Instanton Operators in 5D Gauge Theories

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Motivation

Dramatic result coming out of string/M-theory:

$\exists$ interacting SCFTs in 5D and 6D

$\Rightarrow$ Provide UV fixed points for a variety of 5D gauge theories [Seiberg]

UV theories enjoy larger global or Lorentz symmetries:

- ◇ $\mathcal{N}=1$ SYM theories with $N_f \leq 7$ and $\mathrm{SO}(2N_f)$ symmetry
  $\Rightarrow$ $\mathcal{N}=1$ 5D SCFT with E_{N_f+1} symmetry
- ◇ $\mathcal{N}=2$ SYM theory $\Rightarrow$ $(2,0)$ 6D SCFT

Indirect ways of seeing this enhancement, mainly using index calculations [Kim^2-Lee, Bashkirov, Hwang-Kim^2-Park,...]

Can we find a simpler way? $\Rightarrow$ Draw upon our knowledge of 3D theories where local monopole operators play important role

$\Rightarrow$ Global symmetry and susy enhancement in the IR [Borokhov-Kapustin-Wu, Gaiotto-Witten, ABJM, ...]

Q: Is there an analogue in 5D?

A: We can construct local instanton operators

Outline

- Definition of instanton operators
- Supersymmetry
- Chern-Simons terms
- Applications
- Summary

Definition

Well-known that 5D SYM has conserved current

$$J = \frac{1}{8\pi^2} \mathrm{Tr} \star (F \wedge F)$$

Charged BPS-particle solutions: instanton solitons

Both global and Lorentz symmetry enhancement associated with instanton charge.

Preliminary: An instanton operator is a local operator which creates instanton solitons out of vacuum

The OPE of this current with $\mathcal{I}_n(0)$ is given by

$$J^\mu(x)\mathcal{I}_n(0) \sim \frac{3n}{8\pi^2} \frac{x^\mu}{|x|^5} \mathcal{I}_n(0) + \cdots$$

More formally: Instanton operators, $\mathcal{I}_n(x)$, modify boundary conditions for gauge field in Euclidean path integral:

$$\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle =$$

$$= \int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4_x} F\wedge F=n} [DXDAD\psi]\,\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)e^{-S_E}$$

Fields need to satisfy classical eom near insertion point in $\mathbb{R}^5$

$$D^\mu F_{\mu\nu} = 0\ , \qquad D_{[\mu}F_{\nu\lambda]} = 0$$

but with non-vanishing

$$I = \frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4} F\wedge F$$

In spherical coordinates a simple solution is given by taking $A_r = F_{ri} = 0$ and the angular components satisfying

$$F = \pm \star_{S^4} F$$

This solution for $\mathrm{SU}(2)$ theory was found long ago by Yang, as static $\mathrm{SO}(5)$-symmetric particle in 6D ⇒ Yang monopole

A DBI generalisation for $\mathrm{SU}(N)$ later given by Constable-Myers-Tafjord in context of D1⊥D5 intersections

Alternatively: Instanton operators defined by the condition that the gauge field has a Yang monopole singularity at the insertion point

Instantons on S^4 can be straightforwardly constructed by stereographic projection from $\mathbb{R}^4$

The solutions exhibit some amusing properties:

$$F \wedge F = \frac{8\rho^4 \ \sum_{i=1}^{3} T_i^2}{\Big(1+\rho^2+(1-\rho^2)\cos\theta^1\Big)^4}\sqrt{\gamma}\; d^4\theta$$

with $[T_i, T_j] = 2i\epsilon_{ijk}T_k$ an $N \times N$ representation of $\mathfrak{su}(2)$

When $\rho = 1$ this reduces to the $\mathrm{SO}(5)$-symmetric

$$F \wedge F = \frac{1}{2}\sum_{i=1}^{3} T_i^2 \sqrt{\gamma}\; d^4\theta$$

When the T_i are irreps of $\mathfrak{su}(2)$ then

$$\sum_{i=1}^{3} T_i^2 = (N^2 - 1)\ \mathbb{1}_{N \times N}$$

and $F \wedge F$ is gauge invariant without the trace

By further considering (for generic ρ)

$$I = \frac{1}{8\pi^2} \mathrm{Tr} \int F \wedge F = \frac{N(N^2-1)}{6}$$

Supersymmetry

Are these solutions supersymmetric?

The supervariation of a fermion in the background of the Yang monopole is

$$\delta\psi = \frac{1}{2}\Gamma^{\mu\nu}F_{\mu\nu}\Gamma_5\varepsilon$$

The ε are 32-component spinors which also satisfy

$$\Gamma_{012345}\varepsilon = \varepsilon$$

Using that $F = \pm \star_{S^4} F$ one can satisfy $\delta\psi = 0$ if

$$\Big(\frac{x^\mu}{|x|}\Gamma_\mu\Gamma_5 \pm i\Big)\varepsilon = 0$$

This cannot hold for all x^μ and supersymmetry is broken.

Exception: Singular instantons

⇒ The $\rho = 0$ instanton operators are $\frac{1}{2}$-BPS

CS terms

In $\mathcal{N}=1$ theories we can also add Chern-Simons terms

$$S_{CS}=\frac{k}{24\pi^2}\mathrm{Tr}\int(F\wedge F\wedge A+\frac{i}{2}F\wedge A\wedge A\wedge A-\frac{1}{10}A\wedge A\wedge A\wedge A\wedge A)$$

In the presence of such a term the instanton operators are not always gauge invariant:

$$\delta S_{CS}=\frac{k}{8\pi^2}\mathrm{Tr}\int F\wedge F\wedge\delta A$$

and by considering $\delta A=D\omega$ with $\omega=0$ at ∞ one finds

$$\delta S_{CS}=-\frac{k}{8\pi^2}\mathrm{Tr}\left[\omega(x)\oint_{S^4_x}F\wedge F\right]$$

Inserting this into a correlator

$$\begin{aligned}
&\delta\langle\mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle = \\
&= -\int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4_x} F\wedge F=n} [DXDADD\psi]\;\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\delta S_{CS}e^{-S_E} \\
&= \frac{k}{8\pi^2}\mathrm{Tr}\left[\omega(x)\oint_{S^4_x} F\wedge F\right]\langle\mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle
\end{aligned}$$

Introducing a basis t_a of full gauge group Lie algebra with metric $\kappa_{ab} = \mathrm{Tr}(t_a t_b)$ and symmetric tensor

$$d_{abc} = \mathrm{Tr}(t_{(a}t_{b)}t_c)$$

we can write

$$\delta\mathcal{I}_n = k d_{abc} Q_I^{ab}\omega^c\mathcal{I}_n$$

One needs to understand the properties of

$$Q_I = \frac{1}{8\pi^2} \oint_{S_x^4} F \wedge F$$

Q: Could this play a similar role to $Q_M = \frac{1}{2\pi} \oint_{S^2} F$ in GNO?

For the single instanton irreducible case

$$Q_I = \frac{1}{6} \sum_{i=1}^{3} T_i^2 = \frac{1}{6}(N^2 - 1)\ \mathbb{1}_{N\times N}$$

and Q_I is gauge invariant.

Applications

1. Use instanton operators to relate 6D (2,0) to 5D $\mathcal{N} = 2$ correlators by compactifying on S^1

Implement this by viewing S^1 as orbifold $\mathbb{R}/\Gamma$ and

$$\Gamma : (x, y) \mapsto (x, y + 2\pi R n) \text{ with } n \in \mathbb{Z}$$

For an operator on $\mathbb{R}^5 \times S^1$ write

$$\mathcal{O}(x, y) := \sum_{n \in \mathbb{Z}} \hat{\mathcal{O}}(x, y + 2\pi R n) = \sum_{m \in \mathbb{Z}} e^{imy/R} \mathcal{O}_m(x)$$

where $\mathcal{O}_m$ are Fourier modes

Starting from the two-point function $\langle \mathcal{O}_1(x_1, y_1)\mathcal{O}_2(x_2, y_2)\rangle$, using these facts and the identification $R = g^2/4\pi^2$ one arrives at the following:

- ◇ For the zero modes (pertubative)

$$\langle \mathcal{O}_0(x_1)\mathcal{O}_0(x_2)\rangle = -\frac{c_{12}\pi^{\frac{3}{2}}}{g^2}\frac{\Gamma(\frac{\Delta_1+\Delta_2-1}{2})}{\Gamma(\frac{\Delta_1+\Delta_2}{2})}\frac{1}{|x_{12}|^{\Delta_1+\Delta_2-1}}$$

- ◇ For the non-zero modes (non-pertubative)

$$\langle \mathcal{O}_n(x_1)\mathcal{O}_{-n}(x_2)\rangle$$

$$= -\frac{c_{12}\pi^{\frac{\Delta_1+\Delta_2}{2}}}{2|n|\Gamma(\frac{\Delta_1+\Delta_2}{2})}\left(\frac{2\pi|n|}{g^2|x_{12}|}\right)^{\frac{\Delta_1+\Delta_2}{2}} e^{-\frac{4\pi^2}{g^2}|n||x_{12}|}\left(1+\mathcal{O}\left(\frac{g^2}{|n||x_{12}|}\right)\right)$$

⇒ Lorentz symmetry enhancement via $\mathcal{O}_n(x) := \mathcal{I}_n(x)\mathcal{O}_0(x)$

Compatible with:

- ◇ Momentum conservation on S^1
- ◇ Lack of susy
- ◇ The characteristic e^{-S_n} dependence of the non-zero mode correlators with

$$S_n = \frac{4\pi^2}{g^2}|n||x_1 - x_2|$$

2. Use them to construct broken symmetry currents in 5D IR theories with $\mathcal{N} = 1, 2$ [Tachikawa, Zafrir, Yonekura]

⇒ Turn on fermion zero-modes for one-instanton $\mathrm{SU}(2)$ configuration

$\exists$ fermion zero modes from gluinos plus possible matter zero modes

⇒ in $\mathcal{N} = 1$ leads to a 2^4-dim multiplet of operators in spinor rep of $\mathrm{SO}(2N_f)$

⇒ in $\mathcal{N} = 2$ leads to a 2^8-dim multiplet: KK modes of 6d E-M supermultiplet

3. They can be fully supersymmetrised for an R-twisted version of 5D theory [Rodríguez-Gómez & Schmude]

4. Use them to probe Higgs branch of $\mathcal{N} = 1$ SU(2) theory at infinite coupling [Cremonesi-Ferlito-Hanany-Mekareeya]

$\Rightarrow$ Employs Hilbert series and leads to modification of chiral operator relations

Summary

- Introduced instanton operators in 5D gauge theories
- Looked at some properties
- These are non-BPS in $\mathbb{R}^5$, except for $\rho = 0$
- Various applications including symmetry enhancement