Gravity waves from Kerr/CFT

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based on...

1401.3746	*with:*	A. Strominger
1403.2797	*with:*	S. Hadar, A. Strominger
1504.07650	*with:*	S. Hadar, A. Strominger
1506.08496	*with:*	S. Gralla, N. Warburton

The extreme Kerr throat, NHEK, near-NHEK, etc

- The Kerr metric in Boyer-Lindquist coordinates ($G = c = \hbar = 1$):

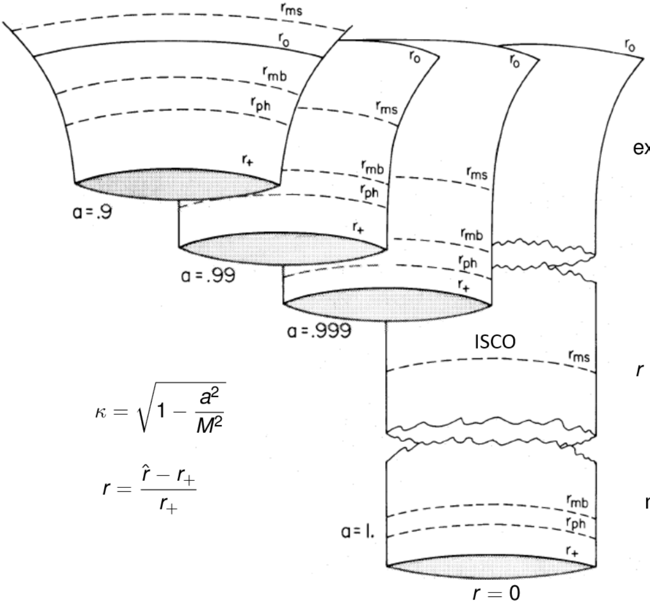
$$ds^2 = -\frac{\Delta}{\hat{\rho}^2}\left(d\hat{t} - a\sin^2\theta d\hat{\phi}\right)^2 + \frac{\sin^2\theta}{\hat{\rho}^2}\left((\hat{r}^2 + a^2)d\hat{\phi} - ad\hat{t}\right)^2 + \frac{\hat{\rho}^2}{\Delta}d\hat{r}^2 + \hat{\rho}^2 d\theta^2\,,$$

$$\Delta = \hat{r}^2 - 2M\hat{r} + a^2\,, \quad \hat{\rho}^2 = \hat{r}^2 + a^2\cos^2\theta\,.$$

- Mass M, angular momentum $J = aM \leq M^2$.
- Near extremality there is a very long throat

$\rightarrow$ decoupling metrics: NHEK, near-NHEK

The extreme Kerr throat, NHEK, near-NHEK, etc



extreme Kerr
$r \sim 1$

NHEK
$r \sim \kappa^p$, $p < 1$

near-NHEK
$r \sim \kappa$

$$\kappa = \sqrt{1 - \frac{a^2}{M^2}}$$

$$r = \frac{\hat{r} - r_+}{r_+}$$

The extreme Kerr throat, NHEK, near-NHEK, etc

- NHEK:

$$ds^2 = 2M^2\Gamma(\theta)\left[-R^2 dT^2 + \frac{dR^2}{R^2} + d\theta^2 + \Lambda(\theta)^2(d\Phi + RdT)^2\right]$$

- near-NHEK:

$$ds^2 = 2M^2\Gamma(\theta)\left[-r(r+2\kappa)dt^2 + \frac{dr^2}{r(r+2\kappa)} + d\theta^2 + \Lambda(\theta)^2(d\phi + (r+\kappa)dt)^2\right]$$

- Isometry group: $SL(2,\mathbb{R})_R \times U(1)_L$. ∂_t in $SL(2,\mathbb{R})_R$, ∂_ϕ is $U(1)$
- Asymptotic symmetry group: Virasoro (L or R), Virasoro-Kac-Moody,...

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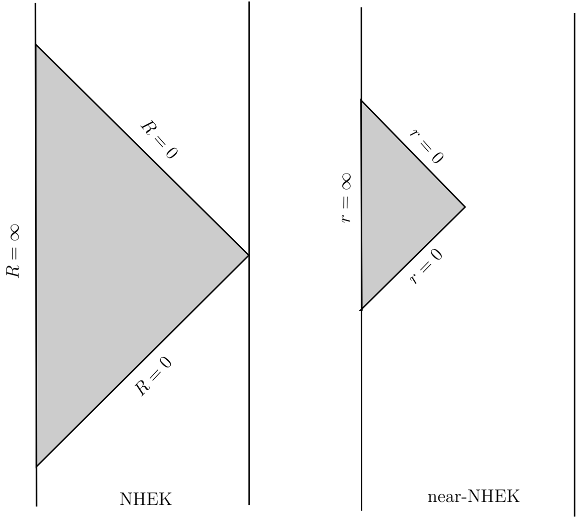


The Kerr/CFT correspondence

‘strong’ Kerr/CFT

the *conjecture* that quantum gravity in the near horizon region of a near-extreme Kerr is dual to a (warped) 2D CFT

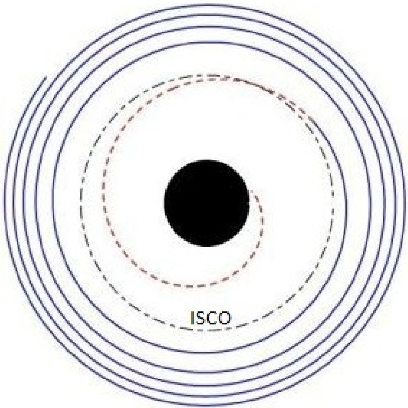
- Relevant for quantum black hole puzzles (e.g. $S_{BH} = S_{Cardy}$)
- Bottom-up: only some dictionary entries known (e.g. c, h)

‘weak’ Kerr/CFT

the *fact* that gravitational dynamics in the near horizon region of a near-extreme Kerr are constrained by an infinite-dimensional conformal symmetry

- Powerful CFT techniques for near-horizon gravitational physics.
- Suffices for interesting questions in observational astronomy.

Extreme-Mass-Ratio-Inspirals (EMRIs)



- A primary gravity waves source for eLISA mission
- Slow vs fast plunges
- So far people do PN approximation or numerics

The plan

- Gravity analysis: Solve linearized Einstein equation with source and compute particle number flux at the horizon

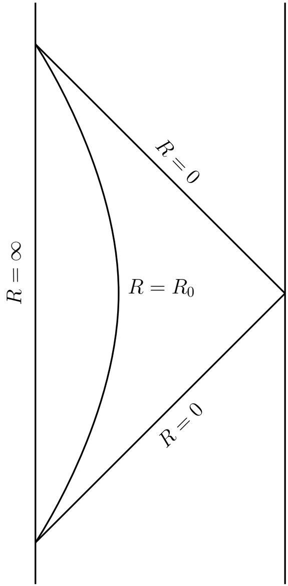
$$\mathcal{F}^{gravity} = \frac{dN}{dt}$$

- CFT analysis: Identify the source deformation of the CFT and compute the transition rate out of the vacuum state

$$\mathcal{R}^{CFT} = \frac{dP}{dt}$$

- Glue asymptotically flat region back and find the flux at future null infinity.

NHEK + circle gravity analysis



NHEK + circle: gravity analysis

- In NHEK, circular orbits at any radius R_0 are marginally stable:

$$
\begin{aligned}
R(T) &= R_0 \\
\Phi(T) &= -\frac{3}{4} R_0\, T + \Phi_0
\end{aligned}
$$

- Consider coupling to a scalar field:

$$\Box\, \Psi = g\, R_0 \delta(R - R_0)\delta(\theta - \pi/2)\delta(\Phi + \frac{3}{4} R_0 T)$$

- Killing flow along $\chi = \partial_T - \frac{3}{4} R_0 \partial_\Phi$, so expand accordingly:

$$\Psi = \sum_{\ell,m} e^{im(\Phi + \frac{3}{4} R_0 T)} S_\ell(\theta) R_{\ell m}(R)$$

where the S_ℓ's are spheroidal harmonics obeying $\mathcal{L}_\theta^{(2)} S_\ell^m = -K_\ell^m S_\ell^m$

NHEK + circle: gravity analysis

- Radial equation:

$$\partial_R(R^2\partial_R R_{\ell m}) + \left(2m^2 - K_\ell + \frac{2\Omega m}{R} + \frac{\Omega^2}{R^2}\right) R_{\ell m} = \frac{M^2}{2\pi} g\, S_\ell(\pi/2)\, R_0 \delta(R - R_0)$$

  where $\Omega = -\frac{3}{4} m R_0$.

- Homogeneous solutions are confluent hypergeometrics, e.g. Whittakers

$$W_{im,h-1/2}(-2i\Omega/R)\,, \qquad M_{im,h-1/2}(-2i\Omega/R)$$

  where

$$h \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + K_\ell - 2m^2}$$

- Boundary conditions:
  @ $R = 0$: $R^{-im} e^{i\Omega/R}$ (ingoing), $R^{im} e^{-i\Omega/R}$ (outgoing)
  @ $R = \infty$: R^{h-1} (Dirichlet), R^{-h} (Neumann), $P\, R^{h-1} + Q\, R^{-h}$ (“leaky”).

NHEK + circle: gravity analysis

With ingoing boundary conditions at the horizon and Neumann or "leaky" at the boundary, we find that, for real h, to leading order, the Klein-Gordon particle number flux down the horizon is given by:

$$\begin{aligned} \mathcal{F}_{\ell m}^{gravity} &= -\int \sqrt{-g}\, J^R_{KG}\, d\theta d\Phi \\ &= \frac{g^2 M^6}{12\pi^2} S_\ell(\pi/2)\, R_0\, m^{-1} e^{-\pi m} \frac{|\Gamma(h+im)|^2}{|\Gamma(2h)|^2} \left|M_{im,h-1/2}(3im/2)\right|^2 \end{aligned}$$

Summary

- In NHEK there are marginally stable circular orbits at every radius R_0.
- The wave equation may be solved analytically in terms of Whittakers.
- With ingoing b.c. at the horizon, we calculate the particle number flux.

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NHEK + circle: CFT analysis

- Q: What is the dual of this gravity situation?
- A: Holographically driven CFT by external source at frequency Ω:

$$S = S_{CFT} + \sum_{\ell} \int d\Phi dT \, J_{\ell}(\Phi, T) \mathcal{O}_{\ell}(\Phi, T)$$

  with

$$J_{\ell}(\Phi, T) = \sum_{m} J_{\ell m} e^{im\left(\Phi + \frac{3}{4} R_0 T\right)}$$

- Kerr/CFT dictionary: operator $\mathcal{O}$ dual to bulk field Ψ, with weight h
- What is $J_{\ell m}$? Bulk solution

$$R_{\ell m} = X \Theta(R_0 - R) R^{in}(R) + Z \Theta(R - R_0) R^{out}(R)$$

- Extend inner solution:

$$R^{ext}_{\ell m} = X \, R^{in}(R)$$

- Read off $J_{\ell m}$ from leading term:

$$R^{ext}_{\ell m} \to J_{\ell m} \, R^{h-1} + \ldots \qquad R \to \infty$$

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NHEK + circle: CFT analysis

Applying Fermi's Golden Rule, the transition rate out of the initial state $|i\rangle$ to any final state $|f\rangle$ is given by the Fourier transform of the two-point function:

$$\begin{aligned}\mathcal{R}^{CFT}_{\ell m} &= 2\pi |J_{\ell m}|^2 \int d\Phi dT\, e^{-im\left(\Phi + \frac{3}{4}R_0 T\right)} \langle \mathcal{O}^\dagger(\Phi, T)\mathcal{O}(0,0)\rangle \\ &= \mathcal{C}^2_{\mathcal{O}} \frac{(2\pi)^2 (3R_0/4)^{2h-1}}{\Gamma(2h)^2} |J_{\ell m}|^2 m^{2h-1} e^{-\pi m} |\Gamma(h+im)|^2\end{aligned}$$

Plugging in $J_{\ell m}$ and normalizing the operators with $\mathcal{C}_{\mathcal{O}} = 2^{2h-1}(2h-1)M/2\pi$ we find:

$$\mathcal{R}^{CFT}_{\ell m} = \frac{g^2 M^6}{12\pi^2} S_\ell(\pi/2)\, R_0\, m^{-1} e^{-\pi m} \frac{|\Gamma(h+im)|^2}{|\Gamma(2h)|^2} \left|M_{im,h-1/2}(3im/2)\right|^2 = \mathcal{F}^{gravity}_{\ell m}$$

Summary

- Dual calculation: holographically driven CFT with source read off from extension of inner solution.
- CFT transition rate matches *perfectly* gravity particle number flux.

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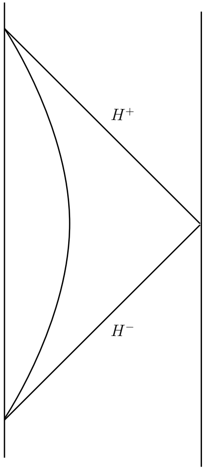
Gluing back asymptotically flat region

e.g.



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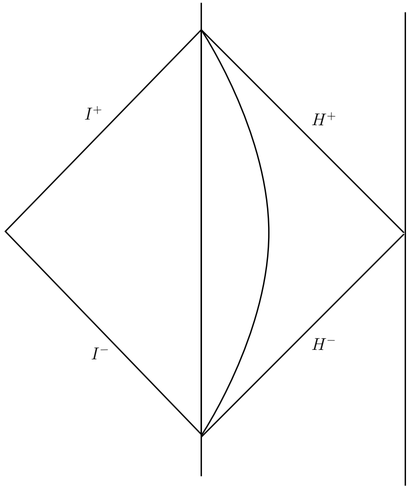


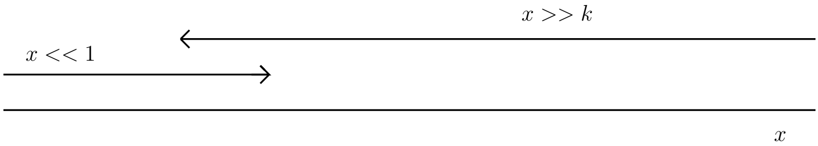
Gluing back asymptotically flat region

Matched Asymptotic Expansions

How it's done: for a suitable parameter $k << 1$,

near solution for $x << 1$, far solution for $x >> k$,

match solutions in $k << x << 1$.



Gluing back asymptotically flat region

For NHEK + circle problem:

- $k = 4M(\hat{\omega} - \frac{m}{2M}) = -\frac{3}{2} m R_0$
- near solution is the NHEK solution
- leaky boundary conditions

In this way we get a full solution everywhere which may be used to calculate explicitly desired observables, e.g. the outgoing radiation flux at future null infinity:

$$\dot{\mathcal{E}}_\infty = \frac{g^2 M^4}{72\pi^2} S_\ell^2(\pi/2) \left| W_{im,h-\frac{1}{2}}(3im/2) \right|^2 (3m^2 R_0/2)^{2\mathrm{Re}[h]} m^{-2} e^{\pi|m|} \times$$
$$\times \frac{|2h-1|^2 |\Gamma(h-im)|^4 / |\Gamma(2h)|^4}{\left| 1 - (3m^2 R_0/2)^{2h-1} \frac{\Gamma(1-2h)^2}{\Gamma(2h-1)^2} \frac{\Gamma(h-im)^2}{\Gamma(1-h-im)^2} \right|^2}$$

NOW THE MAGIC STARTS!

Magic #1: Near extremal ISCO

- Near extremal ISCO is at:

$$\frac{\hat{r} - r_+}{r_+} = 2^{1/3}\kappa^{2/3} + \mathcal{O}(\kappa)$$

- Q: How to get the leading order fluxes in this case?
- A: Just put $R_0 = 2^{1/3}\kappa^{2/3}$ into the extremal formulae!
- E.g. plug in:

$$\dot{\mathcal{E}}_\infty = \frac{g^2 M^4}{72\pi^2} S_\ell^2(\pi/2) \left| W_{im,h-\frac{1}{2}}(3im/2) \right|^2 (3m^2 R_0/2)^{2\mathrm{Re}[h]} m^{-2} e^{\pi|m|} \times \frac{|2h-1|^2 |\Gamma(h-im)|^4 / |\Gamma(2h)|^4}{\left| 1 - (3m^2 R_0/2)^{2h-1} \frac{\Gamma(1-2h)^2}{\Gamma(2h-1)^2} \frac{\Gamma(h-im)^2}{\Gamma(1-h-im)^2} \right|^2}$$

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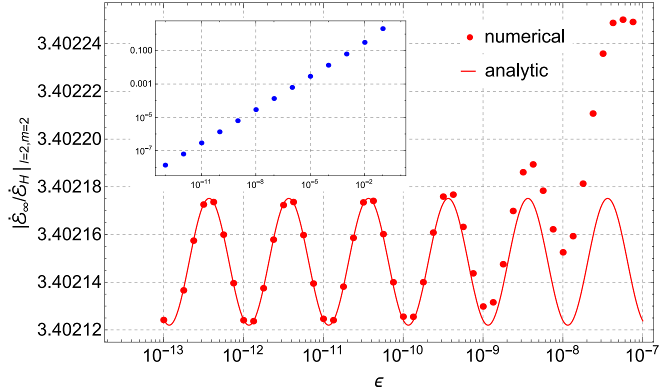
Magic #1: Near extremal ISCO

E.g. this uncovers that what used to look like numerical noise is actually oscillatory behavior in the $\kappa \to 0$ limit



Magic #2: Plunges

Slow plunge in near-NHEK

- Slow plunge orbit that spirals off ISCO

$$
\begin{aligned}
t(r) &= \frac{1}{2\kappa} \ln \frac{1}{r(r+2\kappa)} + t_0 \\
\phi(r) &= \frac{3r}{4\kappa} + \frac{1}{2} \ln \frac{r}{r+2\kappa} + \phi_0
\end{aligned}
$$

  falls into the future horizon in the near-NHEK metric

$$
ds^2 = 2M^2\Gamma(\theta)\left[-r(r+2\kappa)dt^2 + \frac{dr^2}{r(r+2\kappa)} + d\theta^2 + \Lambda(\theta)^2(d\phi + (r+\kappa)dt)^2\right]
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- Q: How to get the solutions in this case?
- A: Do a conformal transformation to map to the NHEK + circle problem!

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Slow plunge in near-NHEK

- Bulk diffeomorphism:

$$\begin{aligned} T &= -e^{-\kappa t}\frac{r+\kappa}{\sqrt{r(r+2\kappa)}} \\ R &= \frac{1}{\kappa}e^{\kappa t}\sqrt{r(r+2\kappa)} \\ \Phi &= \phi - \frac{1}{2}\ln\frac{r}{r+2\kappa} \end{aligned}$$

- Boundary conformal transformation:

$$\begin{aligned} T &= -e^{-\kappa t} \\ \Phi &= \phi \end{aligned}$$

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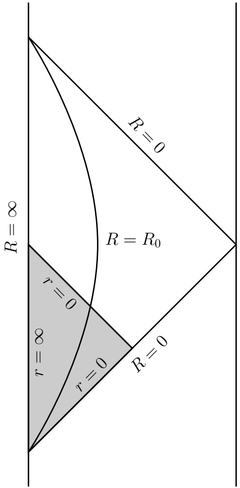
Slow plunge in near-NHEK



Magic #2: Plunges

Fast plunge that spirals off an eccentric last stable orbit