Can AdS backgrounds be classified?

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J. B. Gutowski, GP arXiv:1407.5652;
S. Beck, J. B. Gutowski, GP, arXiv:1410.3431, 1501.07620, 1505.01693

Supersymmetry of AdS backgrounds

- The classification of AdS supergravity backgrounds [Freund-Rubin] is a longstanding problem
- Applications include AdS/CFT, string, M-theory, and supergravity compactifications
- Warped, flux compactification to Minkowski space also arise in the limit of infinite AdS radius.
- Aim 1: Count the number of superymmetries preserved by M-theory, IIA and IIB AdS backgrounds
- Aim 2: Present a paradigm which answers the question posed

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Assumptions and applications

- For AdS_n, $n > 2$, no assumptions are made on the form of the fields apart from imposing on them the symmetries of AdS
- No assumptions are made on the form of Killing spinors
- For AdS_2 in addition it is assumed that the data are such that the maximum principle applies

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M-AdS

The a priori number of supersymmetries preserved by AdS backgrounds in M-theory are

$AdS_n \times_w M^{11-n}$	N
$n=2$	$2k, k<15$
$n=3$	$2k, k<15$
$n=4$	$4k, k\leq 8$
$n=5$	$8, 16, 24$
$n=6$	16
$n=7$	$16, 32$
$n>7$	$-$

Table: The proof for $AdS_2 \times_w M^9$ requires the maximum principle. For the rest, no such assumption is necessary. The bounds on k arise from the non-existence of supersymmetric solutions with near maximal supersymmetry [Gran, Gutowski, GP] and the classification results of [Figueroa-O'Farril, GP].

IIB-AdS

The a priori number of supersymmetries preserved by IIB AdS backgrounds are

$AdS_n \times_w M^{10-n}$	N
$n=2$	$2k, k<14$
$n=3$	$2k, k<14$
$n=4$	$4k, k<7$
$n=5$	$8k, k\leq 4$
$n=6$	16
$n>7$	$-$

Table: IIB backgrounds with more than 28 supersymmetries are maximally supersymmetric and there is a unique plane-wave background, up to a local isometry, with 28 supersymmetries.

IIA-AdS

The a priori number of supersymmetries preserved by (massive) IIA AdS backgrounds are

$AdS_n \times_w M^{10-n}$	N
$n = 2$	$2k, k < 16$
$n = 3$	$2k, k < 16$
$n = 4$	$4k, k \leq 7$
$n = 5$	$8, 16, 24$
$n = 6, 7$	16
$n > 7$	$-$

Table: There are less strict results on the existence of IIA backgrounds with near maximal supersymmetries than those for IIB backgrounds.

Sketching the proof

The warp, flux, AdS_n, $n > 2$, backgrounds can be written as

$$ds^2 = 2du(dr + rh) + A^2(dz^2 + e^{2z/\ell}\sum_{a=1}^{n-3}(dx^a)^2) + ds^2(M^{11-n}) ,$$

with

$$e^+ = du , \quad e^- = dr + rh , \quad h = -\frac{2}{\ell}dz - 2A^{-1}dA ,$$

A is the warp factor and ℓ the AdS radius.

- Solve the KSEs along the lightcone directions r, u
- solve the KSEs along z and then the remaining x^a coordinates
- count the multiplicity of Killing spinors

The solution of the KSEs along the AdS$_n$ directions gives

$$\begin{aligned}\epsilon &= \sigma_+ + \sigma_- + e^{-\frac{z}{\ell}}\tau_+ + e^{\frac{z}{\ell}}\tau_- \\ &\quad -\frac{1}{\ell}\Big(uA^{-1}\Gamma_{+z}\sigma_- + rA^{-1}e^{-\frac{z}{\ell}}\Gamma_{-z}\tau_+ \\ &\quad + \sum_a x^a\Gamma_{az}(\tau_+ + e^{\frac{z}{\ell}}\sigma_-)\Big)\end{aligned}$$

where $\Gamma_\pm\sigma_\pm = \Gamma_\pm\tau_\pm = 0$. The remaining independent KSEs on M^{11-n} are

$$D_i^{(\pm)}\sigma_\pm = 0\,, \quad D_i^{(\pm)}\tau_\pm = 0\,,$$

and

$$\mathcal{A}^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{B}^{(\pm)}\tau_\pm = 0\,,$$

- The integration over z introduces the new algebraic KSEs above

The counting

To count the multiplicity, it turns out that if $\sigma_\pm$ is a solution, so is

$$\tau_\pm = \Gamma_{z\mathrm{a}}\sigma_\pm$$

and vice-versa

Similarly, if σ_+, τ_+ is a solution, so is

$$\sigma_- = A\Gamma_-\Gamma_z\sigma_+ \ , \quad \tau_- = A\Gamma_-\Gamma_z\tau_+$$

and vice-versa.

Furthermore, if σ_- is Killing spinor, then

$$\sigma'_- = \Gamma_{\mathrm{ab}}\sigma_- \ , \quad \mathrm{a} < \mathrm{b} \ ,$$

is also a Killing spinor.

- The number of supersymmetries are derived by counting the linearly independent solutions

New Lichnerowicz theorems

One can establish new Lichnerowicz type theorems as

$$\mathscr{D}^{(\pm)}\sigma_\pm = 0 \Longleftrightarrow D_i^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{A}^{(\pm)}\sigma_\pm = 0\,,$$

These are based on maximum principle formulae

$$D^2 \parallel A^{-1}\sigma_- \parallel^2 + nA^{-1}\partial^i A\partial_i \parallel A^{-1}\sigma_- \parallel^2 = 2A^{-2}\langle \mathbb{D}_i^{(-)}\sigma_-, \mathbb{D}^{(-)i}\sigma_-\rangle + 2\frac{9n-18}{11-n}A^{-2} \parallel \mathcal{A}^{(-)}\sigma_- \parallel^2\,,$$

where $\mathbb{D}_i^{(-)} = D_i^{(-)} + \frac{2-n}{11-n}\Gamma_i\mathcal{A}^{(-)}$ and $\mathscr{D}^{(\pm)} = \Gamma^i\mathbb{D}_i^{(\pm)}$.

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- The Killing spinors of AdS backgrounds do not factorize into Killing spinors on AdS and Killing spinors on M^{11-n}
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Heterotic AdS backgrounds

Consider the heterotic theory up and including two loops in the sigma model perturbation theory.

THEOREM:

- There are no AdS_n supersymmetric backgrounds for $n > 3$
- There are no AdS_2 supersymmetric backgrounds provided that the maximum principle applies
- The warp factor of all AdS_3 backgrounds is constant and preserve 2, 4, 6 and 8 supersymmetries
- Smooth backgrounds that preserve 8 supersymmetries and M^7 is closed are locally isometric to $AdS_3 \times S^3 \times K_3$ or $AdS_3 \times S^3 \times T^4$

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Geometry

The geometry of AdS_3 backgrounds is as follows:

N	M^7	B^k	$fibre$
2	G_2	$-$	$-$
4	$SU(3)$	$U(3)$	S^1
6	$SU(2)$	$self-dual-Weyl$	S^3
8	$SU(2)$	$hyper-Kahler$	S^3

Table: The G-structure of M^7 is compatible with a connection with skew-symmetric torsion. For $N = 4, 6, 8$, M^7 is a local (twisted) fibration over a base space B^k with fibre either S^1 or S^3 .

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Applications

- There are new AdS_3 backgrounds preserving 8 supersymmetries with $B^4 = \mathbb{R}^4$
- There are new AdS_3 backgrounds preserving 4 supersymmetries
- There are new Lichnerowicz type of theorems allowing for curvature square terms
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

Summary

- The fractions of supersymmetry preserved by the most general warped flux AdS backgrounds in M-theory, IIA and IIB supergravities have been determined
- In heterotic theory, there are only AdS_3 solutions, the fractions of supersymmetry preserved have been determined as well as the geometry of the backgrounds
- A new class of Lichnerowicz type of theorems for GL connections has been found
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