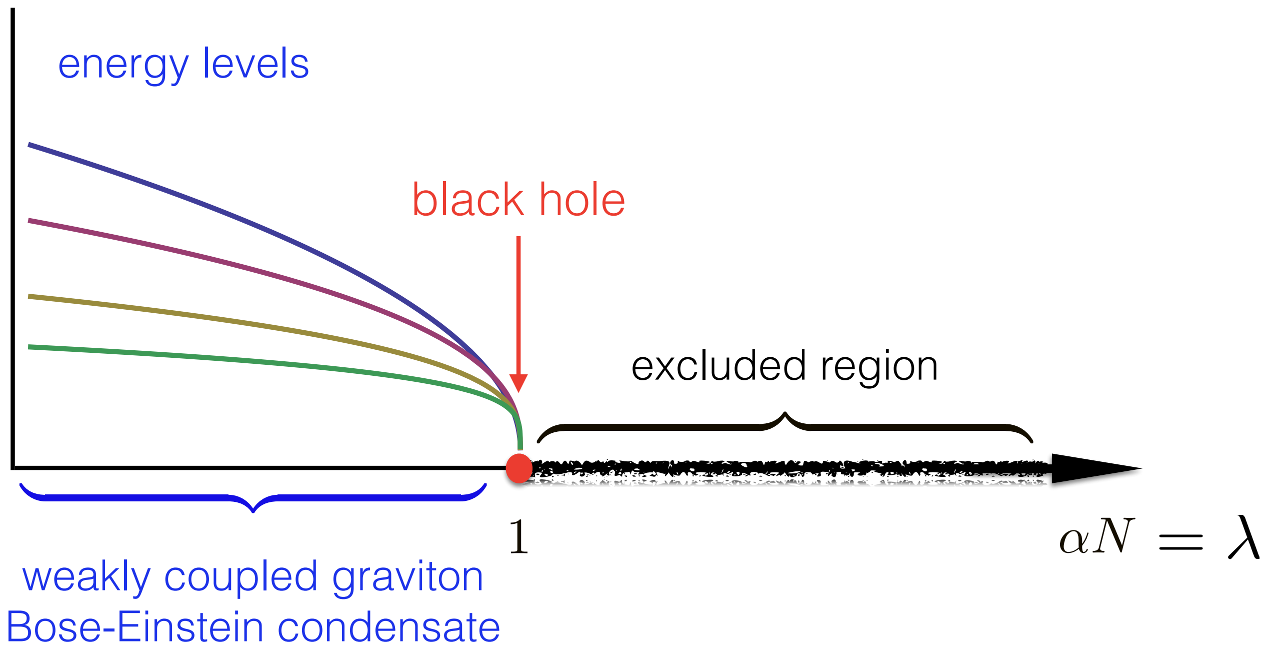
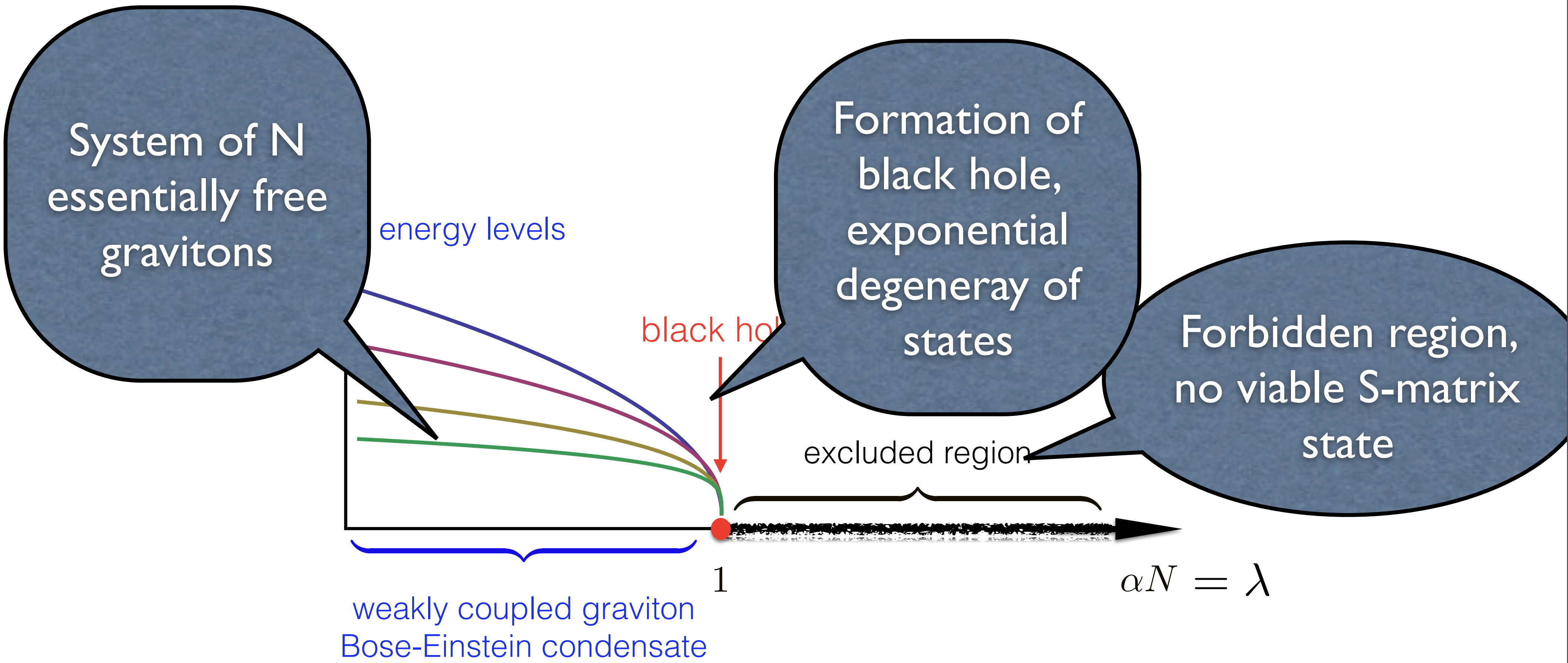
- Non-perturbative enhancement at $\lambda = 1$ due to black hole entropy factor : e^N

$$|M_N^{n.p.}|^2 \simeq \lambda^N$$

(fully saturated at $\lambda = 1$)




energy levels
black hole
excluded region
1
$\alpha N = \lambda$
weakly coupled graviton
Bose-Einstein condensate


System of N essentially free gravitons
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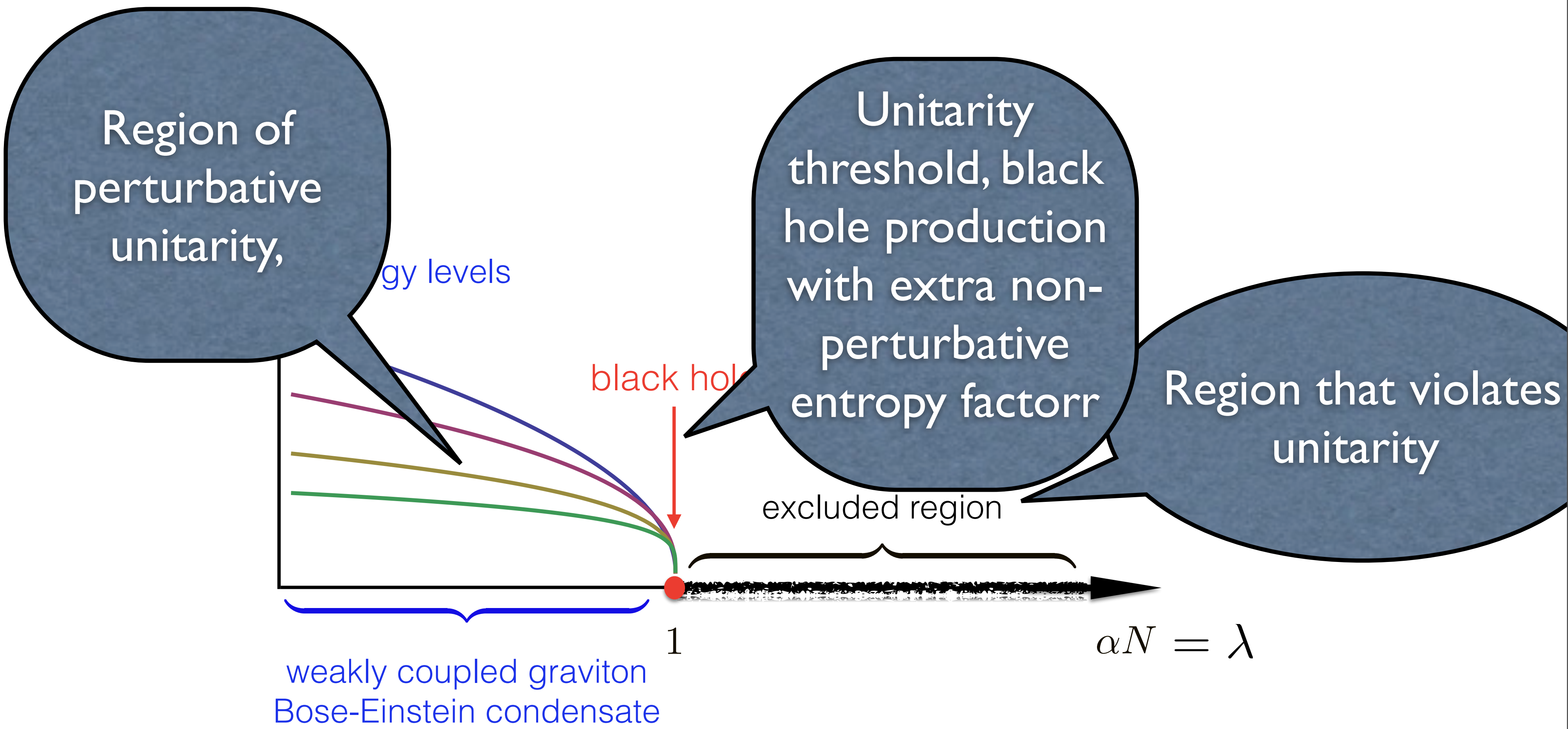
Region of perturbative unitarity,
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Unitarity threshold, black hole production with extra non-perturbative entropy factorr
Region that violates unitarity
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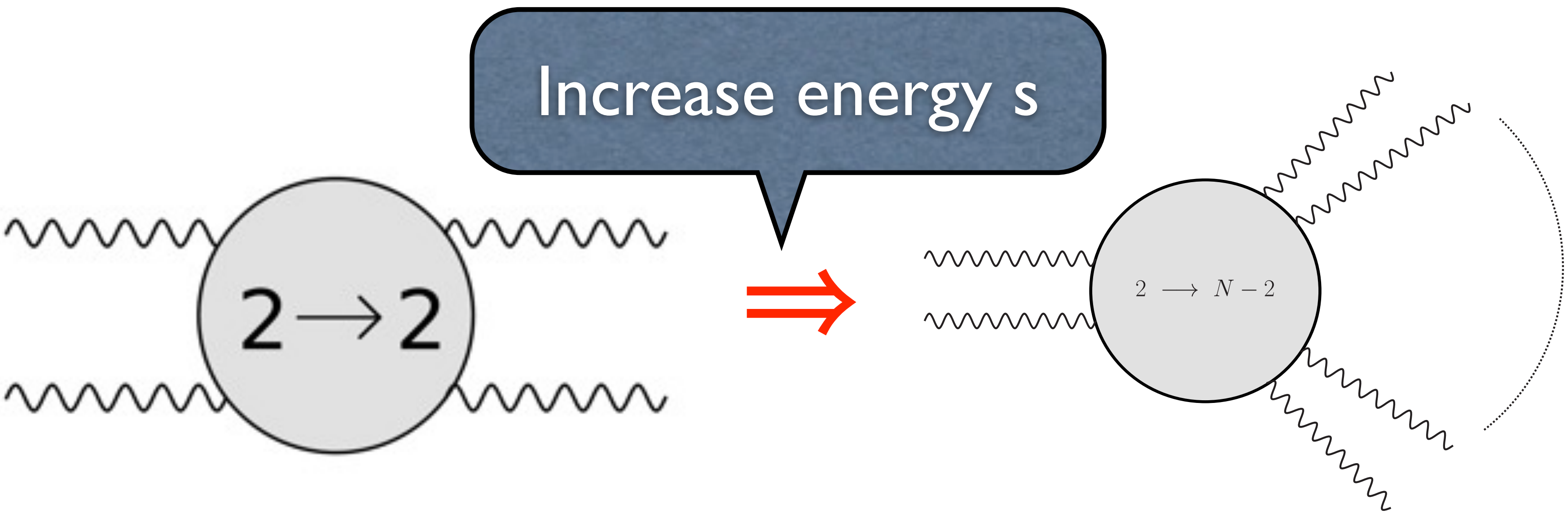
For large s unitarization occurs if N increases appropriately:

This bound implies that $N \gtrsim N_{crit} = sL_P^2$

This is the core of the idea of classicalization!

N should be larger than the corresponding entropy of a black hole with mass equal to the center of mass energy.



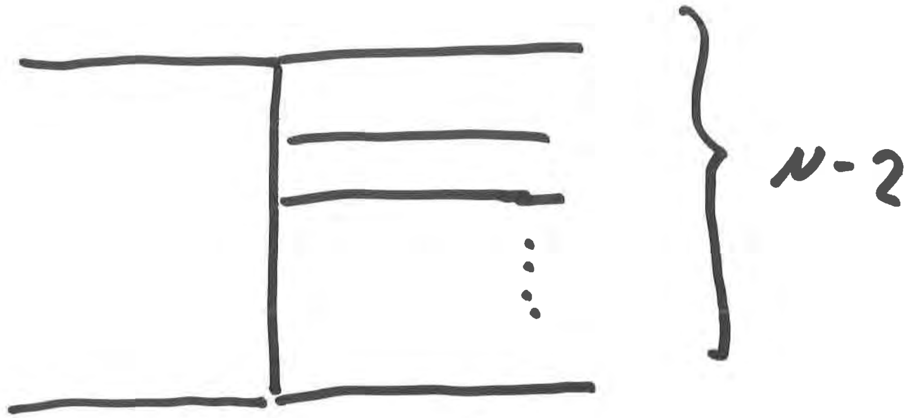
However there remain still some UV problems:

What is happening in the regime where $\lambda > 1$?

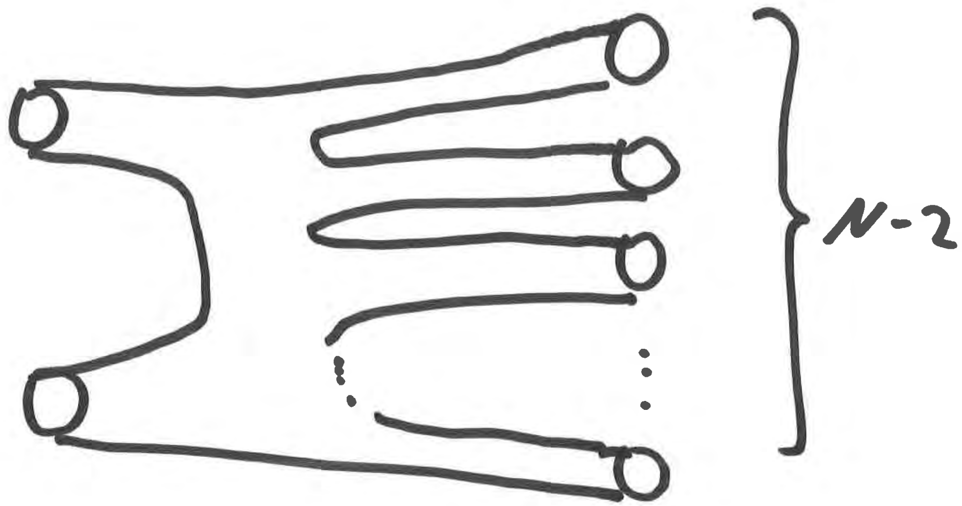
$$N < N_{crit} = sL_P^2$$

(ii) Closed string theory

F.T. :



S.T.



High energy behavior of open/closed string amplitudes shows exponential fall off due to Regge modes.

[Veneziano (1968); Amati, Ciafaloni, Veneziano (1987); Gross, Mende (1987), Gross, Manes (1989)]

Example: 4-point graviton amplitude

$$\mathcal{M}_4 \sim K \frac{\Gamma(-\frac{\alpha'}{4}s)\Gamma(-\frac{\alpha'}{4}t)\Gamma(-\frac{\alpha'}{4}u)}{\Gamma(\frac{\alpha'}{4}s)\Gamma(\frac{\alpha'}{4}t)\Gamma(\frac{\alpha'}{4}u)}$$

$$\longrightarrow_{\alpha'\to\infty} \kappa^2 |A_4|^2 \times 4\pi\alpha' \frac{st}{u} \exp\left\{\frac{\alpha'}{2}(s\ln|s| + t\ln|t| + u\ln|u|)\right\}$$

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Square of
YM-amplitude

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Square of YM-amplitude

Momentum kernel

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Square of YM-amplitude

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String form factor

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Example: 4-point graviton amplitude

$$\mathcal{M}_4 \sim K \, \frac{\Gamma(-\frac{\alpha'}{4}s)\Gamma(-\frac{\alpha'}{4}t)\Gamma(-\frac{\alpha'}{4}u)}{\Gamma(\frac{\alpha'}{4}s)\Gamma(\frac{\alpha'}{4}t)\Gamma(\frac{\alpha'}{4}u)}$$

$$\longrightarrow_{\alpha' \to \infty} \kappa^2 |A_4|^2 \times 4\pi\alpha' \frac{st}{u} \exp\left\{ \frac{\alpha'}{2}(s \ln|s| + t \ln|t| + u \ln|u|) \right\}$$

Square of YM-amplitude

Momentum kernel

String form factor

(Note: this was basically the state of the art before our paper.)

Generalization to arbitrary (large) N:

High energy limit: use of scattering equations:

$$\mathcal{M}(1,\ldots,N) = \kappa^{N-2}\ (4\pi\alpha')^{N-3} \sum_{a=1}^{(N-3)!} \frac{\left(\prod_{i<j}^{N} |z_{ij}^{(a)}|^{\frac{\alpha'}{2}s_{ij}}\right)}{\det{}'\Phi(z^{(a)})^{1/2}\det{}'\Phi(\overline{z}^{(a)})^{1/2}} E_N(\{k,\xi,z^{(a)}\})^2 + \mathcal{O}(\alpha'^{-1})$$

Koba-Nielsen factor

N closed string tree-level amplitude

sum over the (N-3)! solutions of scattering equations

Jacobian/Hessian from saddle point approximation

Again in the classicalization limit we obtain the explicit result for arbitrary N:

$$\mathcal{M}_N = (4\pi\alpha')^{N-3} \prod_{\nu=1}^{N-3} \left(\frac{\nu^\nu (\alpha+\nu)^{\alpha+\nu} (\beta+\nu)^{\beta+\nu}}{(\alpha+\beta+N-3+\nu)^{\alpha+\beta+N-3+\nu}} \right)^{\frac{\alpha' s}{4}}$$

$$\times\ M_N^{FT} + \mathcal{O}\left((\alpha' s)^{-1}\right)$$

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String form factor

Two different energy regimes:

$$(i) \quad \frac{\sqrt{s}}{N} < M_s : \iff \lambda < N g_s^2$$

„infrared“, field theory regime

Field and ST theory amplitudes agree.

This was already conjectured for the MHV case up to 5 points by [Cheung, O´Connell, Wecht (2010)]

$$F_N = 1 \quad \Rightarrow \quad \mathcal{M}_N = M_N^{FT}$$

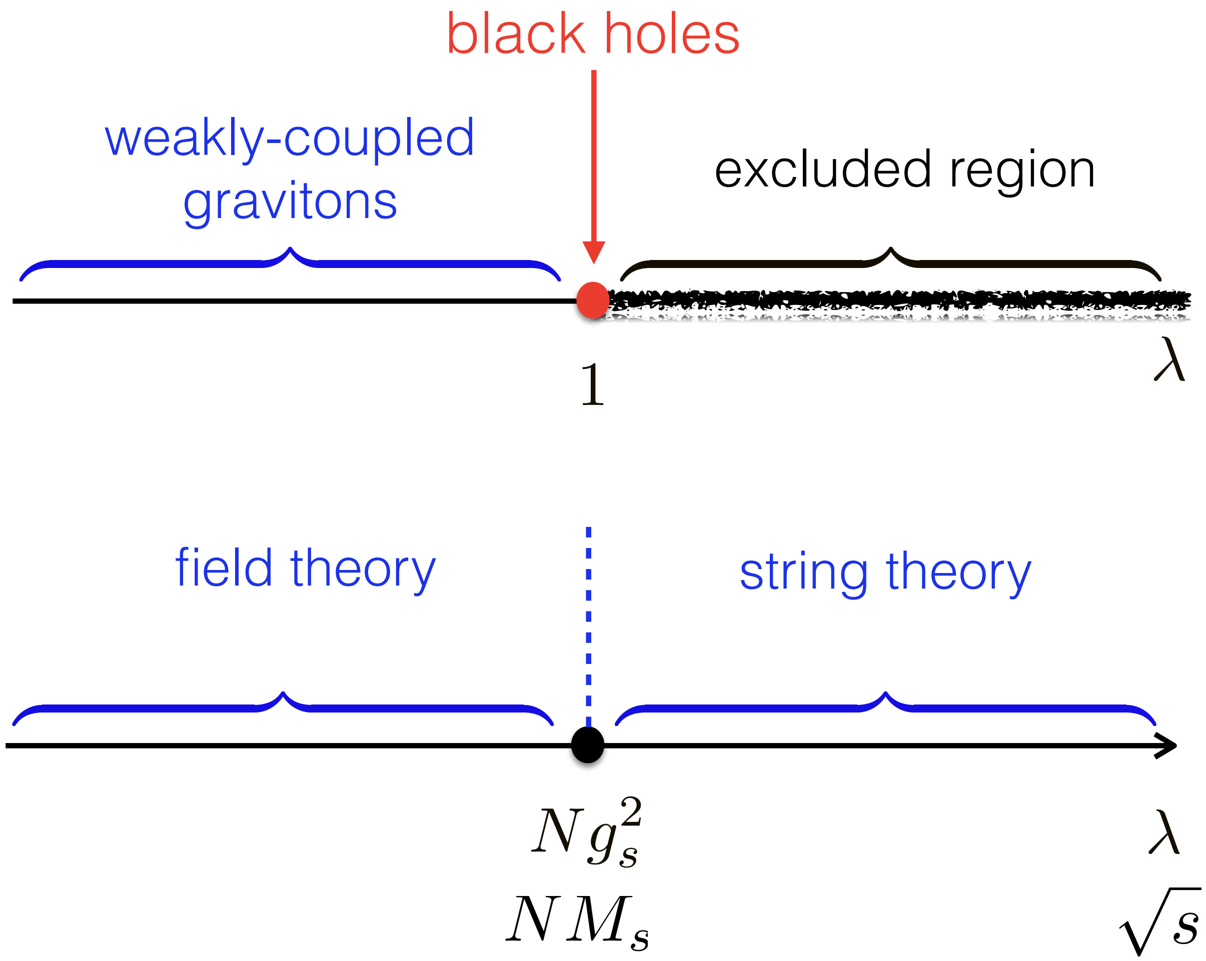
$$(ii) \quad \frac{\sqrt{s}}{N} > M_s : \iff \lambda > N g_s^2$$

„ultraviolet“, string theory regime

$$\mathcal{M}_N \sim \kappa^{N-2} \, {\alpha'}^{N-3} \, s \, e^{-\frac{\alpha'}{2}(N-3) \, s \ln(\alpha' s)}$$

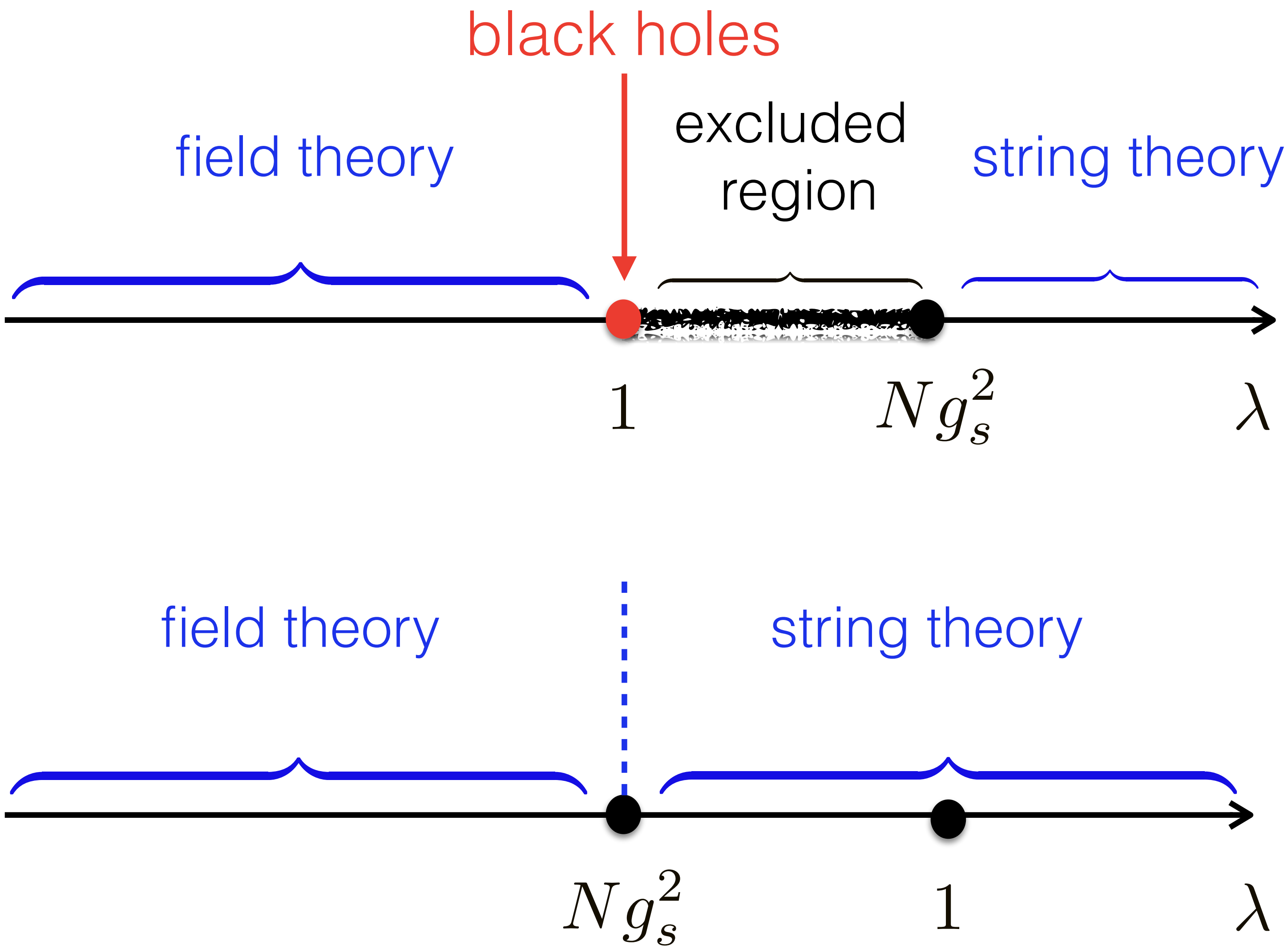
String states dominate.

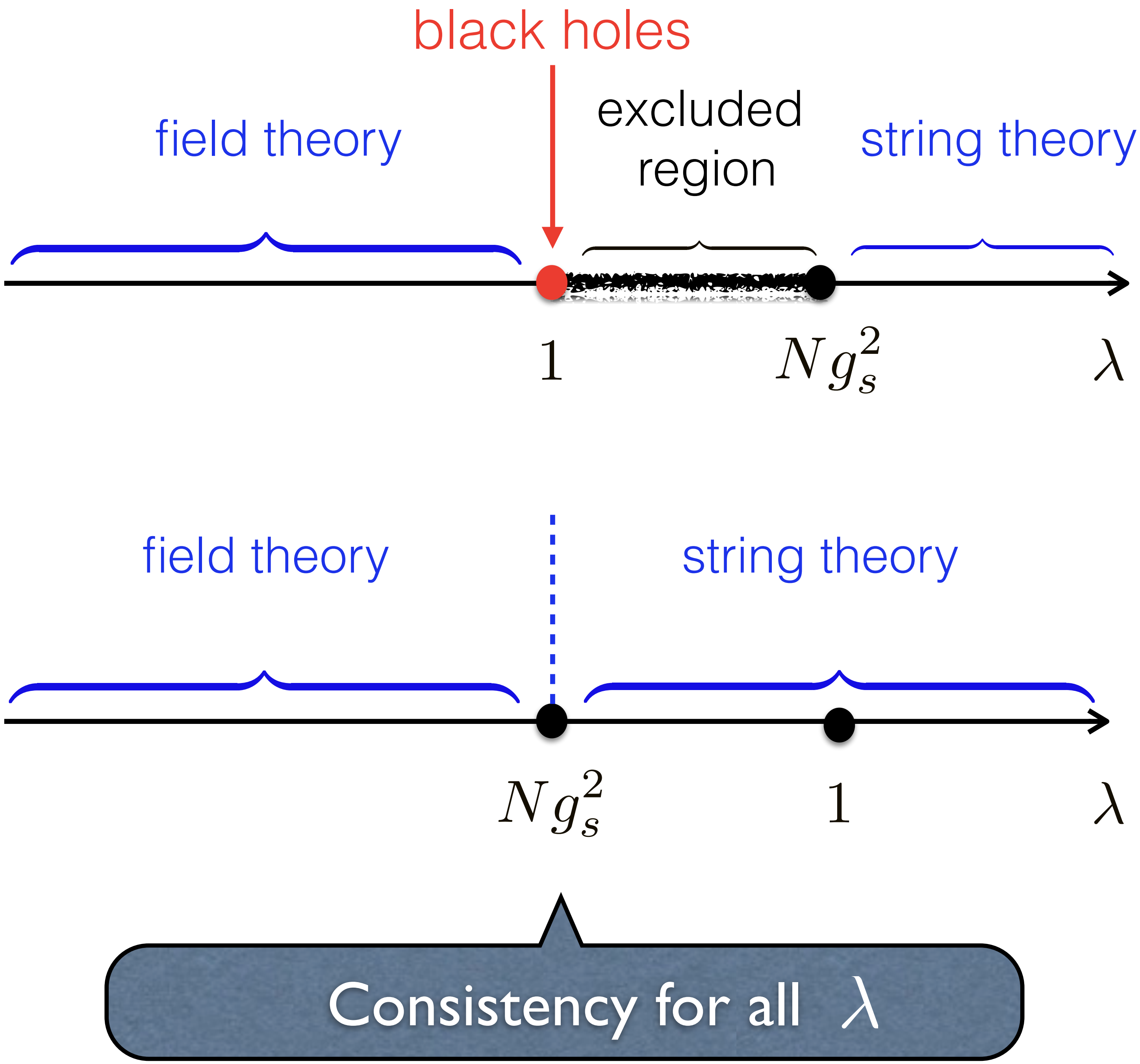
Amplitude gets tamed by string states (Regge modes).



Transition occurs at $E_{\text{IR/UV}} = NM_{\text{string}}$

Gravitons in final state become hard: $E_{final} > M_s$


black holes
field theory
excluded region
string theory
1
Ng_s^2
λ
field theory
string theory
Ng_s^2
1
λ


black holes
field theory
excluded region
string theory
1
Ng_s^2
λ
field theory
string theory
Ng_s^2
1
λ
Consistency for all λ

What is happening at the point $\lambda = N g_s^2 = 1$?

Here the F.T. amplitude agrees with the string amplitude at the critical point $\lambda = 1$.

This the point where the string effects match the amplitude from the F.T. black hole formation.

$g_s = \frac{1}{\sqrt{N}}$ $\Rightarrow$ String - black hole correspondence:

black hole can be described by a state of strings.

[Horowitz, Polchinski (1996); Dvali, D.L. (2009); Dvali, Gomez (2010)]

Here the IR is meeting the UV.

What about loop corrections or higher order gravity (UV) corrections?

They should correspond to 1/N corrections to what we computed:

$$A_{\mathrm{g-loop}} \sim \left(\frac{1}{N}\right)^g \quad , \quad A_{\mathcal{R}^g} \sim \left(\frac{1}{N}\right)^g$$

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There is some recent interest in higher order $\mathcal{R}^g$ gravity:

Scale invariant gravity: $S \sim \int dx^4 \sqrt{-g} R^2$

[Alvarez-Gaume, Kehagias, Kounnas, D.L., Riotto, Toumbas]

- propagating, ghostfree spin-2 only on curved backgrounds (de Sitter or anti-De Sitter).

- flat backgrounds: only scalar mode, no gravitational interaction. Non-trivial interplay between UV/IR !

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- Relation between open and closed string coupling:

$$g_s = g_{open}^2$$

- At point of string-bh correspondence: $g_s = 1/\sqrt{N}$

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Is there possibly any relation between the limit of large number N of gravitons and the large Nc limit in Yang-Mills gauge theories?

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So naively we get: $N = N_c^2$

What is the interpretation of this relation?

Summary:

- New computation of N-point gravity (string) amplitudes in trans-planckian large N region in closed form
- We found evidence for classicalization and black hole production (black hole N-portrait) in field theory.
- We found an interesting trans-Planckian transition between field theory and string theory: string - black hole correspondence.

Next steps:

[Stieberger (2009); Stieberger, Tayor (2014); Cachazo, He, Yuan (2014)]

- Mixed gauge boson (open)/gravity (closed) amplitudes: Bh N-portrait with matter [Dvali, Gomez, D.L. (2013)]
- Bh N-portrait beyond tree level First steps in [Kuhnel, Sundborg (2014)]