THE CHAOTIC EIGENSTATE HYPOTHESIS AND FAST SCRAMBLING ON BH HORIZONS

: A QUANTUM ARNOLD CAT MAP TOY MODEL

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8TH CRETE REGIONAL MEETING
6-11 JULY 2015,NAFPLION,GREECE

I) QM+GR=>BLACK HOLE INFORMATION PARADOX

- STRECHED HORIZON HOLOGRAPHY OF THE BH PHYSICAL PROCESSES– (3+1=>2+1)

  EXPONENTIALLY FAST SPREADING AND MIXING
  OF INFALLING WAVE PACKETS
  TRANSVERSE AND LONGITUTINAL STRECHING OF INFALLING
  STRINGS WRAPPING THE HORIZON

  => CHAOTIC S-MATRIX OF THE STRECHED HORIZON

- COMPLEMENTARITY BETWEEN INFALLING-STATIONARY LOCAL OBSERVERS

- CONSTRAINTS ON THE BH QUANTUM INFORMATION PROCESSING

  SUBADDITIVITY-
  CAUSALITY –NO CLONING
  THE SCRAMBLING TIME BOUND

- II) AIM OF THE PAPER

- CONSTRUCTION OF A CONCRETE TOY MODEL
  FOR A UNITARY AND CHAOTIC S-MATRIX
  IN A **FINITE DIMENSIONAL** HILBERT SPACE,H, OF BLACKHOLE
  MICROSTATES **dimH=Exp[S]**

  APPROPRIATE TO DESCRIBE

- A) THE TRANSITION FROM LOCALIZED WAVE PACKETS TO
  COMPLETELY DELOCALIZED RANDOM PURE STATES
  REALIZING THE EIGENSTATE THERMALIZATION HYPOTHESIS
  OF PAGE-DEUTSCH-BERRY-SREDNICKI

- B) TO ALLOW AN EXPONENTIALLY FAST THERMALIZATION
  SATURATING THE SCRAMBLING TIME BOUND
  OF HAYDEN-PRESKILL, SEKINO-SUSSKIND

- III) SETTING UP THE PROBLEM

- OLD AND LARGE NEAR EXTREMAL BH'S

- ASSUME THAT THE BH MICROSTATES CAN BE DESCRIBED BY FINITE QUANTUM MECHANICS (FQM)

  OF THE MICROSCOPIC DEGREES OF FREEDOM

  LEAVING INSIDE THE STRECHED HORIZON'S UNIVERSAL

  GEOMETRY =AdS2 XΣ , Σ=COMPACT

- RESTRICT TO RADIAL MOTION AND DISCRETIZE
- AdS2[R]=SL[2,R]/SO[1,1,R] ➔ AdS2[N]=SL[2,ZN]/SO[1,1,ZN]

- CONSTRUCT THE AdS2 SUPERCONFORMAL S-MATRIX

  OF PROBE STRING BITS BY SL[2,ZN] ISOMETRY QUANTUM MAPS

- MOTIVATION
- CHECK
- A) IF THE CONSTRUCTION SUPPORTS

  AdS2[N]/CFT1[N] HOLOGRAPHY

B) SL[2,ZN] ISOMETRIES SATISFY

1)THE EIGENSTATE THERMALIZATION HYPOTHESIS

GAUSSIAN PDF OF EIGENSTATE' S PROB VALUES

FLAT PDF OF EIGENSTATE'S PHASES

2) SATURATION OF THE SCRAMBLING TIME BOUND

BASED ON OLD

- E. G. Floratos and J. Iliopoulos, *Phys. Lett. 201 (1988)*237.
- E. G. Floratos, *Phys. Lett. B228 (1989) 335*
- G. G. Athanasiu and E. G. Floratos, *Nucl. Phys. B425 (1994) 343.*
- G. G. Athanasiu, E. G. Floratos and S. Nicolis, *J. Phys. A: Math. Gen. 29 (1996)*6737.
- G.Leontaris,E.G.Floratos,PL B412(1997)35
- G. G. Athanasiu, E. G. Floratos and S. Nicolis, J.Phys.A:Math.Gen.31(1998)L655
- D.Ellinas,E.G.Floratos, *J. Phys. A: Math.Gen.32(1999)L63*

- AND RECENT WORK

- M.Axenides,E.Floratos,S.Nicolis
  JHEP 1402(2014)109

- Chaotic Information Processing by Extremal Black Holes
  M.Axenides,E.G.Floratos,S.Nicolis
  arXiv:1504.00483

- Quantum Cat Map Chaos and Fast Scrambling
  on discretized Blackhole Horizons
  M.Axenides,E.G.Floratos,S.Nicolis
  soon to appear in arXiv-hep/th

HAWKING(1976)- THE BH INFORMATION PARADOX

A) ALICE - FREELY FALLING INTO THE BH
(KRUSKAL COORDINATE FRAME-FULL BH SPACE-TIME)

B) BOB - STATIONARY FAR OUTSIDE THE HORIZON
(SCHWARZILD COORDINATE FRAME-EXTERIOR SOLUTION)

IS THERE A UNITARY CONNECTION OF OBSERVATIONS MADE BY ALICE , BOB AT THEIR RESPECTIVE LABORATORY FRAMES ?
EX: INFALLING WAVE PACKETS OF PARTICLES ,(OR STRINGS)
(PURE STATES)

- **HAWKING(1976): ANSWER NO**
- **WE MUST EXTEND QUANTUM MECHANICS TO DESCRIBE EVOLUTION BETWEEN PURE AND MIXED STATES**
  INGOING PURE STATE-OUTGOING HAWKING RADIATION!

- **T'HOOFT(1984-90): ANSWER YES**
- **WE MUST ENCODE ALL THE 3D INFALLING INFORMATION ON THE STRECHED HORIZON BY A 2D HOLOGRAPHIC UNITARY SCATTERING MATRIX**
  AND THE OUTGOING HAWKING RADIATION IS NOT REALLY THERMAL BUT ENCODES ALL
  THE INFALLING INFORMATION THROUGH ITS CORRELATIONS

HORIZON HOLOGRAPHY -COMPLEMENTARITY – CHAOS

SUSSKIND-’T HOOFT (1993-1994):

BLACK HOLE COMPLEMENTARITY PRINCIPLE =>
INFALLING STRING AT STRING SCALES FROM THE HORIZON SUFFERS LARGE TRANSVERSE AND LONGITUTINAL STRECHING AND WRAPS EXPONENTIALLY FAST THE STRECHED HORIZON =>CHAOS AND HOLOGRAPHY

1) FOR BOTH OBSERVERS , TOTAL HILBERT SPACE H=HA x HB
NO VIOLATION OF ANY PHYSICAL LAW

2) THEIR OBSERVABLES ARE COMPLEMENTARY LIKE POSITION AND MOMENTUM

THOSE OF THE INFALLING OBSERVER CAN BE RECONSTRUCTED FROM THE OBSERVABLES OF THE STATIONARY ‘s ONE (and vice versa).

THE CORRESPONDING “FOURIER TRANSFORM”
IS A CHAOTIC UNITARY MATRIX-RELATED TO THE S-MATRIX of
PURE INFALLING STATES TO UNITARY THERMAL HAWKING RADIATION

3) ALL THE INFORMATION OF THE INFALLING OBSERVER IS PROCESSED BY THE INTERIOR OF THE BLACK HOLE CHAOTIC DYNAMICS .

BECAUSE OF NO-LOSS OF INFORMATION AND UNITARITY IT SHOULD BE HOLOGRAPHICALLY STORED ON THE STRECHED HORIZON (THROUGH CHAOTIC MIXING)

dimH(Hilbert space of Horizon's microstates)

=Exp[A/4] , A=AREA OF THE HORIZON

4) THE OUTGOING HAWKING RADIATION AFTER HORIZON THERMALIZATION, IT ENCODES THE HORIZON'S STORED INFORMATION IN ITS CORRELATIONS

CONSTRAINTS ON THE BH QUANTUM INFORMATION PROCESSING AND THE SCRAMBLING TIME BOUND

1) FROM THE LAWS OF BLACK HOLE THERMODYNAMICS
IT FOLLOWS THAT THE MICROSCOPIC DEGREES OF FREEDOM
ON THE STRECHED HORIZON MUST BE IN THERMAL EQUILIBRIUM
(MAXIMUM OF ENTROPY)

2) $S[AC]+S[BC]>S[ABC]+S[B]$ A=BLACK HOLE INTERIOR,B=BLACK HOLE EXTERIOR,C=STRECHED HORIZON

3)ANY NEW QUBIT NEEDS TIME FOR THERMALIZATION
BEFORE INFORMATION TRANSER TO HAWKING RADIATION
CAN TAKE PLACE

4) FROM THE GENERAL LAWS OF QUANTUM INFORMATION PROCESSING
THERE ARE CONSTRAINTS ON THE TIME DURATION OF THE PROCESSING-

WE NEED FAST HOLOGRAPHIC STORAGE OF INFORMATION ON THE STRECHED HORIZON THROUGH STRONG CHAOTIC MIXING

- (PAGE 1993-94) BASIC ASSUMPTION :
  CHAOTIC UNITARY THERMALIZATION OF THE SUBSYSTEMS IN THE INTERIOR OF BH HAAR MEASURE UNITARY ENSEMBLE=>

  PAGE TIME =R/2,(RADIATION OF HALF MASS)

  TIME TO ENCODE INFORMATION (THROUGH QUANTUM ENTANGLEMENT) IN HAWKING RADIATION
  TIME AFTER WHICH THERE IS MAXIMUM ENTANGLEMENT FOR EACH ADDITIONAL QUBIT OF INFALLING INFORMATION

- **2007 PRESKILL-HAYDEN**

MUCH FASTER ENTANGLEMENT OF THE BH INTERIOR AND EXTERIOR REGIONS
IN ORDER TO AVOID QUANTUM CLONING OF THE SAME QUBITS
BY EXTERIOR AND INTERIOR OBSERVERS
(NICE-SLICE OBSERVERS EXIST!)

IT IMPLIES FAST CHAOTIC MIXING ON THE STRECHED HORIZON
EXPONENTIALLY FAST DIFFUSION(BUT HOW?DYNAMICS)

BH SCRAMBLING TIME = Log[R/lp]

- **2008 SUSSKIND-SEKINO**

SCRAMBLING TIME BOUND CONJECTURE

CHAOTIC SCRAMBLING-NON LOCAL HYPERDIFUSION
MATRIX MODEL INTERACTIONS FOR THE MICROSCOPIC DOF'
(CAUSAL SATURATION OF SCRAMBLING TIME)

BLACK HOLES ARE THE UNIVERSE'S FASTEST SCRAMBLERS!

EXTREMAL BH'S
NEAR HORIZON AdS2 UNIVERSAL GEOMETRY
AND
SUPERCONFORMAL DYNAMICS

EXTREMAL BLACK HOLES, U[1] CHARGES=> MASS=CHARGE

- TEMPERATURE=ZERO, NO HAWKING RADIATION
- FINITE ENTROPY=AREA=FUNCTION OF U[1] CHARGES(?)
- SINGLE HORIZON,TIME LIKE INTERIOR AND EXTERIOR REGIONS
- UNIVERSAL AdS2 RADIAL AND TIME GEOMETRY,IN THE NEAR HORIZON REGION=>SL[2,R] ISOMETRY , DOUBLE BOUNDARY

- SCQM FOR BPS PARTICLES IN THE NEAR HORIZON REGION

  (TOWNSEND,STROMINGER,KALOSH,…)

- GROUND STATE FOR REISNER -NOESTROM , ALL HAWKING RADIATION GONE - ENERGY(M-Q)

- VAFA-STROMINGER -MICROSCOPIC COUNTING PARTITION FUNCTIONS OF SUSY BH ,BOUND STATES OF BPS- STRINGS, D-BRANES
- CLASSIFICATION OF EXTREMAL SUSY BH IN 4D AND 5D SUPERGRAVITIES (FERRARA,SEN,VERLINDE,MOORE)

  LATTICE ORBITS OF THE DUALITY GROUPS,WALL CROSSING
- UNIVERSAL ATTRACTOR MECHANISM

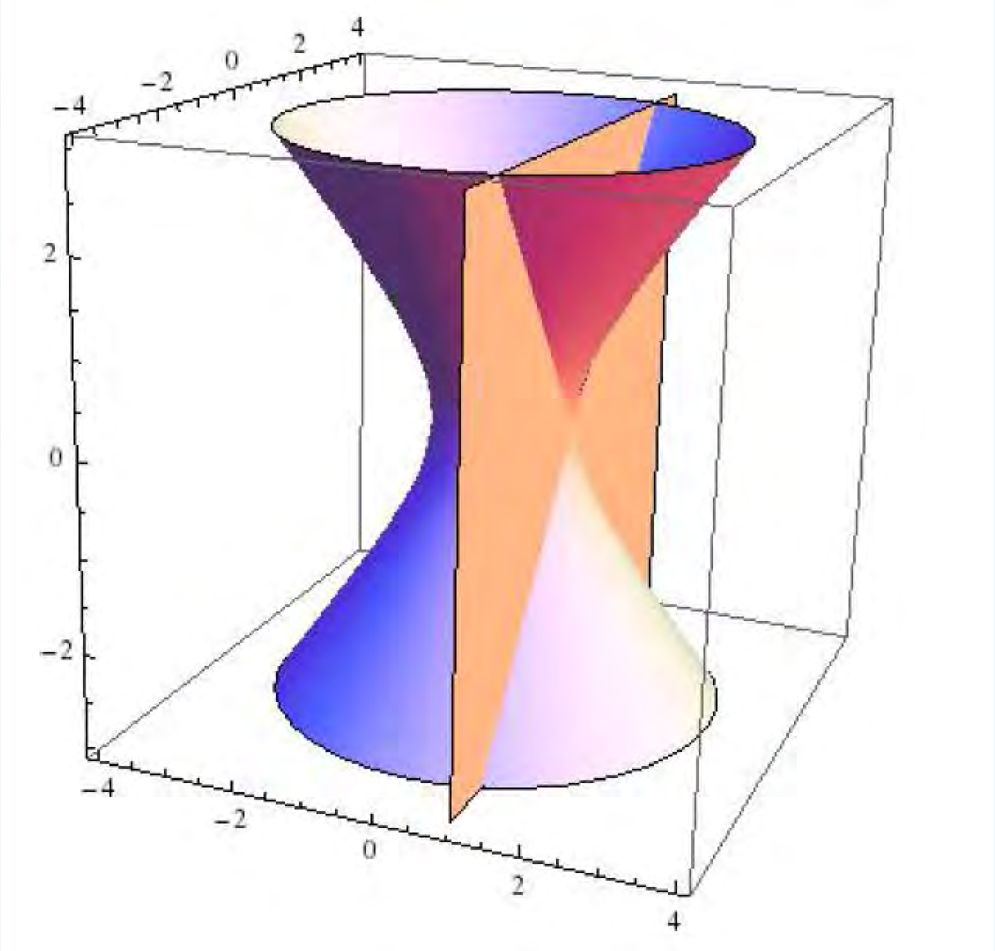
  (FERRARA,KALOSH,SEN..)
- AdS2/CFT1 (SL[2,R]=1D CONFORMAL GROUP)

  (STROMINGER,SEN, …)
- CLASSICAL ENTROPY AS ENTANGLEMENT ENTROPY OF THE TWO BOUNDARIES (TAKAYANAGI 2008)
- EXACT QUANTUM ENTROPY –AS QUANTUM ENTANGLEMENT OF THE TWO BOUNDARY CONFORMAL 1D SYSTEMS

  (SEN,DABLOKHAR,PIOLINE)

AdS2=SL[2,R]/SO[1,1]

$$x_0^2 + x_1^2 - x_2^2 = 1$$



WEYL ACTION OF SL[2,R] ON AdS2

To every point $x_\mu \in \mathrm{AdS}_2$, $\mu = 0, 1, 2$, we assign the traceless and real, 2×2 matrix

$$\mathsf{M}(x) \equiv \begin{pmatrix} x_0 & x_1 + x_2 \\ x_1 - x_2 & -x_0 \end{pmatrix} \tag{2.3}$$

Its determinant is $\det \mathsf{M}(x) = -x_0^2 - x_1^2 + x_2^2 = -1$.

The action of any $\mathsf{A} \in SL(2,\mathbb{R})$ on AdS_2 is defined through the non-linear mapping

$$\mathsf{M}(x') = \mathsf{A}\mathsf{M}(x)\mathsf{A}^{-1} \tag{2.4}$$

This induces an $SO(1,2)$ transformation on $(x_\mu)_{\mu=0,1,2}$,

$$x' \equiv \mathsf{L}(\mathsf{A})x \tag{2.5}$$

Choosing as the origin of coordinates the base point $\boldsymbol{p} \equiv (1,0,0)$, its stability group $SO(1,1)$ is the group of Lorentz transformations in the $x_0 = 0$ plane of $\mathscr{M}^{1,2}$ or equivalently, the “scaling” subgroup D of $SL(2,\mathbb{R})$

$$\mathsf{D} \ni \mathsf{S}(\lambda) \equiv \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix} \tag{2.6}$$

for $\lambda \in \mathbb{R}^*$.

For this choice of the stability point, we define the coset h_A by decomposing A as

$$\mathsf{A} = h_\mathsf{A}\mathsf{S}(\lambda_\mathsf{A}) \tag{2.7}$$

Thus, we associate uniquely to every point $x \in \mathrm{AdS}_2$ the corresponding coset representative $h_\mathsf{A}(x)$.

ARITHMETIC DISCRETIZATION OF AdS2=SL[2,R]/SO[1,1] =>AdS2[N]=SL[2,Z[N]]/SO[1,1,[Z[N]]

X0^2+X1^2-X2^2=1 MOD[N]

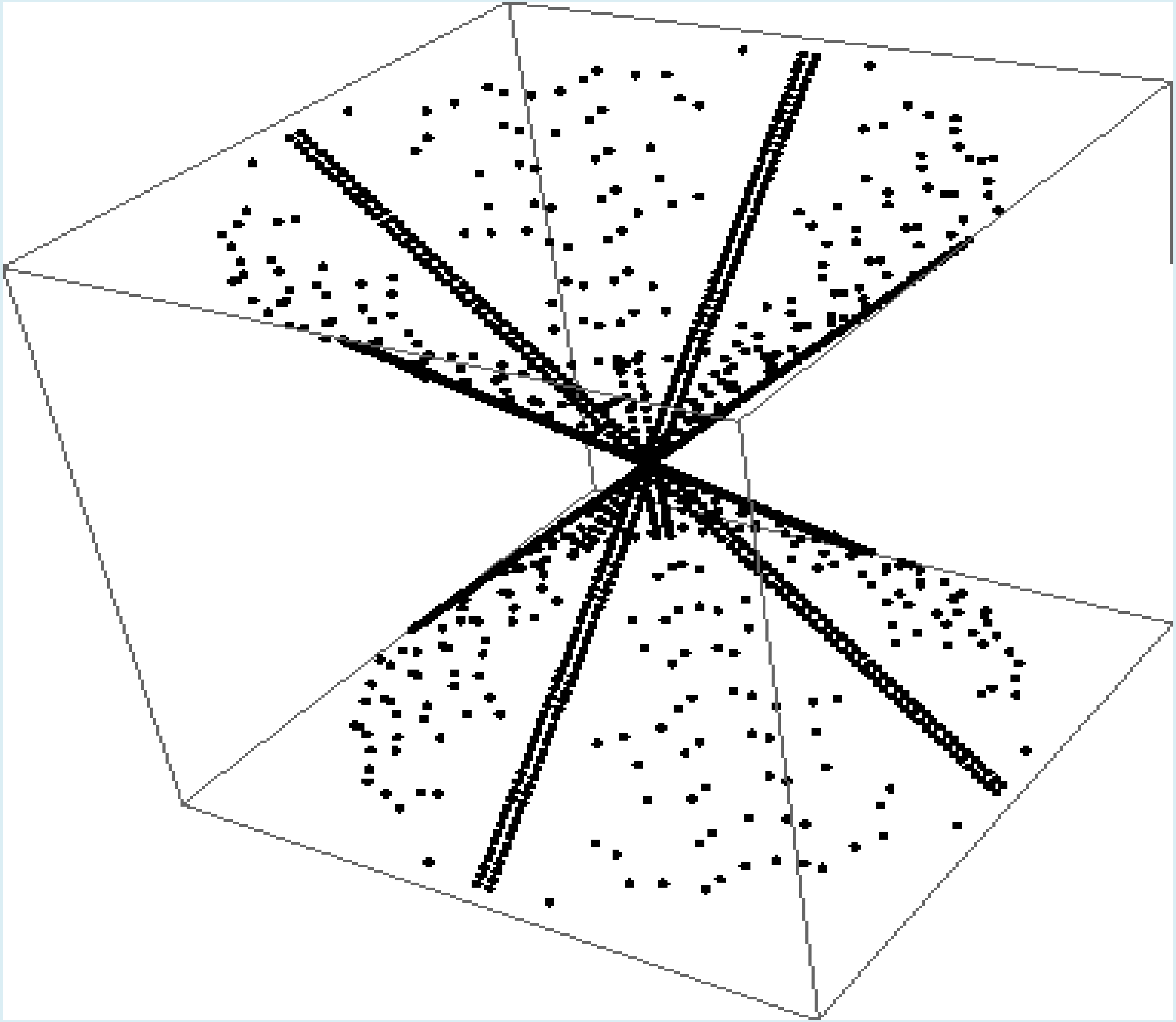
ALL INTEGER SOLUTIONS MOD[N]=> DISCRETE SET OF POINTS=ADS2[N]

X0=A- M B,
X1=B+M A
X2=M A,B,M=0,1,..,N-1

A^2+B^2=1 MOD[N], DISCRETE CIRCLE S1[N]

ROTATING THE 'LINE', { X0=0,X1=M,X2=M M=0,1,2,..,N-1} AROUND S1[N]

SO[1,1,Z[N]] STABILITY GROUP OF P: X0=1,X1=X2=0



DISCRETE CONFORMAL DYNAMICS

ARNOLD CAT MAP A={{1,1},{1,2}},

WEYL ACTION ON AdS2[N]

PROPERTIES

a)STRONG ARITHMETIC CHAOS

(ARNOLD,FORD,BERRY VOROS,VIVALDI,DI VIZENZO)

b)HOLOGRAPHY=>NON LOCAL REDUNDANT STORAGE OF INFORMATION

c)MIXING TIME -LIAPUNOV EXPONENT

d)GENERATION OF KOLMOGOROV –SINAI ENTROPY

BASIC PROPERTY OF ARNOLD CAT MAP =FIBONACCI CHAOS

A^n={{f(2n-1),f(2n)},{f(2n),f(2n+1)}},

f(n) FIBONACCI INTEGERS

f[n+1]=f[n]+f[n-1];f[0]=0,f[1]=1

FOR ANY INTEGER N

Periods of A MOD[N] A^(T(N))=IdentityMatrix MOD[N]

DYSON:IF N=f[m] → T(N)=2 m

BUT SINCE FOR m->INFINITY f[m]->Exp[c m],c=log[Goldenratio]

WE OBTAIN T[N]->Log[N]

LOGARITHMIC TIME CHAOTIC MIXING(SCRAMBLING)
For FIBONACCI SEQUENCE OF INTEGER HILBERT SPACE DIMENSIONS

KOLMOGOROV ENTROPY

PROCESSING OF INFORMATION BY AREA PRESERVING MAPS

GENERALIZED ARNOLD CAT MAPS A={{a,b},{c,d}} in SL[2,Z[N]]
(r,s)->(r,s)A ,ModN

MIXING TIME= ½ OF THE PERIOD OF THE MATRIX A

ARNOLD CAT MAP MIXES THE ENTIRE PHASE SPACE IN LOGARITHMIC TIME

T[MIXING]=LOG[N]=S[KOLMOGOROV ENTROPY]

EXACT QUANTIZATION OF CAT MAPS

ω=Exp[2 π i/N], Planck constant , h=2 π/N

Q=Diag{1, ω, ω^2,.., ω^(N-1)}
P={{0,0,..,1},{1,0,0,..,0},{0,1,..,0},..{0,0,..,0,1,0}}

QP=ωPQ   Heis-Weyl CR'S

1- QBIT PER MINIMUM PHASE SPACE AREA-
Q-HALL EFFECT ,UNIT OF FLUX 1/N

QF=FP,

FINITE FOURIER TRANSFORM: FFT , F, UNITARY NxN MATRIX,(F^4=I)

F(k,l)= ω^(k l) /Sqrt[N] k,l=0,1,2,…,N-1

FINITE –DISCRETE QUANTUM MECHANICS

J[r,s]=ω^(rs/2)P^r Q^s,

Heis-Weyl group

QUANTIZATION 0F CAT MAPS A=>U[A],

UNITARY N DIM IRREP OF SL[2,Z[N]]

U[A]J[r,s]U[A]^(-1)=J[(r,s)A] (WEIL)

U[AB]=U[A]U[B] FOR N=PRIME INTEGER->EXACT- NOT PROJECTIVE!

ONE-TIME STEP EVOL OF STATES=>SCHRODINGER EQUATION FOR DISCRETE MAPS

|n+1>=U[A]|n> , |0> A GIVEN INITIAL STATE

- <u>U[A](k,l)=1/Sqrt[N] ω^[-1/2b (ak^2+d l^2-2 k l)], k,l,=0,1,2,..,N-1</u>

  U[A]^n=U[A^n],

EXAMPLES

1)HARMONIC OSCILLATOR   A={{0,-1},{1,0}}

U[A](k,l)=1/Sqrt[N] ω^(kl)=F , Q-FOURIER=FFT

HARMONIC OSCILLATOR GROUP SO[2,Z[N]]
(BALIAN-ITZYKSON 1986)

A={{a,-b},{b,a}}   a^2+b^2=1 MOD[N},a,b=0,1,2,..,N-1
FOR N=prime integer=4 k+(-)1,ORDER OF THE GROUP 4k.

QUANTIZATION OF THE AdS2[N] STRECHED HORIZON

- POINTS =>h =COSETS OF SL[2,N]/SO[1,1,N]
- QUANTUM POINTS |h>=U[h] |0>
- |0> GROUND STATE OF U[SO[1,1,N]]
- NON-COMMUTATIVE MANIFOLD

  THROUGH COHERENT STATES(H.GROSSE)

  CLASSICAL ACTION OF ISOMETRIES

  |h>=>U[A]|h>=|Ah>

ANY INTEGER (COARSE GRAINING)FACTORIZATION,

- N=N1xN2 ,N=Exp[R^2], R^2=>R1^2 + R2^2
- SL[2,Z[N]]=SL[2,Z[N1]]xSL[2,Z[N2]]
- A[N]=A[N1]A[N2]
- H[N]=H[N1]+H[N2], SCHWINGER HEIS-WEYL FACTORIZATION
- U[A[N]]=U[A[N1]]U[A[N2]]

FAST QUANTUM MAPS N^2->N LogN

- S[N]=S[N1]+S[N2] ADDITIVITY OF COARSE GRAINED ENTROPIES

EIGENSTATE THERMALIZATION SENARIO
PAGE,DEUTSCH,BERRY,SREDNICKI

IF THE EIGENSTATES OF A CLOSED QM SYSTEM ARE RANDOM

(RANDOM PHASES AND GAUSSIAN DISTRIBUTED AMPLITUDES)

THEN ANY INITIAL PURE STATE OF A SUBSYSTEM THERMALIZES TO
THE THERMAL DENSITY MATRIX OF THE SUBSYSTEM

RELATION TO THE INFORMATION PARADOX

2013 SREDNICKI TALK TO KITP

ARNOLD QUANTUM CAT MAP A={{1,1},{2,1}}

EXACT CONSTRUCTION OF THE SPECTRUM AND EIGENSTATES FOR N=p,prime,

LINEAR SPECTRUM

RANDOM EIGENSTATES ->LINEAR COMBINATIONS OF MULTIPLICATIVE CHARACTERS OF GF[P]

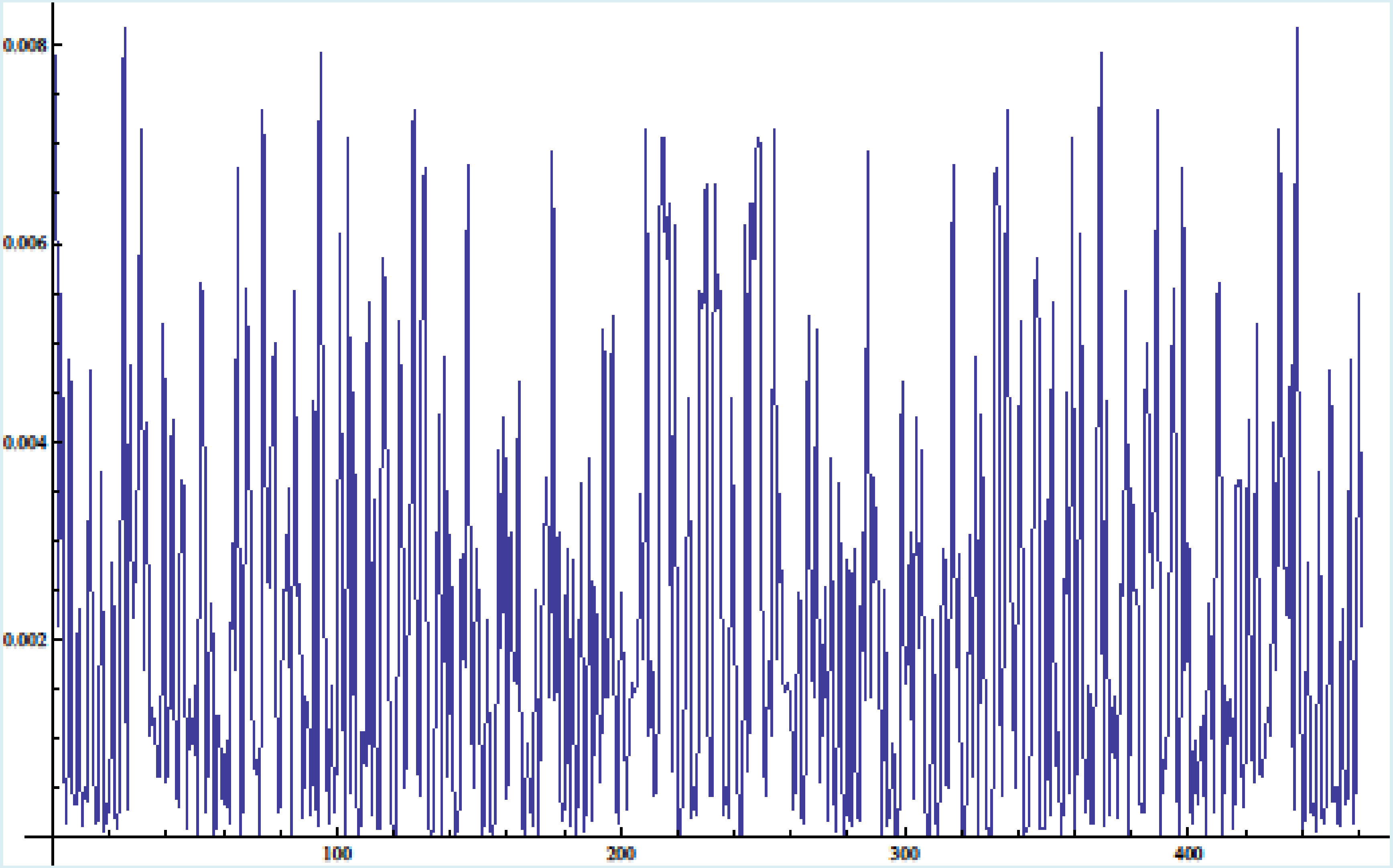
1)RANDOM PHASES
2)GAUSSIAN RANDOM AMPLITUDES

BUT SCARS(VOROS.,NONEMACHER)F
FOR SEQUENCES OF N's WITH SHORT PERIODS

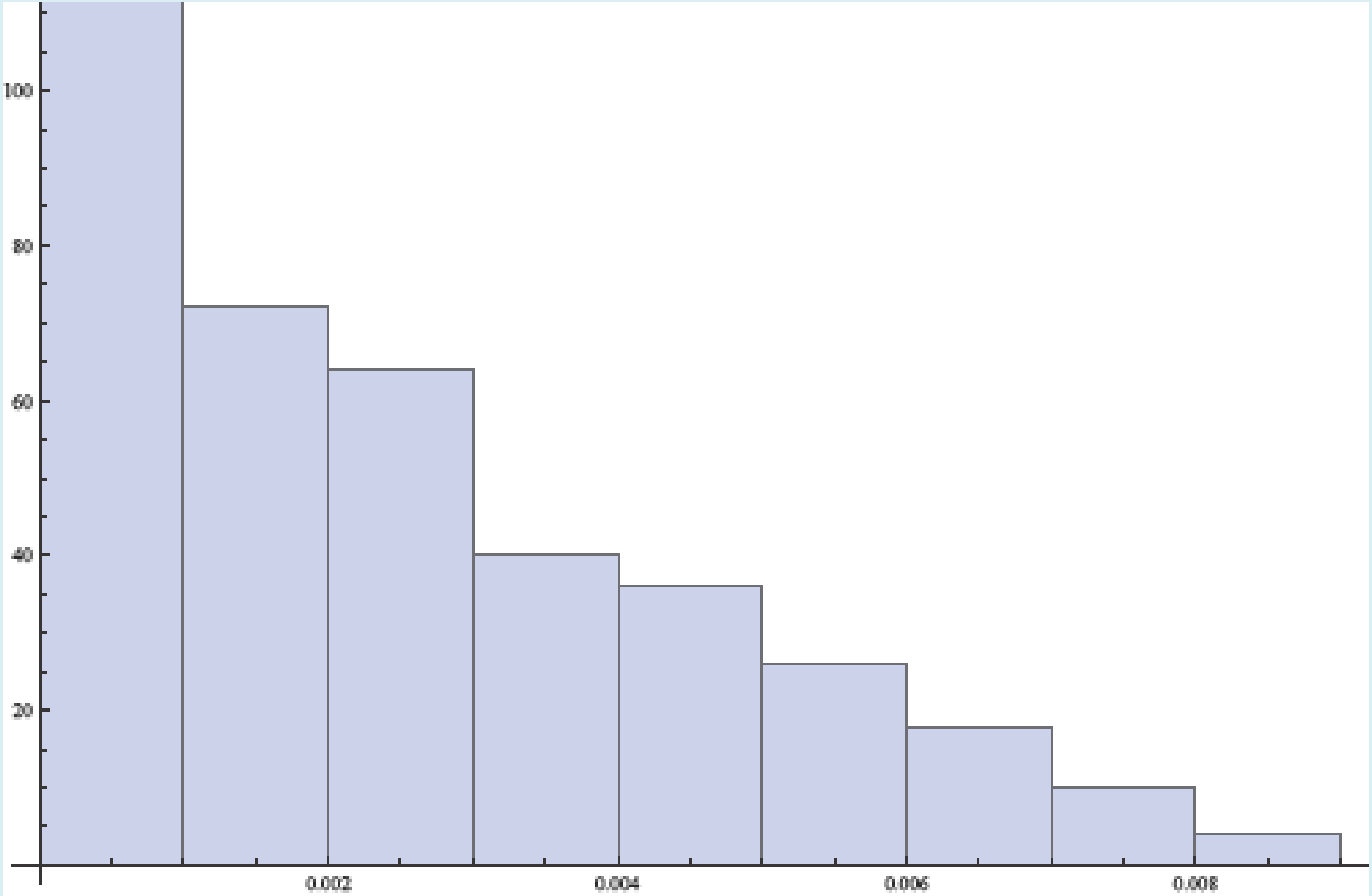
QUANTUM CHAOS, STRONG MIXING

- FACTORIZATION FOR ARNOLD CAT MAPS IMPLIES LOGARITHMIC IMPROVEMENT FROM N^2->N LOGN
- USING QUANTUM CIRCUITS FOR THE IMPLEMENTATION OF THE QUANTUM MAP AND COUNTING THE NUMBER OF GATES NLOGN->(LOGN)^2
- EXACTLY AS FOR THE QUANTUM FOURIER FACTORIZATION ALGORITHM OF SHOR.

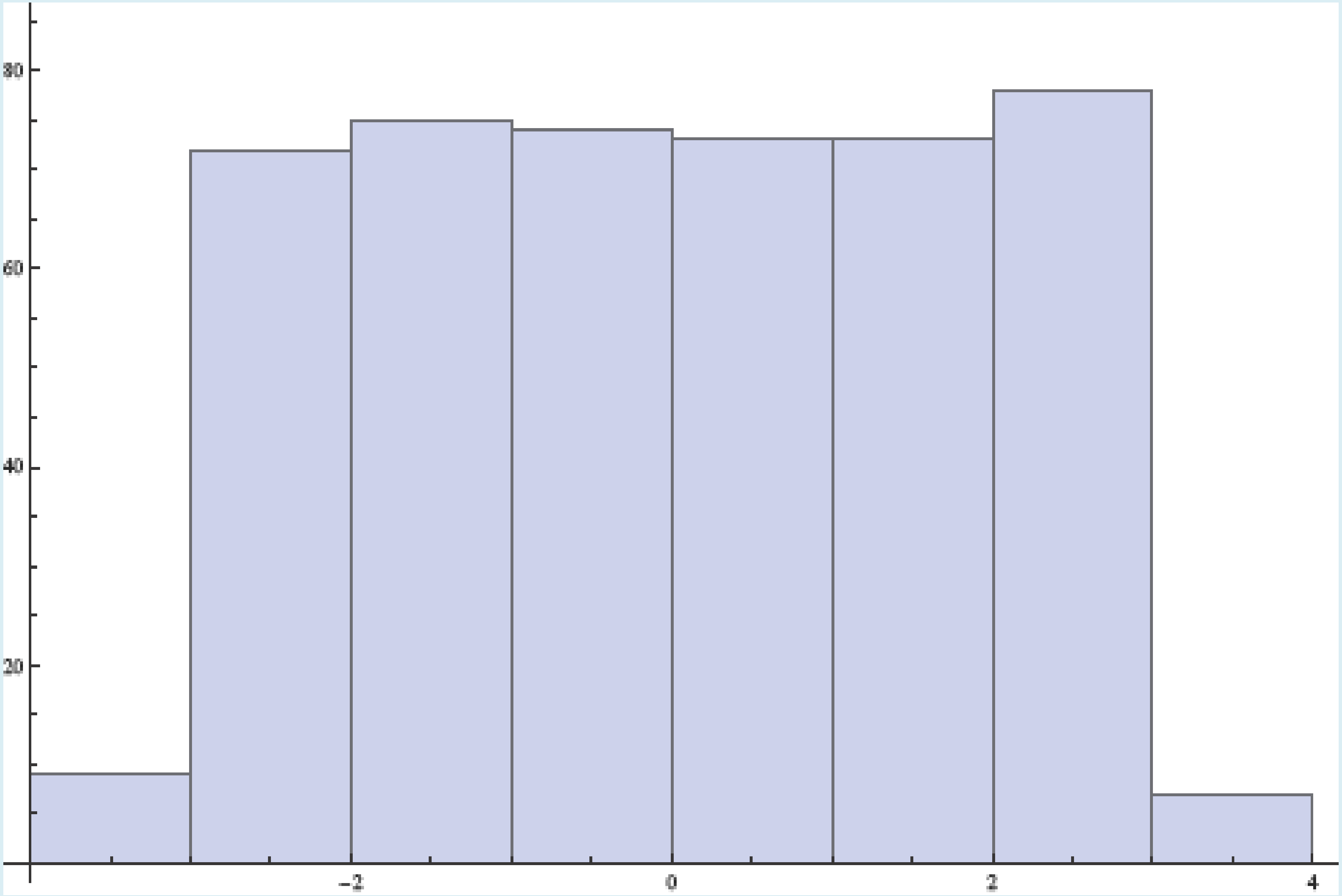
- N=461,T[461]=23 ,GROUND STATE,PHI=0



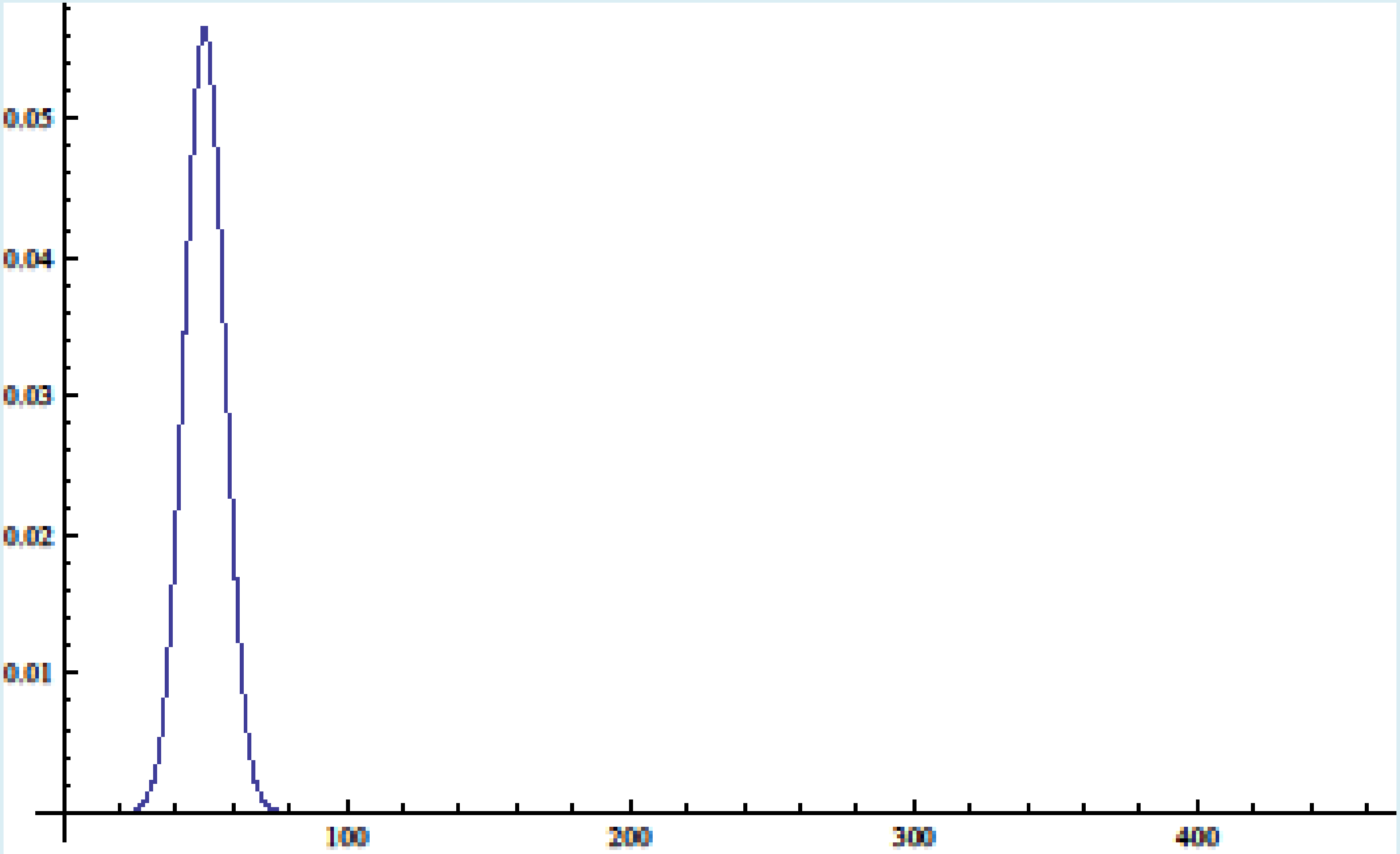
- GROUND STATE AMPLSQUARE DF



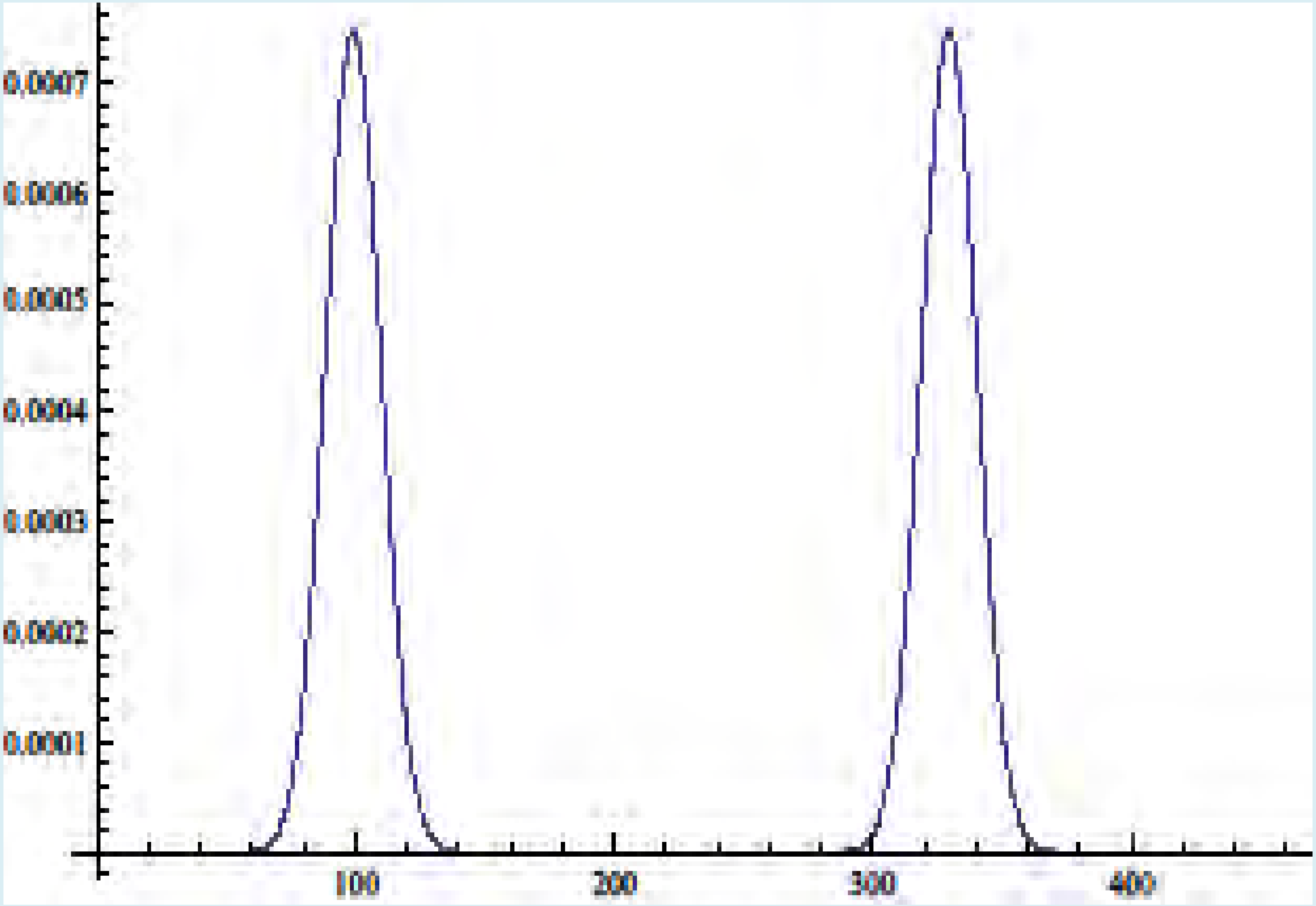
- GROUND STATE PHASE DF



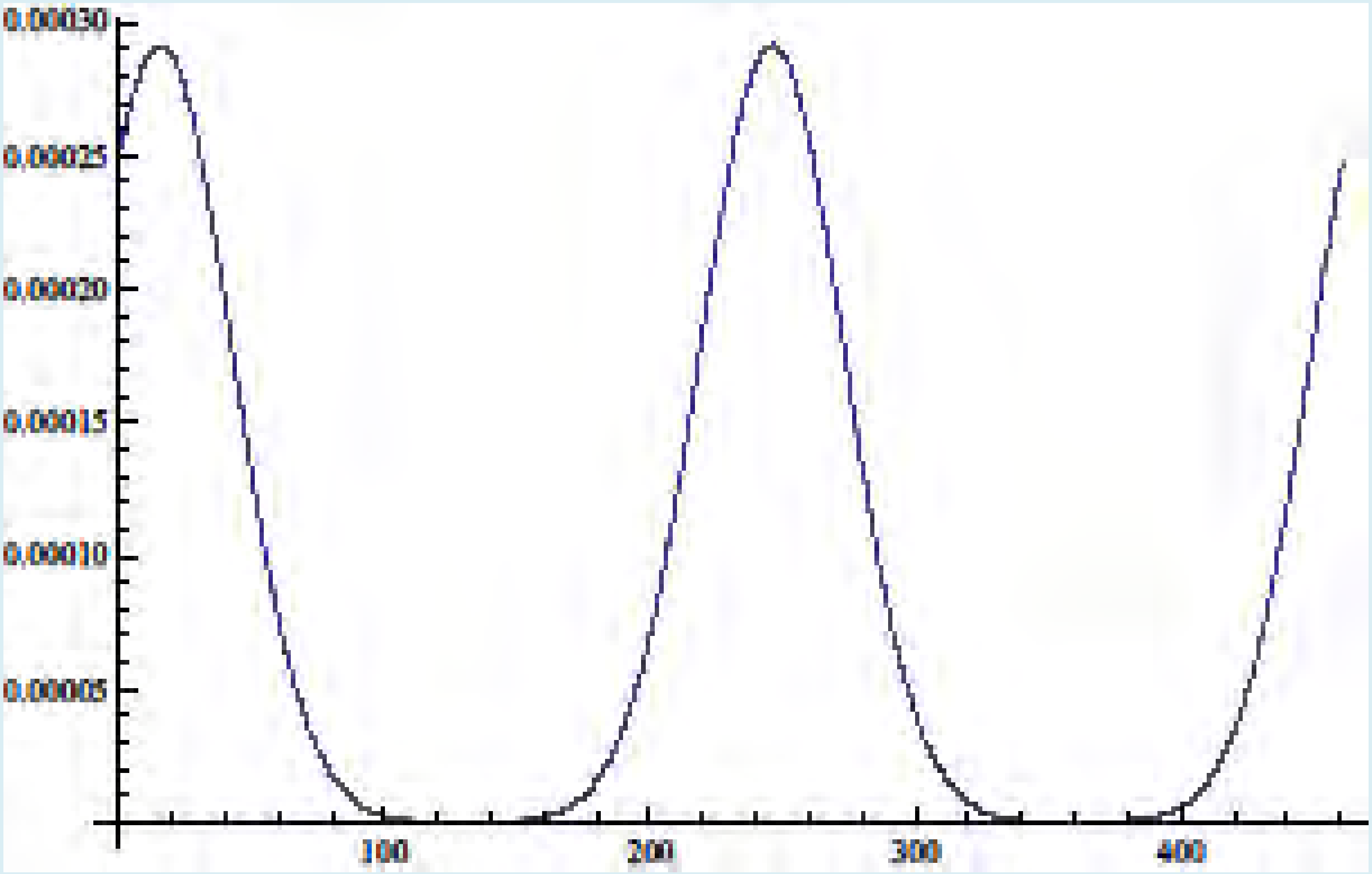
- SCATTERING EXPERIMENT, N=p=461
- GAUSSIAN WF AT T=0



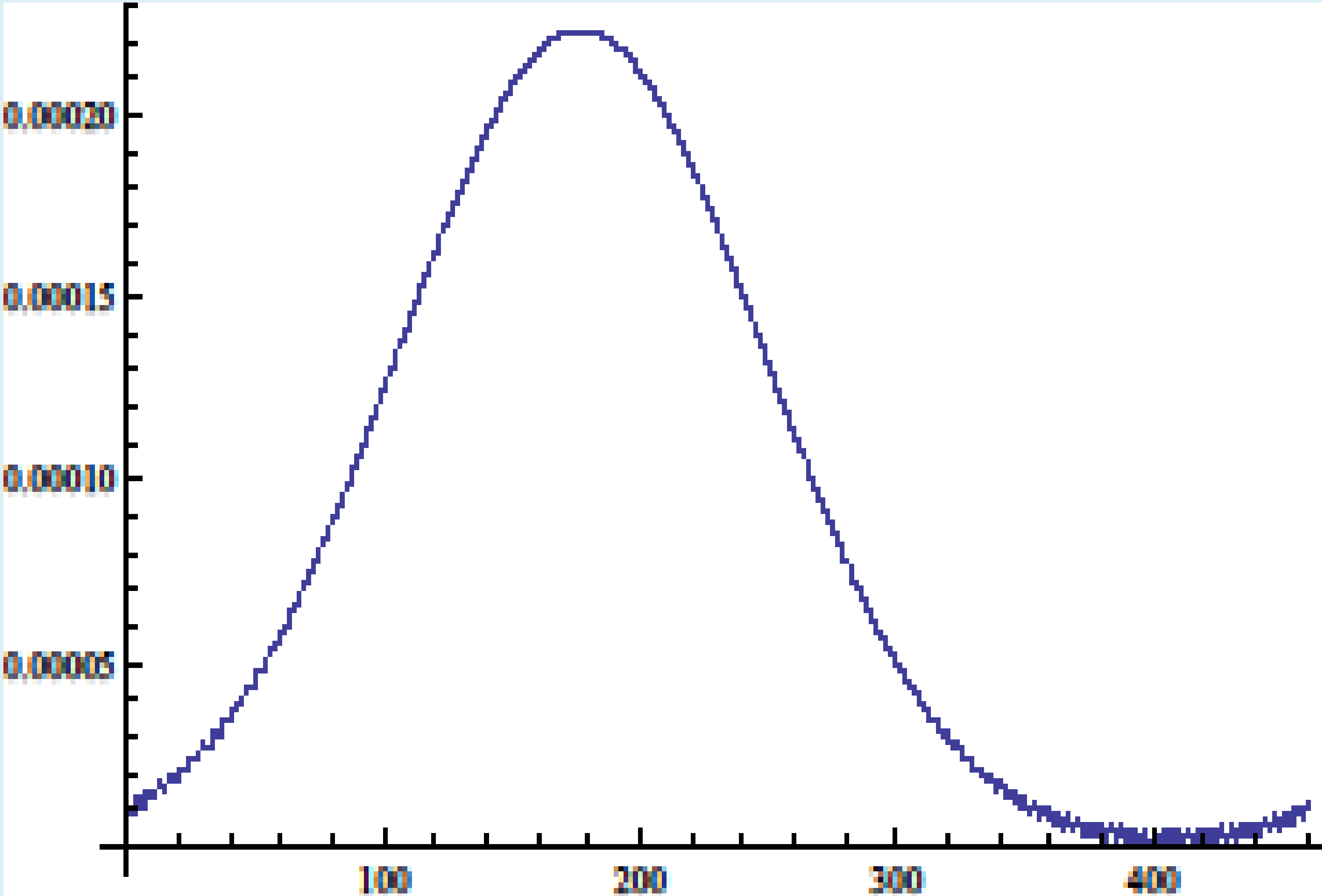
- TIME=1



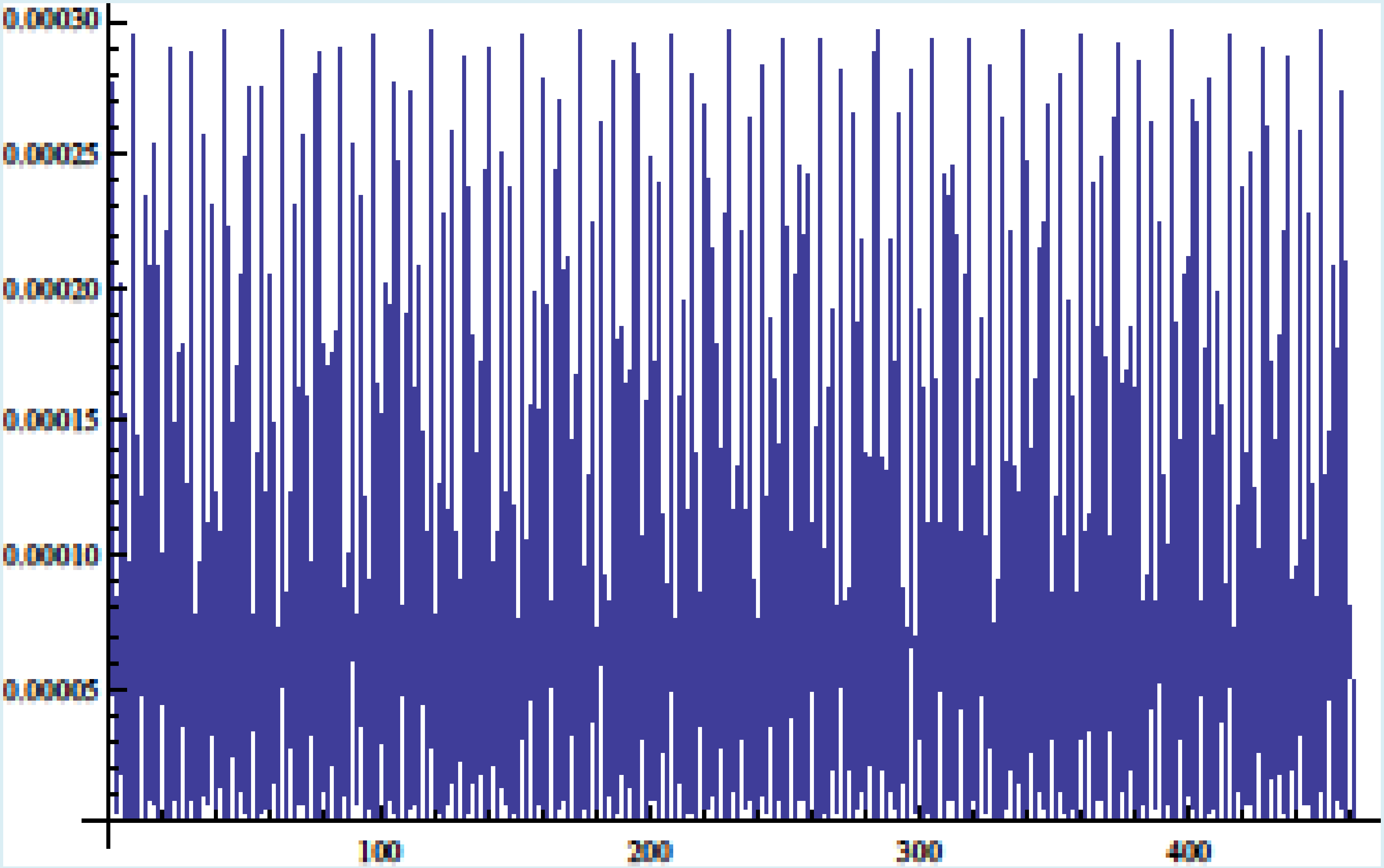
- T=3



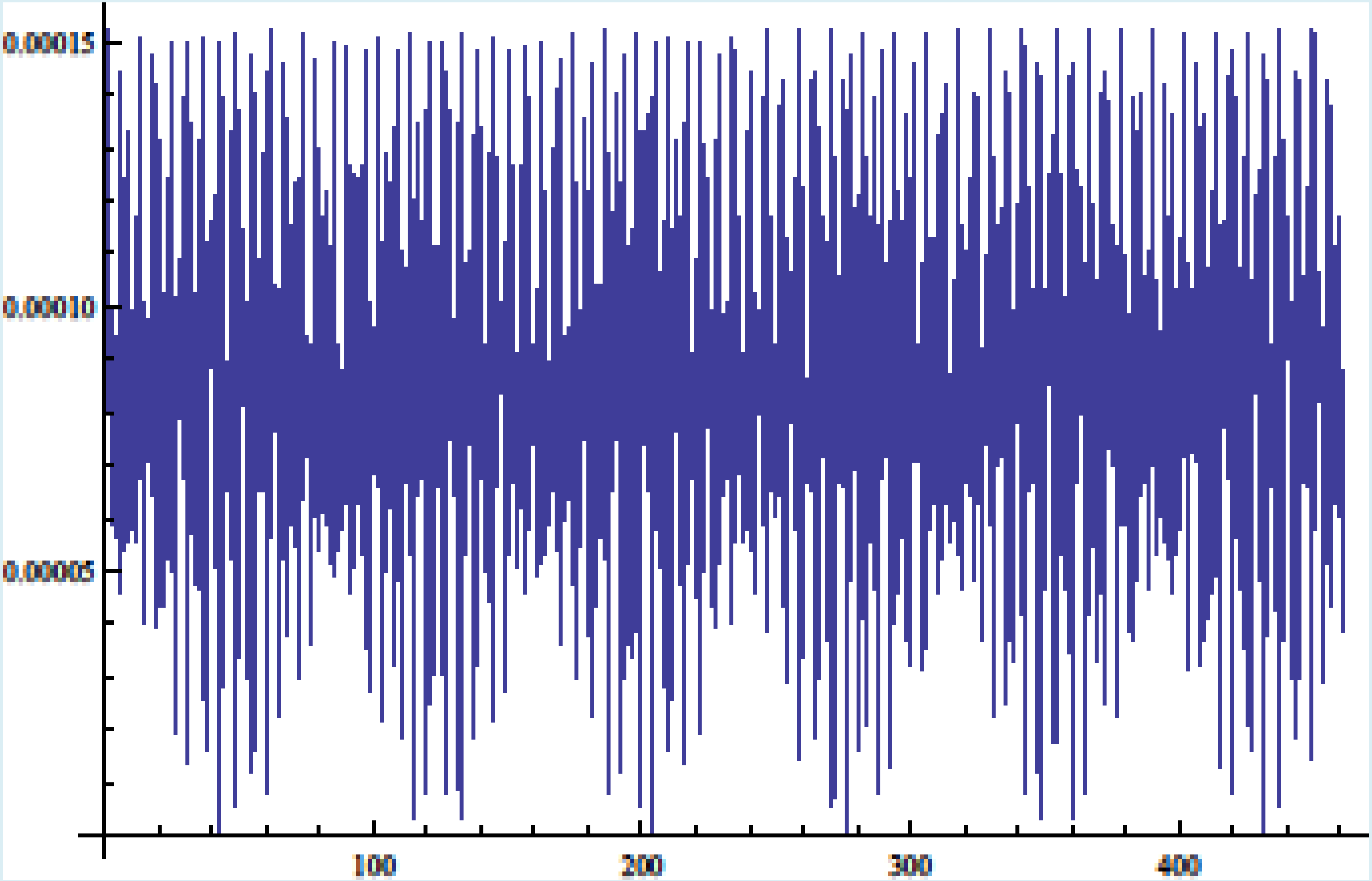
- T=3



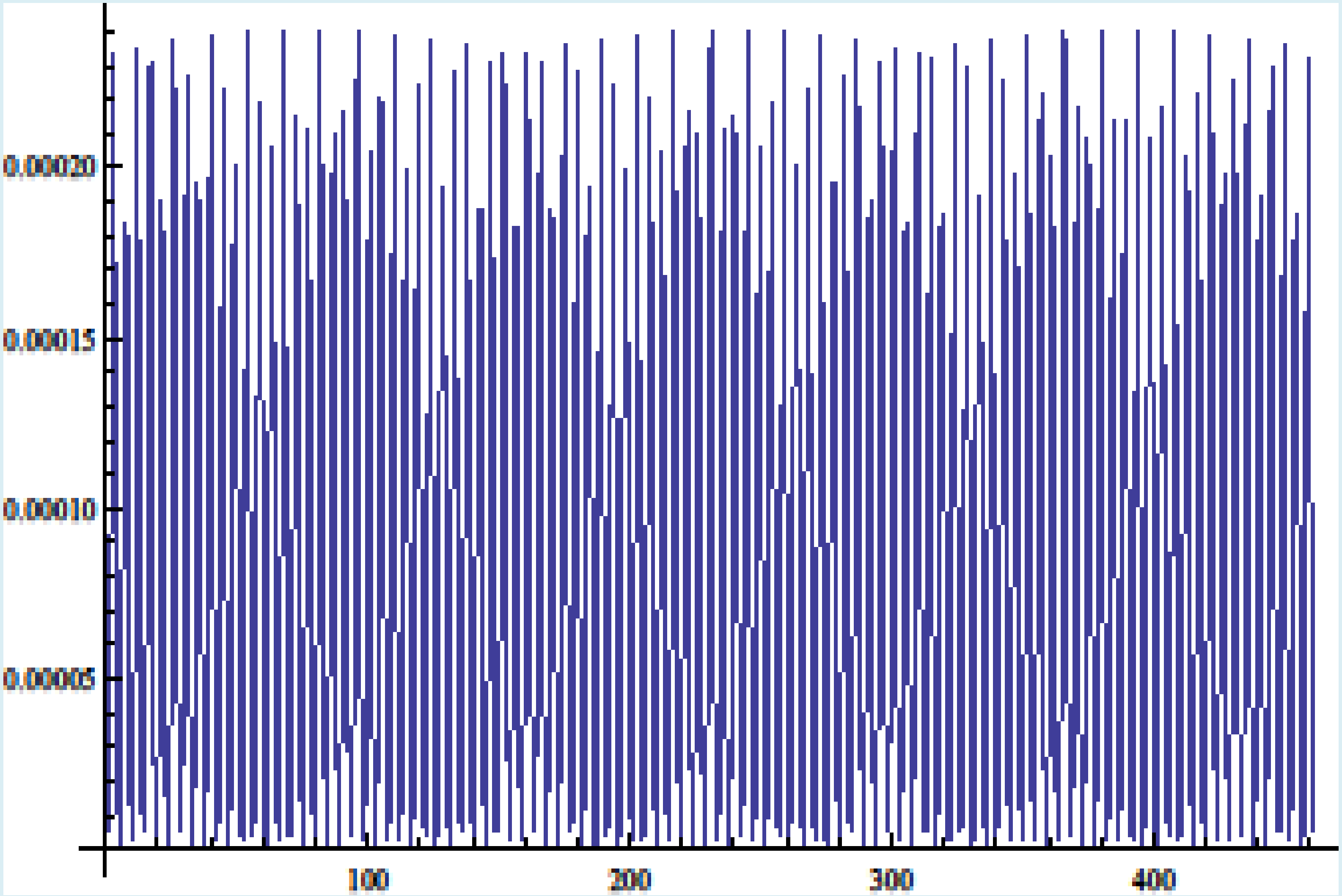
- T=4



- T=6



- T=7



CONCLUSIONS

COMPLEMENTARITY IS BASED ON THE ASSUMPTION OF THE SCRAMBLING TIME BOUND IN ORDER TO AVOID QUANTUM CLONING AND ENCODE HOLOGRAPHICALLY FAST ENOUGH THE INFALLING INFORMATION ON THE CORRELATIONS OF

THE EMMITED HAWKING RADIATION

DISCRETIZING AdS2 RADIAL AND TIME NEAR HORIZON GEOMETRY OF EXTREMAL BLACKHOLES AND AT THE SAME TIME PRESERVING THE ALGEBRAIC STRUCTURE OF THE ISOMETRIES NECESSARILY LEADS TO THE MOD[N] ARITHMETIC DISCRETIZATION WITH HOLOGRAPHY AdS2[N]/CFT1[N],THIS CLASSICAL STRUCTURE IS LIFTED AT THE QUANTUM LEVEL

THROUGH FINITE QUANTUM MECHANICS

THE QUANTUM CAT MAP CHAOTIC DYNAMICS

ON THE DISCRETIZED HORIZON AdS2[N] ,

THERMALIZES THE INFORMATION ,DUE TO THEIR RANDOM EIGENSTATES,(ETH),

IN LOGARITHMIC TIME AND THUS IT SATURATES

THE SCRAMBLING TIME BOUND