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Holographic compactifications of 6d (1,0) theories from massive IIA supergravity

based on arXiv:1502.06616, 1502.06620, 1506.05462 [hep-th]
in collaboration with F. Apruzzi, M. Fazzi, A. Rota and A. Tomasiello

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6d SCFT's

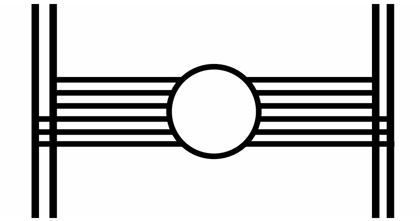
| interacting QFT's in $d > 4$ dimensions known to exist due to string/M-theory

prominent example: 6d $(2,0)$ SCFT capturing the dynamics of N coincident M5-branes

no known Lagrangian | $\sim N^3$ degrees of freedom

certain 6d $(1,0)$ SCFT's from M5-branes on orbifolds; another example from

NS5-D6-D8 brane intersections [Hanany, Zaffaroni '97] [Brunner, Karch '97]



Compactifying the 6d theories

interesting phenomena upon compactification

| $(2,0)$ theory: M5-branes wrapping supersymmetric cycles

many examples: Riemann surfaces $\subset CY_3$, associative 3-cycles $\subset G_2$-manifolds, Kähler 4-cycles $\subset CY_4$, Cayley 4-cycles $\subset$ Spin(7)-manifolds ... [Maldacena, Nùnẽz '00] [Acharya, Gauntlett, Kim '01] [Gauntlett, Kim, Waldram '01]

⋆ novel $\text{SCFT}_{d\leq 4}$'s – e.g. class $\mathcal{S}$ SCFT_4's [Gaiotto '09] – and AdS_{d+1} gravity duals

⋆ the covariant derivative of the supersymmetry parameters on the worldvolume is twisted by the normal bundle connection

can we expect similar phenomena for the $(1,0)$ theories?

we take a step on the gravity side

AdS$_7$ solutions of massive IIA supergravity

infinitely many analytic [Apruzzi, Fazzi, Rosa, Tomasiello '13] [Apruzzi, Fazzi, AP, Rota, Tomasiello '15]

$$ds^2_{10} = e^{2A} ds^2_{\mathrm{AdS}_7} + ds^2_{M_3} \,, \qquad ds^2_{M_3} = dr^2 + \frac{1}{16} e^{2A}(1-x^2) ds^2_{S^2}$$

$$H = -\left(6e^{-A} + F_0\, e^{\phi} x\right) \mathrm{vol}_{M_3} \,, \qquad F_2 = \tfrac{1}{16} e^{A-\phi} \sqrt{1-x^2} \left(F_0\, e^{A+\phi} x - 4\right) \mathrm{vol}_{S^2}$$

$A = A(r)$, $\phi = \phi(r)$, $x = x(r)$; determined by a single function $\beta(y)$ subject to an ODE

$$(q^2)' = \frac{2}{9} F_0 \,, \qquad q \equiv -\frac{4y\sqrt{\beta}}{\beta'} \,, \qquad \left(dr = \left(\tfrac{3}{4}\right)^2 \tfrac{e^{3A}}{\sqrt{\beta}} dy\right)$$

.dual 6d (1,0) SCFT's

engineered by NS5-D6-D8 brane intersections [Gaiotto, Tomasiello '14] The isometry of S^2 corresponds to the SU(2) R-symmetry

AdS_7 solutions of massive IIA supergravity

. a simple case

$$\beta = \frac{8}{F_0}(y - y_0)(y + 2y_0)^2$$

The metric on M_3 reads

$$ds^2_{M_3} = \sqrt{-\frac{y_0}{6F_0}}\left(\frac{d\tilde{y}^2}{(1-\tilde{y})\sqrt{\tilde{y}+2}} + \frac{4}{3}\frac{(1-\tilde{y})(\tilde{y}+2)^{3/2}}{8-4\tilde{y}-\tilde{y}^2}ds^2_{S^2}\right) , \qquad \tilde{y} \equiv \frac{y}{y_0} \in [-2, 1]$$

Near the endpoints

$$ds^2_{M_3} = \begin{cases} \lim\limits_{\tilde{y}\to 1} & d\rho^2 + \rho^2 ds^2_{S^2} \qquad \texttt{regular endpoint} \\ \lim\limits_{\tilde{y}\to -2} & \rho^{-\frac{1}{2}}\left(d\rho^2 + \rho^2 ds^2_{S^2}\right) \qquad \texttt{D6 brane} \end{cases}$$

flux quantization $\int_{S^2}(F_2 - F_0 B) = n_{\text{D6}} \;\Rightarrow\; y_0 = -\frac{3}{8}\frac{n^2_{\text{D6}}}{F_0}$

. free energy

$$\mathcal{F}_{0,6} = \frac{512}{45}\pi^4 n^2_{\text{D6}} N^3 , \qquad N = \frac{1}{4\pi^2}\int H$$

AdS$_7$ solutions of massive IIA supergravity

. general case

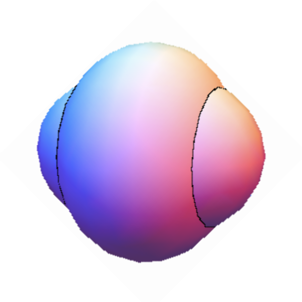
$$\beta = \frac{y_0^3}{b_2^3 F_0}\left(\sqrt{\hat{y}} - 6\right)^2 \left(\hat{y} + 6\sqrt{\hat{y}} + 6b_2 - 72\right)^2 , \qquad \hat{y} \equiv 2b_2\left(\frac{y}{y_0} - 1\right) + 36$$

$b_2 < 12$: a D6 stack at each enpoint

$b_2 > 12$: a D6 stack at one endpoint, an O6 singularity at the other

$b_2 = 12$: simple case

⋆ additional D8 branes wrapping the S^2 at a fixed $y = y_{\mathrm{D8}}$, as F_0 discontinuities



AdS_5 and AdS_4 compactifications

| It is possible to compactify the AdS_7 solutions to AdS_4 [Rota, Tomasiello '15] and AdS_5 [Apruzzi, Fazzi, AP, Tomasiello '15] on quotients of h y p e r b o l i c s p a c e s

$$ds^2_{10} = X^{\frac{15}{2}} e^{2A} ds^2_7 + X^{\frac{5}{2}} \left(dr^2 + \frac{1}{16} \frac{e^{2A}(1-x^2)}{w} Ds^2_{S^2} \right) \, , \qquad w \equiv X^5(1-x^{''}) + x^2$$

$$ds^2_7 = \begin{cases} ds^2_{\mathrm{AdS}_5} + \frac{1}{3} ds^2_{\mathbb{H}^2/\Gamma} \\ ds^2_{\mathrm{AdS}_4} + \frac{4}{5} ds^2_{\mathbb{H}^3/\Gamma} \end{cases} \qquad X^5 = \begin{cases} \dfrac{3}{4} \\ \dfrac{5}{8} \end{cases} \qquad \Gamma \subset \begin{cases} \mathrm{PSL}(2,\mathbb{R}) \\ \mathrm{PSL}(2,\mathbb{C}) \end{cases}$$

⋆ the S^2 metric is " t w i s t e d " by the s p i n c o n n e c t i o n of $\mathbb{H}^2$ or $\mathbb{H}^3$

⋆ can be obtained by a simple map

$$e^{2A} \to X^{\frac{15}{2}} e^{2A} \, , \qquad r \to X^{\frac{5}{4}} r \, , \qquad x \to \frac{x}{w} \, , \qquad e^{\phi} \to X^{\frac{5}{4}} \frac{e^{\phi}}{\sqrt{w}}$$

.detour

AdS_5 solutions of massive IIA supergravity

| [Apruzzi, Fazzi, AP, Tomasiello '15] classification of supersymmetric $\mathrm{AdS}_5 \times M_5$ solutions

$$ds^2_{10} = e^{2A} ds^2_{\mathrm{AdS}_5} + ds^2_{M_5}$$

⋆ supersymmetry ⇒ system of differential equations for an identity structure

local metric determined

$$ds^2_{M_5} = ds^2_{\mathcal{C}} + \frac{1}{9} b^2 D\psi^2 + \frac{e^{-6A+2\phi}}{b^2}\left[b^2(e^{-2A}dx^2 + e^{2A}dy^2) + (a_2 e^{-A}dx + a_1 e^{A}dy)^2\right]$$

$$b^2 = 1 - a_1^2 - a_2^2\,, \qquad a_1 = -\frac{1}{2}e^{-3A+\phi}y\,, \qquad D\psi = d\psi + \rho$$

. properties

- $\xi = 3\,\partial_\psi$ generates the U(1) R-symmetry
- S^1 fibered over $\mathcal{C}$

⋆ solutions characterized by 3 functions $\{a_2\,,\,\phi\,,\,A\}$ of 4 variables subject to 6 PDE's

AdS_5 solutions of massive IIA supergravity

| assumptions

- $ds^2_{10} = e^{2A}[\, ds^2_{AdS_5} + ds^2_{\Sigma}(x_1, x_2)\,] + ds^2_{M_3}$
- no dependence on x_1, x_2

| consequences

- Σ has constant curvature $\rightarrow$ compact Riemann surface Σ_g
- The PDEs reduce to an ODE for a single function

⋆ For zero Romans mass we recover known solutions:

[Maldacena, Núñez '00] [Bah, Beem, Bobev, Wecht '12] [Itsios, Núñez, Sfetsos, Thompson '13]

⋆ For non-zero Romans mass:

- a couple of globally problematic solutions
- the $AdS_5 \times \mathbb{H}^2/\Gamma$ solutions presented earlier

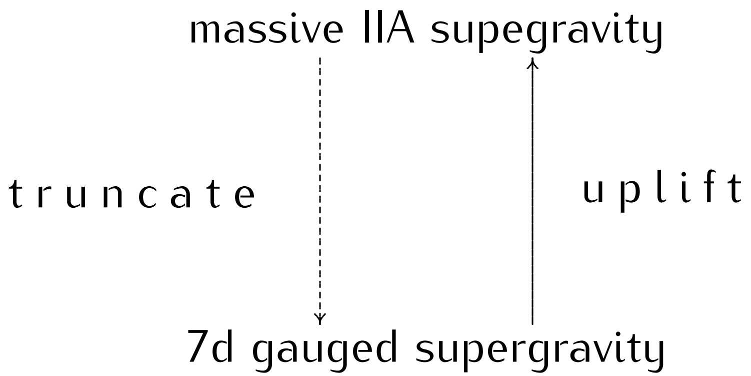
.end of detour

.advance

| study asymptotically anti-deSitter solutions e.g. holographic RG flows

convenient to work in lower dimensions; construct a consistent truncation

[AP, Rota, Tomasiello '15]



The compactification solutions hint a reduction Ansatz

- introduce 7d SU(2) gauge vector fields, gauging the isometry of S^2
- promote X to a 7d scalar field

7d minimal gauged supergravity

[Townsend, van Nieuwenhuizen '83] [Mezincescu, Townsend, van Nieuwenhuizen '84]

$$\mathcal{L}_7 = R - \tfrac{1}{2} * d\varphi \wedge d\varphi - V(\varphi) * 1 - \tfrac{1}{2} e^{\frac{4}{\sqrt{10}}\varphi} * \mathcal{F}_4 \wedge \mathcal{F}_4 - \tfrac{1}{2} e^{-\frac{2}{\sqrt{10}}\varphi} * \mathcal{F}_2^i \wedge \mathcal{F}_2^i$$
$$+ \tfrac{1}{2} \mathcal{F}_2^i \wedge \mathcal{F}_2^i \wedge \mathcal{A}_3 - h \mathcal{F}_4 \wedge \mathcal{A}_3 + \text{fermions}\,(\psi_\mu^a\,,\ \lambda^a)$$

where $V(\varphi)$ is the scalar potential

$$V(\varphi) = 2h^2 e^{-\frac{8}{\sqrt{10}}\varphi} - 4\sqrt{2} h g e^{-\frac{3}{\sqrt{10}}\varphi} - 2g^2 e^{\frac{2}{\sqrt{10}}\varphi}$$

for $\dfrac{g}{h} > 0$ two extrema:

- $e^{-\frac{5}{\sqrt{10}}\varphi} = \frac{1}{2\sqrt{2}} \dfrac{g}{h}$ `supersymmetric max.`
- $e^{-\frac{5}{\sqrt{10}}\varphi} = \frac{1}{\sqrt{2}} \dfrac{g}{h}$ `non-supersymmetric min.`

⋆ obtained as a consistent truncation of M-theory on S^4 [Lü, Pope '99]

Consistent truncation: equations of motion

.reduction Ansatz Neveu-Schwarz sector

$$\ell^{-1}ds^2_{10} = \tfrac{1}{8}g^2X^{-\frac{1}{2}}e^{2A}ds^2_7 + X^{\frac{5}{2}}ds^2_{M_3}\ , \qquad ds^2_{M_3} = dr^2 + \frac{1}{16}\frac{e^{2A}(1-x^2)}{w}Ds^2_{S^2}\ , \qquad \left(\ell \equiv \tfrac{8\sqrt{2}}{g^3}\right)$$

$$Ds^2_{S^2} \equiv \sum_i \left(dy^i + \epsilon^{ijk}y^j g\mathcal{A}^k\right)^2\ , \qquad \{y^i \in \mathbb{R}^3 : y^i y^i = 1\}$$

dilaton: $e^{2\Phi} = \ell \dfrac{X^{\frac{5}{2}}}{w}e^{2\phi}$

NS-NS potential: $\ell^{-1}B = \dfrac{1}{16}\dfrac{e^{2A}x\sqrt{1-x^2}}{w}\mathrm{vol}_2 - \dfrac{1}{2}e^A dr \wedge (\omega - \tfrac{1}{2}y^i\mathcal{A}^i)\ , \qquad \left(d\omega = -\tfrac{1}{2}\mathrm{vol}_2\right)$

Consistent truncation: equations of motion

subsitute the reduction Ansatz in the 10d equations of motion

$$\text{NS-NS sector} \Rightarrow \begin{cases} \text{Einstein equations} \\ \text{e.o.m. of } X \end{cases} \qquad \text{R-R sector} \Rightarrow \begin{cases} \text{e.o.m. of } \mathcal{A}^i \\ \text{e.o.m. of } \mathcal{A}_3 \end{cases}$$

identified with the e.o.m. of 7d minimal gauged supegravity

$$X = \exp\left(\frac{\varphi}{\sqrt{10}}\right) , \qquad h = \frac{g}{2\sqrt{2}}$$

for any solution β: universal

⇝ any solution of 7d minimal gauged supergravity can be uplifted on M_3 to an exact solution of 10d massive IIA supergravity

Consistent truncation: supersymmetry

is supersymmetry preserved by the uplifting process?

reduction Ansatz for the supersymmetry parameters schematically

$$\epsilon_1 \sim \xi \otimes \chi_1 + \xi^c \otimes \chi_1^c \,, \qquad \epsilon_2 \sim \xi \otimes \chi_2 - \xi^c \otimes \chi_2^c$$

ξ: 7d spinor | χ_1, χ_2 spinors on M_3

$$\chi_1 = -i e^{\frac{A}{2}} e^{-i\frac{\pi}{2}\sigma_3} e^{i\frac{\alpha}{2}\sigma^3} \chi_{S^2} \,, \qquad \chi_2 = e^{\frac{A}{2}} e^{-i\frac{\alpha}{2}\sigma^3} \chi_{S^2} \,, \qquad \sin\alpha = \frac{x}{w}$$

⋆ vanishing 10d fermion susy variations ⇒ set of equations for ξ

⇝ any spinor ξ such that the 7d fermion supersymmetry variations vanish can be uplifted so that the 10d fermion supersymmetry variations vanish as well

Holographic RG flows

.domain wall solutions

$$ds_7^2 = e^{2f_1(r)}(dr^2 + ds^2_{\mathbb{R}^{5-d,1}}) + e^{2f_2(r)} ds^2_{\Sigma_d}$$

asymptotically $\lim_{r\to 0} f_1 \sim \lim_{r\to 0} f_2 \sim \log r$: AdS_7 with $\mathbb{R}^{5-d,1} \times \Sigma_d$ boundary

preserved supersymmetry $\rightsquigarrow$ ODE's for f_1, f_2 and X

numerical solutions for $d = 2, 3$ [Acharya, Gauntlett, Kim, '01] [Maldacena, Núñez '00] [Bah, Beem, Bobev, Wecht '12]

$$\begin{array}{c} AdS_7 \\ \uparrow\downarrow \\ AdS_{(7-d)} \times \Sigma_d \end{array}$$

.free energy

$$\frac{\mathcal{F}_{0,6-d}}{\mathcal{F}_{0,6}} = X_{\text{IR}}^{20}\,\text{Vol}(\Sigma_d)$$

AdS_3 solutions

new supersymmetric AdS_3 solutions of massive IIA supergravity
dual to $(0,1)$ and $(0,2)$ $SFCT_2$'s

$\cdots\ \mathcal{N}=1\ AdS_3 \times \mathbb{H}^4\ \cdots$

$$ds_7^2 = \frac{2}{g^2} X^{-2}\left(ds^2_{AdS_3} + \frac{4}{7} ds^2_{\mathbb{H}^4}\right) , \qquad X^5 = \frac{7}{12} , \qquad \mathcal{F}_4 = \frac{3\sqrt{2}}{g^3} \mathrm{vol}_{\mathbb{H}^4}$$

- $SU(2)$ gauge field $\equiv$ self-dual component of the $SO(4)$ spin connection

$\cdots\ \mathcal{N}=2\ AdS_3 \times M_4\ \cdots$

$$ds_7^2 = \frac{2}{g^2} X^{-2}\left(ds^2_{AdS_3} + \frac{4}{3} ds^2_{M_4}\right) , \qquad X^5 = \frac{4}{3} , \qquad \mathcal{F}_4 = \frac{\sqrt{2}}{g^3} \mathrm{vol}_{M_4}$$

- M_4 : Kähler–Einstein of $R=-4$
- $U(1)$ gauge field $\equiv$ $U(1)$ component of the $U(2)$ spin connection

Conclusions and Outlook

.conclusions

- AdS$_4$ and AdS$_5$ solutions of massive IIA supergravity dual to field theories in three and four dimensions which are twisted compactifications of 6d (1, 0) SCFT's
- universal consistent truncation of massive IIA supergravity, to 7d minimal gauged supergravity; any solution of the lower-dimensional theory uplifts to a 10d solution; supersymmetry is preserved in this process

.future work

- The Kaluza–Klein spectrum of the $\mathrm{AdS}_7 \times M_3$ backgrounds, beyond the massless modes, can be used to analyse the spectrum of operators of the dual SCFT's

.THANK YOU