Quantum corrections to effective couplings in string theories with spontaneous supersymmetry breaking

Ioannis Florakis



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Based on work with

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+ work in progress

07/07/2015 8th Regional meeting in String Theory

### Introduction

- 20 years of significant progress in string theory
- Semi-realistic vacua possessing basic desirable features
- Reconstruction of low energy effective action with  $\mathcal{N}=1~$  SUSY at tree level
- Quantitative comparison with low energy data : incorporate loop corrections

But what about SUSY breaking ?



Introduction

(stringy) Scherk-Schwarz mechanism

- Spontaneous breaking of SUSY with exactly tractable worldsheet description
- Worldsheet description in terms of freely-acting orbifolds

Scherk, Schwarz 1979 Rohm 1984 Kounnas, Porrati 1988 Atick, Witten 1988 Kounnas, Rostand 1990

When SUSY is (spontaneously) broken but the vacuum is classically stable

it is meaningful and important to study one-loop radiative corrections to couplings in the low energy effective action



Thresholds to non-abelian gauge couplings in 4d heterotic vacua  $\mathcal{N} = 2 \longrightarrow \mathcal{N} = 0$ 

Universality

More realistic constructions  $\mathcal{N} = 1 \longrightarrow \mathcal{N} = 0$  breaking and chirality



Vacuum configuration







vacuum is classically stable (W=0)

Deformation of vertex operators / fields by symmetry  $\boldsymbol{Q}$ 

 $\Phi(X_5)$ 



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$$\Phi(X_5 + 2\pi R) = e^{iQ} \Phi(X_5)$$

$$\Phi(X_5)$$



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$$\Phi(X_5 + 2\pi R) = e^{iQ} \Phi(X_5)$$

$$\Phi(X_5)$$

$$\Phi(X_5) = e^{iQX_5/2\pi R} \sum_{m \in \mathbb{Z}} \Phi_m e^{imX_5/R}$$

Kaluza-Klein spectrum of charged states is shifted

 $m \to m + Q/2\pi$ 

$$M_{\rm KK} = \frac{|Q|}{2\pi R}$$



'Flat' gauging of  $\mathcal{N}=2$  supergravity : completely fixed by string vacuum

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2) \times \mathrm{SO}(2)} \times \frac{\mathrm{SO}(4,4+n)}{\mathrm{SO}(4) \times \mathrm{SO}(4+n)}$$

$$S \qquad T,U \qquad T_i, U_i, \Phi^A$$
(BPS states  $n = \infty$ )

 $f_{ijk} = \langle \mathcal{V}_i \mathcal{V}_j \mathcal{V}_k \rangle_{\text{string}}$ 

action up to 2 derivatives fixed by the couplings among vectors and hypers





Keeping lightest BPS states  $\Phi^A \in \mathbb{R}$ 

scalar potential 
$$V = \frac{|T - 2U|^2 \Phi^A \Phi_A + 2(|T - 2U|^2 + |T - 2\overline{U}|^2) (\Phi^A \Phi_A)^2}{S_2 T_2 U_2}$$



Identify extra massless states in string spectrum

$$O_4 O_4 \bar{V}_{12} \bar{O}_4 \bar{V}_{16} \times \frac{1}{2} \left( \Gamma_{2,2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \Gamma_{2,2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

bi-fundamental (12, 16) of  $SO(12) \times SO(16)$ 

$$m_{\rm BPS}^2 = \frac{|T/2 - U|^2}{T_2 U_2} = |P_R|^2$$

no longer annihilating spacetime supercharges !

stringy Scherk-Schwarz continuously deforms masses of all states



Gauge thresholds

Running coupling associated to gauge group  $\,\mathcal{G}\,$ 

$$\frac{16\pi^2}{g_{\mathcal{G}}^2(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_{\mathcal{G}} \log \frac{M_s^2}{\mu^2} + \Delta_{\mathcal{G}} + \frac{16\pi^2}{g_s^2} +$$





 $-\frac{1}{4g_{\mathcal{G}}^2} F_{\mu\nu} F^{\mu\nu}$  BPS saturated term only BPS states run in the loop

$$\sum_{\text{BPS states}} \operatorname{Str} \left( \frac{1}{12} - s^2 \right) \left( Q^2 - \frac{1}{4\pi\tau_2} \right) q^{\frac{1}{4}|P_L|^2 + N_{\text{osc}} - \frac{1}{2}} \bar{q}^{\frac{1}{4}|P_R|^2 + \bar{N}_{\text{osc}} - 1}$$

$$q = e^{2\pi i \tau}$$

$$\sum_{\text{BPS states}} \operatorname{Str} \left( \frac{1}{12} - s^2 \right) \left( Q^2 - \frac{1}{4\pi\tau_2} \right) q^{\frac{1}{4}|P_L|^2} \bar{q}^{\frac{1}{4}|P_R|^2 + \bar{N}_{\text{osc}} - 1}$$

left moving oscillators cancel out





left moving oscillators cancel out



Difference of thresholds for gauge group factors  $\,\mathcal{G}_1\,$  ,  $\,\mathcal{G}_2\,$ 

$$\Delta_{\mathcal{G}_{1}} - \Delta_{\mathcal{G}_{2}} = \int_{\mathcal{F}} d\mu \ \Gamma_{2,2}(T,U) \Phi(\bar{\tau})$$
holomorphic modular function (invariant)
regular everywhere

$$\Phi(\bar{\tau}) = \frac{c_{-1}}{\bar{q}} + c_0 + c_1 \bar{q} + \dots$$
$$= c_{-1} j(\bar{\tau}) + c_0$$

the pole corresponds to the bosonic vacuum of the heterotic string



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Difference of thresholds for gauge group factors  $\, \mathcal{G}_1 \,$  ,  $\, \mathcal{G}_2 \,$ 

$$\Delta_{\mathcal{G}_1} - \Delta_{\mathcal{G}_2} = \delta\beta_{12} \,\int_{\mathcal{F}} \,\frac{d^2\tau}{\tau_2^2} \,\Gamma_{2,2}(T,U)$$

 $= -\delta\beta_{12} \log \left( T_2 U_2 |\eta(T) \eta(U)|^4 \right) + \text{constant}$ 

Dixon, Kaplunovsky, Louis '91

Independently of the details of the vacuum (almost)

universality



Supersymmetry is spontaneously broken



explicitly non-holomorphic



Threshold for SO(16) gauge group with K3 realized as  $T^4/\mathbb{Z}_2$  orbifold

$$\begin{split} \Delta_{\mathrm{SO}(16)} &= \int_{\mathcal{F}} d\mu \, \left\{ -\frac{1}{48} \, \Gamma_{2,2} [^{0}_{0}] \, \frac{\hat{E}_{2} \, \bar{E}_{4} \, \bar{E}_{6} - \bar{E}_{6}^{2}}{\bar{\eta}^{24}} \quad \text{BPS subsector} \right. \\ &+ \Gamma_{2,2} [^{0}_{1}] \, \left[ -\frac{1}{4N \times 144} \, \frac{\Lambda^{\mathrm{K3}} [^{0}_{0}]}{\eta^{12} \, \bar{\eta}^{24}} (\vartheta_{3}^{8} - \vartheta_{4}^{8}) \, \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{4}^{4} \, \left( (\hat{E}_{2} - \bar{\vartheta}_{3}^{4}) \, \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{4}^{4} + 8 \bar{\eta}^{12} \right) \right] \quad \begin{array}{c} \text{dependence} \\ &\text{on hypers} \end{array} \\ &+ \Gamma_{2,2} [^{0}_{1}] \left[ -\frac{1}{96} \, \frac{\bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{4}^{4} (\bar{\vartheta}_{3}^{4} + \bar{\vartheta}_{4}^{4}) \, \left[ (\hat{E}_{2} - \bar{\vartheta}_{3}^{4}) \, \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{4}^{4} + 8 \bar{\eta}^{12} \right] \quad \begin{array}{c} \text{BPS subsector} \\ &\text{(exceptionally for } \mathbb{Z}_{2} ) \end{array} \\ &- \frac{1}{144} \, \frac{\vartheta_{2}^{4} \, (\vartheta_{3}^{8} - \vartheta_{4}^{8})}{\eta^{12}} \, \left( \frac{\hat{E}_{2} - \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{4}^{4} + 8 \bar{\eta}^{12} \right] \\ &- \eta^{12} \, \frac{\vartheta_{2}^{4} \, (\vartheta_{3}^{8} - \vartheta_{4}^{8})}{\eta^{12}} \, \left( \frac{\hat{E}_{2} - \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{3}^{4} \, \bar{\vartheta}_{4}^{4} + 8 \bar{\eta}^{12} \right] \\ &+ (S \cdot \tau) + (ST \cdot \tau) \right\} \quad \text{non-holomorphic} \end{split}$$



These expressions do not look very friendly...



BUT may still be computed explicitly!



"The anatomy of an orbifold"





The "hard" N=4 to N=2 BPS subsector

$$\begin{bmatrix} 0,0\\0,1 \end{bmatrix} \\ \begin{bmatrix} 0,1\\0,1 \end{bmatrix} \begin{bmatrix} 0,1\\0,1 \end{bmatrix} \begin{bmatrix} 0,1\\0,1 \end{bmatrix} = \frac{1}{2} \times (b_{\mathrm{E}_8} - b_{\mathrm{E}_7}) \times \int_{\mathcal{F}} d\mu \ \Gamma_{2,2}(T,U) = 72 \log T_2 U_2 |\eta(T)\eta(U)|^4$$

the theory is effectively  $K3xT^2$ 

$$b_{\rm E_8} = -60 \qquad \qquad b_{\mathcal{G}} = \operatorname{STr}\left(\frac{1}{12} - s^2\right) Q_{\mathcal{G}}^2$$
$$b_{\rm E_7} = -84$$



The spontaneous N=4 to N=2 BPS subsector

$$\begin{bmatrix} 0,0\\1,1 \end{bmatrix} = \frac{1}{2} \times (b_{SO(16)} - b_{SO(12)}) \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0\\1 \end{bmatrix} (T,U) \\ = -8 \ \log T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4$$

the theory is effectively the  $Z_2$  orbifold

$$\hat{g} = g (-1)^{F_1 + F_2}$$

$$g : \begin{cases} Z^1 \to -Z^1 \\ Z^2 \to -Z^2 \\ Z^3 \to Z^3 + \frac{1}{2} \end{cases}$$

$$b_{\rm SO(16)} = 4$$
  
 $b_{\rm SO(12)} = -12$ 

Spontaneous N=2 to N=0 non-BPS subsector





Spontaneous N=2 to N=0 non-BPS subsector

$$\begin{bmatrix} 0,1\\1,0 \end{bmatrix} \begin{bmatrix} 0,1\\1,1 \end{bmatrix} = \frac{1}{2} \times \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0\\1 \end{bmatrix} (T,U) \frac{1}{2} \times \left( \begin{bmatrix} 0,1\\1,0 \end{bmatrix} + \begin{bmatrix} 0,1\\1,1 \end{bmatrix} \right)$$
$$\begin{bmatrix} 1,1\\1,0 \end{bmatrix}$$

$$\# \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}[{}^0_1] \left\{ -\frac{\vartheta_2^8 |\vartheta_3^4 + \vartheta_4^4|^2 \,\bar{\vartheta}_3^4 \,\bar{\vartheta}_4^4}{\eta^{12} \,\bar{\eta}^{12}} - \frac{\vartheta_2^4 \,\vartheta_4^4 |\vartheta_2^4 - \vartheta_4^4|^2 \,\bar{\vartheta}_3^4 \,\bar{\vartheta}_4^4}{\eta^{12} \,\bar{\eta}^{12}} + \frac{\vartheta_2^4 \,\vartheta_3^4 \,|\vartheta_2^4 + \vartheta_3^4|^2 \,\bar{\vartheta}_3^4 \,\bar{\vartheta}_4^4}{\eta^{12} \,\bar{\eta}^{12}} \right\}$$

highly non-holomorphic

write in terms of Kac-Moody characters

$$12(O_8^2 V_8 + 3V_8^3)(\bar{O}_8^2 \bar{V}_8 - \bar{V}_8^3)$$
 factorizes



Spontaneous N=2 to N=0 non-BPS subsector

$$\begin{bmatrix} 0,1\\1,0 \end{bmatrix} \begin{bmatrix} 0,1\\1,1 \end{bmatrix} = \frac{1}{2} \times \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0\\1 \end{bmatrix} (T,U) \frac{1}{2} \times \left( \begin{bmatrix} 0,1\\1,0 \end{bmatrix} + \begin{bmatrix} 0,1\\1,1 \end{bmatrix} \right)$$
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highly non-holomorphic

write in terms of Kac-Moody characters

$$12 (O_8^2 V_8 + 3V_8^3) (\bar{O}_8^2 \bar{V}_8 - \bar{V}_8^3)$$
 factorizes  
= 8



Spontaneous N=2 to N=0 non-BPS subsector

$$\begin{bmatrix} 0,1\\1,0 \end{bmatrix} \begin{bmatrix} 0,1\\1,1 \end{bmatrix} = \frac{1}{2} \times \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0\\1 \end{bmatrix} (T,U) \frac{1}{2} \times \left( \begin{bmatrix} 0,1\\1,0 \end{bmatrix} + \begin{bmatrix} 0,1\\1,1 \end{bmatrix} \right)$$
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highly non-holomorphic

write in terms of Kac-Moody characters

CERN

$$= 8 \qquad \text{MSDS identities} \qquad \text{I.F. & Kounnas 2009}$$

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$$\begin{bmatrix} 0,1\\1,0 \end{bmatrix} \begin{bmatrix} 0,1\\1,1 \end{bmatrix} = \frac{1}{2} \times \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0\\1 \end{bmatrix} (T,U) \frac{1}{2} \times \left( \begin{bmatrix} 0,1\\1,0 \end{bmatrix} + \begin{bmatrix} 0,1\\1,1 \end{bmatrix} \right)$$
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highly non-holomorphic

$$= \frac{1}{2} \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T,U) \left[ \ \delta b + \delta b' \ \left( \frac{\vartheta_2(\tau)}{\eta(\tau)} \right)^{12} \right]$$
 cancellations among the right moving oscillators !

right moving oscillators !

$$\delta b = -rac{32}{3} \qquad \qquad \delta b' = -rac{2}{3}$$

hidden spectral flow

a word about modular integrals

$$\int_{\mathcal{F}} d\mu \ \Gamma_{2,2}(T,U) = -\log T_2 U_2 |\eta(T)\eta(U)|^4$$

$$\int d\mu \ \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T,U) = -\log T_2 U_2 |\vartheta_4(T) \vartheta_2(U)|^4 \qquad \text{generaliz}$$

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$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}(T,U) \, j(\tau) = -\log |j(T) - j(U)|^2$$

$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}[{}^0_1](T,U) \ \frac{\vartheta_2^{12}}{\eta^{12}} = -2\log|j_2(T/2) - j_2(U)|^4 \qquad \text{new result}$$

# based on a generalized Borcherds product formula for $\Gamma_0(2)$

$$\prod_{\substack{K>0\\L\in\mathbb{Z}}} \left( \frac{(1-q_T^K q_U^L)^N}{1-q_T^{NK} q_U^{NL}} \right)^{c(KL)} = \left( \frac{j_2(T) - j_2(U)}{j_2(T) - 24} \right)^2$$

$$\frac{\vartheta_2^{12}}{\eta^{12}} + 24 = \sum_n c(n) \, q^n$$
a word about modular integrals

$$\int_{\mathcal{F}} d\mu \ \Gamma_{2,2}(T,U) = -\log T_2 U_2 |\eta(T)\eta(U)|^4$$

Dixon, Kaplunovsky, Louis 1991

$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2} \begin{bmatrix} 0\\1 \end{bmatrix} (T,U) = -\log T_2 U_2 \ |\vartheta_4(T) \ \vartheta_2(U)|^4 \qquad \text{generalization}$$

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$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}(T,U) \, j(\tau) = -\log|j(T) - j(U)|^2$$

Cardoso, Lüst, Mohaupt 1995 Harvey, Moore 1996

generalization

 $\mathbf{2}$ 

$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}[{}^0_1](T,U) \ \frac{\vartheta_2^{12}}{\eta^{12}} = -2\log|j_2(T/2) - j_2(U)|^4 \qquad \text{new result}$$

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$$\frac{\vartheta_2^{12}}{\eta^{12}} + 24 = \sum_n c(n) \, q^n$$

a word about modular integrals

$$\int_{\mathcal{F}} d\mu \ \Gamma_{2,2}(T,U) = -\log T_2 U_2 |\eta(T)\eta(U)|^4$$

$$\int_{\mathcal{F}_0(2)} d\mu \, \Gamma_{2,2} \big[{}_1^0\big](T,U) = -\log T_2 U_2 \, |\vartheta_4(T) \, \vartheta_2(U)|^4$$

$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}(T,U) \, j(\tau) = -\log|j(T) - j(U)|^2$$

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$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}[^0_1](T,U) \ \frac{\vartheta_2^{12}}{\eta^{12}} = -2\log|j_2(T/2) - j_2(U)|^4$$

Dixon, Kaplunovsky, Louis 1991

Cardoso, Lüst, Mohaupt 1995 Harvey, Moore 1996

generalization

#### new result

Angelantonj, I.F., Pioline 2015

#### based on a generalized Borcherds product formula for $\Gamma_0(2)$

$$\prod_{\substack{K>0\\L\in\mathbb{Z}}} \left( \frac{(1-q_T^K q_U^L)^N}{1-q_T^{NK} q_U^{NL}} \right)^{c(KL)} = \left( \frac{j_2(T) - j_2(U)}{j_2(T) - 24} \right)^2$$

$$\frac{\vartheta_2^{12}}{\eta^{12}} + 24 = \sum_n c(n) \, q^n$$

Full Result for  $K3 \sim T^4/\mathbb{Z}_2$ 

$$\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} = 72 \log[T_2 U_2 |\eta(T) \eta(U)|^4] - \frac{8}{3} \log[T_2 U_2 |\vartheta_4(T) \vartheta_2(U)|^4] + \frac{2}{3} \log|j_2(T/2) - j_2(U)|^4$$



Full Result for  $K3 \sim T^4/\mathbb{Z}_2$ 

$$\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} = 72 \, \log[T_2 U_2 \, |\eta(T) \, \eta(U)|^4] - \frac{8}{3} \, \log[T_2 U_2 \, |\vartheta_4(T) \, \vartheta_2(U)|^4] \\ + \frac{2}{3} \, \log|j_2(T/2) - j_2(U)|^4 \\ \downarrow \\ j_2(\tau) = \left(\frac{\eta(\tau)}{\eta(2\tau)}\right)^{24} + 24$$



Full Result for  $\mathrm{K3} \sim T^4/\mathbb{Z}_N$ 

 $\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} = \alpha \log[T_2 U_2 |\eta(T) \eta(U)|^4] + \beta \log[T_2 U_2 |\vartheta_4(T) \vartheta_2(U)|^4] + \gamma \log|j_2(T/2) - j_2(U)|^4$ 

$$(\alpha, \beta, \gamma) = (72, -\frac{8}{3}, \frac{2}{3}) \qquad \mathbb{Z}_2 \& \mathbb{Z}_3$$
$$(\alpha, \beta, \gamma) = \frac{5}{8}(72, -\frac{8}{3}, \frac{16}{15}) \qquad \mathbb{Z}_4$$
$$(\alpha, \beta, \gamma) = \frac{35}{144}(72, -\frac{8}{3}, \frac{1}{3}) \qquad \mathbb{Z}_6$$

Independently of the details of the vacuum (almost)



Full Result for  $\mathrm{K3} \sim T^4/\mathbb{Z}_N$ 

 $\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} = \alpha \log[T_2 U_2 |\eta(T) \eta(U)|^4] + \beta \log[T_2 U_2 |\vartheta_4(T) \vartheta_2(U)|^4] + \gamma \log|j_2(T/2) - j_2(U)|^4$ 

$$(\alpha, \beta, \gamma) = (72, -\frac{8}{3}, \frac{2}{3}) \qquad \mathbb{Z}_2 \& \mathbb{Z}_3$$
$$(\alpha, \beta, \gamma) = \frac{5}{8}(72, -\frac{8}{3}, \frac{16}{15}) \qquad \mathbb{Z}_4$$
$$(\alpha, \beta, \gamma) = \frac{35}{144}(72, -\frac{8}{3}, \frac{1}{3}) \qquad \mathbb{Z}_6$$

Independently of the details of the vacuum (almost)



universality

Full Result for  $\mathrm{K3} \sim T^4/\mathbb{Z}_N$ 

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logarithmic singularity at T/2 = U (plus images)

extra charged massless states

Drastically different than the supersymmetric case



Full Result for  $\mathrm{K3}\sim T^4/\mathbb{Z}_N$ 

difference of beta function coefficients

$$\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} = \alpha \log[T_2 U_2 |\eta(T) \eta(U)|^4] + \beta \log[T_2 U_2 |\vartheta_4(T) \vartheta_2(U)|^4] + \gamma \log|j_2(T/2) - j_2(U)|^4$$



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jump in beta function coefficients



What is the origin of this unexpected universality ?



$$\Delta\Phi[^{H,h}_{G,g}] = \frac{i}{8\pi} \sum_{k,\ell,\rho,\sigma\in\mathbb{Z}_2} (-)^{(k+\rho)G+(\ell+\sigma)H} \bar{\vartheta}[^k_{\ell}]^6 \,\bar{\vartheta}[^{k+h}_{\ell+g}] \,\bar{\vartheta}[^{k-h}_{\ell-g}] \,\bar{\vartheta}[^{\rho}_{\sigma}]^8 \,\partial_{\bar{\tau}} \log \frac{\bar{\vartheta}[^{\rho}_{\sigma}]}{\bar{\vartheta}[^k_{\ell}]}$$



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spectral flow  
(repeated Riemann identities) = 
$$4\left(2(-1)^{(1+H)(1+G)} - 1\right) \bar{\eta}^{18} \bar{\vartheta}^{[1+h]}_{[1+g]} \bar{\vartheta}^{[1-h]}_{[1-g]}$$



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left-right "mirror" of BPS property : projects on right-moving ground states !

What is the origin of this unexpected universality ?



Universality : special property of "N=2 sectors"

depends on the orbifold action on the bosonic side of the heterotic string



What is the origin of this unexpected universality ?



Suppose that SUSY could be restored :

$$(-1)^{F_{\text{s.t.}}}(-1)^{F_1+F_2}\delta \to (-1)^{F_1+F_2}\delta$$

the left-moving contribution drops out

IF there is no (vector/hyper) enhancement in the bulk of T,U

$$\frac{\delta\Phi[{}^{0,1}_{1,g}]}{\bar{\eta}^{18}\,\bar{\vartheta}[{}^{0}_{1+g}]^2} = \text{constant}$$



What is the origin of this unexpected universality ?



reflects the presence of extra massless states

Additional contributions to the thresholds : generalised universality

$$\int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}[{}^0_1] \left| \frac{\vartheta_2^{12}}{\eta^{12}} \right|^2 \quad + \quad \int_{\mathcal{F}_0(2)} d\mu \ \Gamma_{2,2}[{}^0_1] \left| \frac{E_6 + E_4 X_2}{\eta^{12}} \right|^2$$

Does non-supersymmetric Universality arise in chiral models ?

What about singularities due to extra massless scalars ?



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Work with CY in the singular limit realised as  $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$  with standard embedding

YES !

 $G = \mathcal{E}_6 \times \mathcal{E}_8 \times \mathcal{U}(1)^2$ 

#### Charged massless spectrum : Chiral

g <sub>1</sub> - twisted sector	16 chiral multiplets in	( <b>27</b> , <b>1</b> ) + (1,1)
g <sub>2</sub> - twisted sector	16 chiral multiplets in	( <b>27</b> , <b>1</b> ) + (1,1)
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g1g2 - twisted sector	16 chiral multiplets in	( <b>27</b> , <b>1</b> ) + (1, 1)



Turn on a Scherk-Schwarz flux on top  $\mathbb{Z}_2' = (-1)^{F_{\mathrm{s.t.}} + F_1 + F_2} \delta$ 

 $G = SO(10) \times SO(16)$  CY with Scherk-Schwarz flux (N=0)

Charged massless spectrum : Chiral

twisted sectors

16 N=1 chiral multiplets 
$$(16, 1) + (10, 1) + (1, 1)$$

16 complex scalars (10, 1) + 2(1, 1)

**16** fermions (16, 1)



Universality in Models with Chirality







CERN

"Blue orbits"



$$-\log T_2^{(i)} U_2^{(i)} |\eta(T^{(i)}) \eta(U^{(i)})|^4$$

$$-\log T_2^{(1)} U_2^{(1)} |\vartheta_4(T^{(1)}) \vartheta_2(U^{(1)})|^4$$





$$-\log\left|j_2(T^{(1)}/2) - j_2(U^{(1)})\right|^4$$

universality

Mone-loop radiative corrections to gauge couplings in heterotic strings

- One-loop radiative corrections to gauge couplings in heterotic strings
- Supersymmetry spontaneously broken by Scherk-Schwarz flux

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- Model is a standard stand Standard stand Standard stan
- Mathematical end unexpected Universality structure
- Can arise in chiral models
- Opens possibilities for string model building

## Outlook

Outlook



## Outlook



Semi-realistic string model building ?
## Thank you !

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