Three charge black holes and quarter BPS states in little string theory

A. Giveon, J. Harvey, DK, S. Lee

Introduction

In this talk we will discuss the system of k NS5-branes in type II(B) string theory. The fivebranes will be taken to wrap $R^4 \times S^1$. The theory on the fivebranes preserves 16 supercharges and lives in 4+1 non-compact dimensions (later we will discuss the case where R^4 is replaced by T^4).

This theory is known as Little String Theory (LST). It can be studied using holography, by focusing on the near-horizon geometry of the fivebranes, which is an asymptotically linear dilaton spacetime.

We will mostly focus on states in this theory that carry momentum p and winding w around the circle, while preserving a quarter of the supersymmetry of the LST.

These states can be thought of as the three charge black holes studied by Strominger, Vafa and many others in the context of providing a microscopic interpretation of black hole entropy.

They also figure prominently in the fuzzball program, which attempts to describe these microstates by horizonless geometries.

Our main interest will be in the dependence of the spectrum of these states on the positions of the fivebranes. We will see that it is qualitatively different when the fivebranes are separated by any finite distance, and when they are coincident. The two cases are separated by a string-black hole transition.

This is surprising, since separating the fivebranes corresponds in the low energy theory to Higgsing a non-abelian gauge group, and one would expect that if the W-boson mass scale is low, the physics of high mass states, such as the ones we will study, should not be affected. We will discuss why it nevertheless happens, and comment on some implications.

Near-horizon geometry of NS5-branes

Callan, Harvey and Strominger showed that the near-horizon geometry of k NS5-branes is described by an exactly solvable worldsheet CFT,

$$R_\phi \times SU(2)_k \times R^{5,1}$$

where R_ϕ represents the radial direction away from the fivebranes, and corresponds to a free scalar field with linear dilaton $\Phi = -Q\phi/2, Q^2 = \alpha'/k$; the three-sphere transverse to the fivebranes is described by a level k SU(2) WZW model.

In this background, the string coupling varies with the distance from the fivebranes. In terms of the coordinate ϕ (which is proportional to $\log r$), one has

$$g_s^2 \simeq e^{-Q\phi}$$

Thus, at large distance from the fivebranes, $\phi \to \infty$, the string coupling goes to zero. This is the boundary of the near-horizon geometry, the analog of the boundary of AdS for gauge/gravity duality.

At the same time, as one approaches the fivebranes, $\phi \to -\infty$, the string coupling diverges. Hence, the exact background above is not useful for calculations – to make it useful we need to do something about the strong coupling singularity.

There are two ways of dealing with it, both of which will be useful for us. We next describe them.

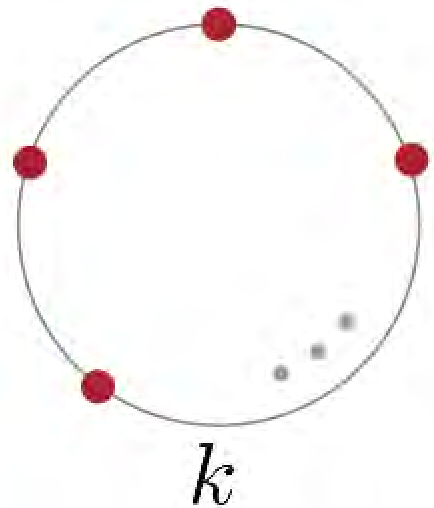
Double Scaled LST

One way is to separate the fivebranes. It is clear from the results of CHS that a single fivebrane does not have a linear dilaton throat. Thus, in any configuration of the fivebranes in which no two coincide, the coupling is bounded.

One can arrange the separations such that the coupling is everywhere small. This amounts to demanding that the masses of D-strings stretched between different NS5-branes, M_W , satisfy the condition

$$M_W \gg m_s$$

A particularly nice configuration of fivebranes that can be analyzed exactly is:



Fivebranes spread equidistantly around a circle.

The reason this configuration is nice is that it is described by an exactly solvable worldsheet CFT,

$$\left(\frac{SL(2)}{U(1)} \times \frac{SU(2)}{U(1)}\right)/Z_k \times R^{4,1} \times S^1$$

By taking the radius of the circle to be sufficiently large, we can arrange for the string coupling to be everywhere small. In that case, the dynamics of the theory can be studied using perturbative string techniques.

In particular, the states we are interested in, that carry momentum and winding on the S^1 and preserve ¼ of the supersymmetry, are standard perturbative BPS states, for which the right-movers on the worldsheet are in the ground state, while the left-movers are in a general excited state. Thus, they satisfy:

$$N_R = 0; \quad N_L = N = pw$$

$$\mathrm{M} = \left|\frac{p}{R} + \frac{wR}{\alpha'}\right|$$

The spectrum of these states is encoded in the elliptic genus of the worldsheet CFT. In our case, the non-trivial part of the background is

$$\left(\frac{SL(2)_k}{U(1)} \times \frac{SU(2)_k}{U(1)}\right)/\mathbb{Z}_k$$

Its elliptic genus, defined as

$$\mathcal{E}_{\mathrm{DSLST}} = \mathrm{Tr}_{\mathcal{H}_{\mathrm{RR}}}\left[(-1)^F q^{L_0-\frac{c}{24}} \bar{q}^{\bar{L}_0-\frac{\bar{c}}{24}} e^{2\pi i z(2J^3_{\mathrm{R}})}\right]$$

can be calculated using standard worldsheet techniques. One gets

$$\mathcal{E}_d = +\frac{i\vartheta_{11}(\tau,z)}{k\eta(q)^3}\sum_{\beta,\gamma=1}^{k} e^{2\pi i\frac{\beta\gamma}{k}} q^{\frac{\beta^2}{k}} \left(e^{2\pi i z}\right)^{\frac{2\beta}{k}} \mathcal{A}_{1,k}(\tau, \frac{z+\beta\tau+\gamma}{k})$$

where

$$\mathcal{A}_{1,k}(\tau,z) = \sum_{t\in\mathbb{Z}} \frac{q^{kt^2}\left(e^{2\pi i z}\right)^{2kt}}{1-\left(e^{2\pi i z}\right)q^t}$$

is the Appell-Lerch sum.

Actually, the expression on the previous slide is not the full story. It only includes the contributions of states in the above CFT that are normalizable. Since the target space is non-compact, there are also delta-function normalizable states, and it turns out that they too contribute to the elliptic genus.

Unlike the contribution of the normalizable states, that of the continuum is not holomorphic (in q). This takes one in the direction of Mock-modular forms – the contribution of the normalizable modes is holomorphic but not modular, while the full thing is modular but not holomorphic. We will not pursue it here.

Our interest is in the entropy of ¼ BPS states that the elliptic genus gives rise to. The number of states with given p, w can be read off the coefficient of q^N in the elliptic genus, where $N = pw$. This can be obtained by standard manipulations and gives rise for $N \gg 1$ to the entropy

$$S = 2\pi\sqrt{\left(2 - \frac{1}{k}\right)pw}$$

While this result was obtained for fivebranes placed equidistantly around a circle, it is actually independent of the positions of the fivebranes. This is a general property of the elliptic genus.

Thus, one might be tempted to conclude that it is also valid in the limit where the fivebranes coincide. Indeed, from the point of view of the theory on the fivebranes, separating them corresponds to Higgsing an $SU(k)$ gauge theory to $U(1)^{k-1}$.

When the mass of the W-bosons, M_W, is small, one might expect it to not influence the physics of massive states such as the ¼ BPS states we are studying.

We will next show that this expectation is not realized.

Black holes versus Strings

When the fivebranes are all coincident (i.e. at the origin of moduli space), the DSLST analysis breaks down due to strong coupling and we need to use other tools.

(in fact, it breaks down before that point, when the coupling becomes of order one, but we believe that as long as the fivebranes are not coincident, this is a technicality)

Exactly at the origin, there is another candidate for a state that has the same quantum numbers as the fundamental string states discussed above. This state is the (extremal) two dimensional black hole (the two dimensions being t, ϕ), charged under the $U(1)$ gauge fields obtained from reduction from three dimensions on the S^1.

This black hole has an exact worldsheet CFT description as a coset

$$\frac{SL(2) \times U(1)}{U(1)} \times SU(2) \times R^4$$

- The $U(1)$ that is being gauged is a combination of the CSA of $SL(2)$ and the extra $U(1)$. This combination depends on the charges p, w and non-extremality parameter (which we will set to zero).

- For large k one can describe it as a solution of Einstein-Maxwell gravity.

- In fact, this black hole is nothing but the three charge black hole of Strominger and Vafa, except we are viewing it as a state in the LST and not in the full string theory.

- The entropy of this black hole can be computed exactly. In the extremal (1/4 BPS case) it is given by the familiar

$$S = 2\pi\sqrt{kpw}$$

This looks qualitatively similar, but is different (larger) than the result for separated fivebranes we got before.

What is going on?

Before answering this, we need to revisit a point that we were a little careless about so far. So far we took the fivebrane worldvolume to be $R^4 \times S^1$. In that case the two dimensional string coupling, which is related to the mass of the black hole, M, is finite, but the six dimensional string coupling is infinite. Thus, to control the theory we need to replace the R^4 by a compact space, say T^4.

But now, the theory on the fivebranes lives in 0+1 dimensions, i.e. it is quantum mechanics. The positions of the fivebranes can no longer be fixed; instead, the vacuum is characterized by a wavefunction on the moduli space.

Superficially, the ground state wavefunction would be expected to spread over the whole moduli space, with points where fivebranes coincide being special points in the middle of moduli space.

What we have effectively discovered is that this is not the case. The QM one gets by compactifying LST on $S^1 \times T^4$ has non-trivial vacuum structure.

One vacuum corresponds to the quantization of the moduli space of distinct fivebranes. That branch has the high energy entropy of strings,

$$S = 2\pi\sqrt{\left(2 - \frac{1}{k}\right)pw}$$

Another vacuum corresponds to the quantization of the system of coincident fivebranes. This branch has the high energy entropy of black holes

$$S = 2\pi\sqrt{kpw}$$

And there are other vacua, characterized by numbers of coincident fivebranes $(k_1, k_2, \cdots, k_n)$.

Comments

- Note that the picture proposed above couldn't possibly be correct if instead of LST we had a local QFT. However, LST is not a local QFT, and the vacuum structure we found is directly related to this fact. In particular, it is a manifestation of UV-IR mixing in this theory. Classically, the different vacua are related by sending an IR scale (the mass of W-bosons) to zero, and yet they differ in their high energy behavior.

- It is instructive to generalize the discussion above to the non-extremal case. The entropy formulae we wrote down before have a simple generalization to that case. For strings one finds

$$S_{\text{pert}} = \pi\sqrt{2-\frac{1}{k}}\,l_s\left(\sqrt{M^2-q_L^2}+\sqrt{M^2-q_R^2}\right)$$

For black holes

$$S_{bh} = \pi\sqrt{k}\,l_s\left(\sqrt{M^2-q_L^2}+\sqrt{M^2-q_R^2}\right)$$

But now, the story is more interesting. The positions of the fivebranes are no longer moduli in this non-extremal case. A configuration of separated fivebranes is time dependent – the fivebranes attract each other and eventually collide.

If the non-extremality parameter is small, the timescale of this process is long, and we have the following picture. For a long period, we can use the non-extremal string entropy. However, for late times the string coupling grows, this description breaks down and the thermodynamics becomes that of black holes.

The sharp string-black hole transition observed in the BPS case can be understood by taking the late time and BPS limits in different orders.

- Relation to other work:

- String-black hole transition of A. Giveon, DK, E. Rabinovici, A Sever (2005).
- Witten's Coulomb and Higgs branch CFT.
- Fuzzballs.
- Horowitz-Polchinski string-black hole correspondence.
- Critical string thermodynamics (Atick-Witten).
- The previous talks in this session.