String (non) geometry from F-theory

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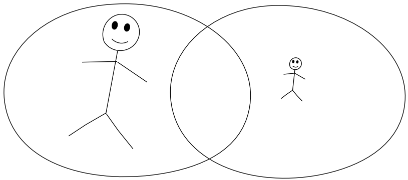
Motivation

- Generalized/doubled geometry program;
- Uplift of duality orbits in lower dimensional gauged supergravities (cosmological billiards, exotic branes, non-geometric fluxes, . . .);
- Non-commutativity/non-associativity ?
- Non-geometric black hole microstates;
- Ingredients for constructing dS vacua.



Monodromy and mapping tori

Consider a T^2 fibered over a base $\mathcal{B}$.

- The simplest case if $\mathcal{B} = S^1$. The total space of the fibration (known as the mapping torus) can be written as

$$\mathcal{N}_\phi = \frac{T^2 \times [0,1]}{(x,0) \sim (\phi(x),1)} \, ,$$

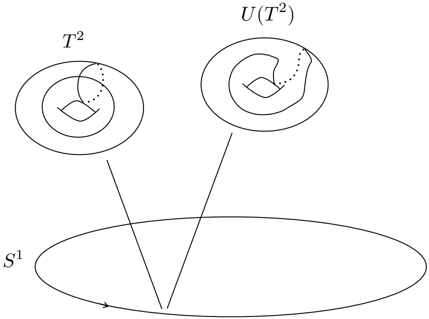
  where the monodromy ϕ is an element of the mapping class group of T^2, $MCG(T^2) = SL(2,\mathbb{Z})$.
- All such ϕ can be written as a product of Dehn twists:

$$MCG(T^2) \approx \langle U, V | UVU = VUV, (UV)^6 = 1 \rangle \, .$$

Monodromy and mapping tori

We can take Dehn twists around the homology basis:

$$U = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \qquad V = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$



Monodromy and mapping tori

The geometry of $\mathcal{N}_\phi$ is closely related to the monodromy ϕ [Thurston]

- ϕ parabolic (reducible) $\rightarrow$ Nil-geometry
- ϕ elliptic (periodic) $\rightarrow$ Euclidean-geometry
- ϕ hyperbolic (Anosov*)$\rightarrow$ Sol-geometry

We note that this trichotomy of torus diffeomorphisms can be generalized to arbitrary genus.

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*This contains Arnold's cat map on the torus.

T-folds

String theory on T^2 has a T-duality group

$$O(2,2,\mathbb{Z}) = SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\rho \times \mathbb{Z}_2^2 \, ,$$

It is tempting to consider a generalization of mapping tori where the fibers are glued with a monodromy in $SL(2,\mathbb{Z})_\rho$ (recall that $\rho = B + iV$). This can be seen as an element of the mapping class group of an auxiliary T^2_ρ:

Duality twist = Dehn twist

- The resulting space is a non-geometric T-fold [Hull];
- Such spaces can arise from globally obstructed T-dualities.

[See talk by C. Hull]

Metrics

It is easy to construct a local metric for such spaces, with arbitrary monodromy in $SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\rho$:

$$ds^2 = d\theta^2 + H(\theta)_{ab} dx^a dx^b \,,$$

where

$$H(\theta) = \frac{\rho_2(\theta)}{\tau_2(\theta)} \begin{pmatrix} 1 & \tau_1(\theta) \\ \tau_1(\theta) & |\tau(\theta)|^2 \end{pmatrix} \,.$$

Given a matrix $M \in SL(2,\mathbb{Z})$ we set

$$\tau(\theta) = M(\theta)[\tau(0)]$$

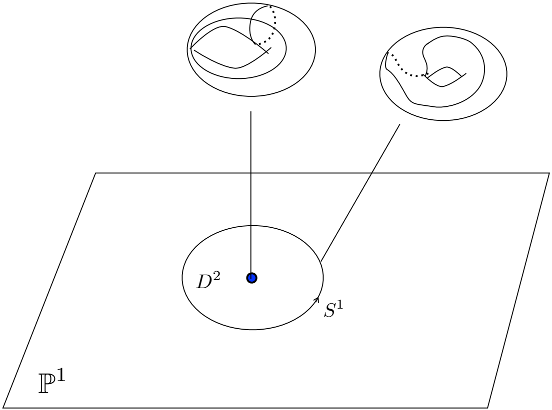
where the action is via Möbius transformation and

$$M(\theta) = \exp\left[\log M \cdot \frac{\theta}{2\pi}\right] ,$$

and analogously for $\rho(\theta)$.

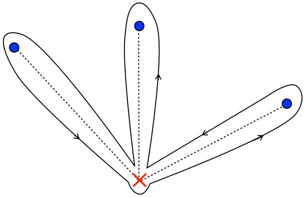
Fibrations on $\mathbb{P}^1$

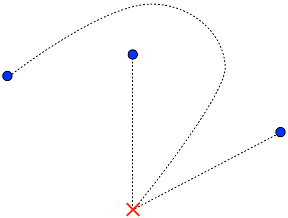
We consider the case of a two dimensional base, say $\mathcal{B} = \mathbb{P}^1$.



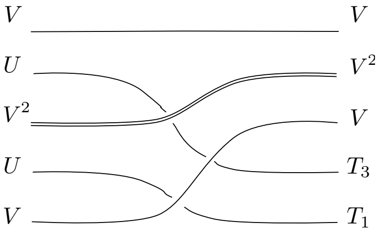
Fibrations on $\mathbb{P}^1$

We can again classify all local solutions on small disks by considering the monodromy on $\partial D^2 = S^1$. Multiple degenerations are in correspondance with factorisations of the total monodromy M, up to local and global conjugations by duality elements.





Braid action



The local freedom is just a braid action on the factorisation of M in terms of Dehn twists.

This agrees with a prescription given by “moving branch cuts” and the familiar ABC decomposition used in F-theory.

Local solutions and T-defects

A useful tool to study the geometry of torus fibrations is the semi-flat approximation [SYZ]: preserve $U(1)^2$ isometries of T^2.

- Here we also consider a varying Kähler modulus and we allow degenerations with monodromy in $SL(2,\mathbb{Z})_\rho$: [Hellerman,McGreevy,Willliams]

$$
\begin{aligned}
ds^2 &= \eta_{\mu\nu} dx^\mu dx^\nu + e^{2\varphi_1} \tau_2 \rho_2 dz d\bar{z} + H_{ab}(z) dx^a dx^b \,, \\
B_2 &= \rho_1 dx^8 \wedge dx^9 \,, \qquad e^{2\Phi} = \rho_2 \,, \quad a, b = 8, 9 \,.
\end{aligned}
$$

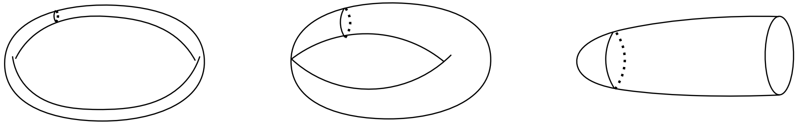
- Eom fix τ, ρ, ϕ to be meromorphic in $\mathcal{B}$: “stringy cosmic fivebranes” [Greene, Shapere, Vafa, Yau].
- By solving a Riemann-Hilbert problem, we obtain local solutions for the fields with arbitrary T-duality monodromy.

Example I

The simplest example is a monodromy $\tau \to \tau + 1$, namely $\tau(z) = \frac{i}{2\pi} \log\left(\frac{\mu}{z}\right)$. The solution is:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{2\pi} \log\left(\frac{\mu}{r}\right) \left[d\theta^2 + r^2 dr^2 + (dx^8)^2\right] + \frac{2\pi}{\log\left(\frac{\mu}{r}\right)} \left(dx^9 + \frac{\theta}{2\pi} dx^8\right)^2 .$$

As it is well known, this is a semi-flat approximation of a KK monopole. The full metric arises by re-summing exponential corrections near the degeneration [Ooguri, Vafa].



Example II

- The mirror $\rho \to \rho + 1$ is an NS5 brane smeared on the torus. Corrections to the semi-flat approximation can be derived in gauge theory [Diaconescu, Seiberg], [Becker, Sethi].
- If we consider a monodromy $(\tau \to \tau + 1, \rho \to \rho + 1)$, we obtain a smeared NS5 brane on top of the Taub-NUT space (by harmonic superposition).

Example III

If we consider a monodromy $\rho \to \frac{\rho}{\rho+1}$, namely a (0,1) brane, the solution is globally non-geometric: "exotic brane"

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h(r)(dr^2 + r^2 d\theta^2) + \frac{h(r)}{h(r)^2 + \theta^2} ds^2_{89} \,,$$

$$B_2 = \frac{\theta}{h(r)^2 + \theta^2} dx^8 \wedge dx^9 \,, \qquad e^{2\Phi} = \frac{h(r)}{h(r)^2 + \theta^2} \,.$$

where $h(r) = \log\left(\frac{\mu}{r}\right)$.

- This can be obtained by T-dualizing a smeared KKM.
  [de Boer, Shigemori], [Hassler, Lüst]

Example IV

We consider an elliptic monodromy $(\tau \to -1/\tau, \rho \to -1/\rho)$. The torus metric, dilaton and B-field are:

$$G_{11} = \tau_1^2 + \tau_2^2 = \frac{r + \mu^2 - 2r\mu\cos(\theta + 2\sigma)}{\left[r + \mu + 2\sqrt{r\mu}\cos\left(\theta/2 + \sigma\right)\right]^2},$$

$$G_{12} = \tau_1 = \frac{\sin(\theta/2 + \sigma)}{\cos(\theta/2 + \sigma) + \cosh\left[\frac{1}{2}\log\left(\frac{r}{\mu}\right)\right]},$$

$$e^{2\Phi} = \rho_2 = -\frac{\sinh\left[\frac{1}{2}\log\left(\frac{r}{\mu}\right)\right]}{\cos\left(\frac{\theta}{2} + \sigma\right) + \cosh\left[\frac{1}{2}\log\left(\frac{r}{\mu}\right)\right]},$$

$$B = \rho_1 = \tau_1 = G_{12}\,.$$

- The asymmetric orbifold at the fixed point was described by [Condeescu, Florakis, Kounnas, Lüst].
- Not T-dual to a geometric solution.

Heterotic T-folds

We can describe the previous situation with two independent elliptic fibrations. We then have the Kodaira classification of singular fibers. Interestingly, only a class of monodromies arise in this case: local solutions with hyperbolic (Anosov) monodromies do not arise [Matsumoto, Montesinos-Amilibia].
In the Heterotic theory with unbroken gauge group, the auxiliary fibration is physical: Heterotic/F-theory duality map both τ and ρ fibrations to a geometric CY compactification of F-theory [McOrist, Morrison, Sethi].

Elliptic fibrations (remainder)

Recall that the fibers $T^2_{\tau,\rho}$ can be described by a pair of Weierstrass equations

$$y^2 = x^3 + f_4 x + g_6 \, , \quad \tilde{y}^2 = \tilde{x}^3 + \tilde{f}_4 \tilde{x} + \tilde{g}_6$$

and (τ, ρ) are determined by the Klein's j-invariant:

$$j(\tau) \propto \frac{f^3}{4f^3 + 27g^2} \, , \quad j(\rho) \propto \frac{\tilde{f}^3}{4\tilde{f}^3 + 27\tilde{g}^2} \, .$$

Elliptic fibrations (remainder)

The F-theory dual is an elliptically fibered K3 surface:

$$y^2 = x^3 + a\,z^4 x + x^5 + c\,z^6 + z^7\,.$$

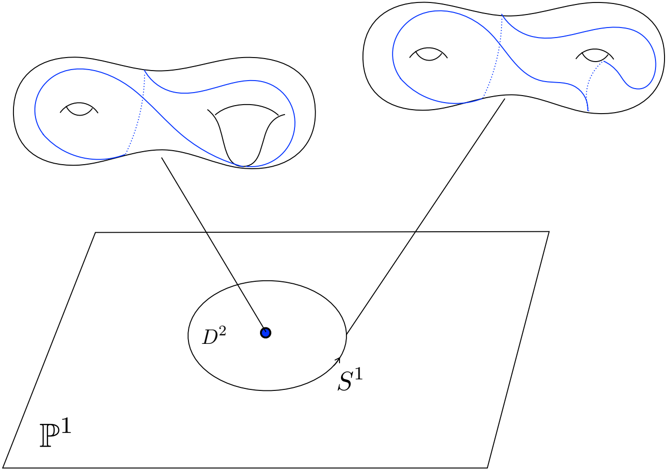
The duality is given by [Cardoso, Curio, Lüst, Mohaupt]

$$j(\tau)j(\rho) = -1728^2\,\frac{a^3}{27}\,,$$
$$(j(\tau) - 1728)(j(\rho) - 1728) = 1728^2\,\frac{c^2}{4}\,.$$

Heterotic T-folds

We are not restricted to the $E_8 \times E_8$ group. A similar picture exists if a single Wilson line is turned on [Clingher,Doran], [Malmendier,Morrison], [Jockers,Gu]. We can understand the non-geometric twists of $O(2,3,\mathbb{Z})$ as elements of the mapping class group of a genus-2 surface Σ.

Heterotic T-folds



Heterotic T-folds

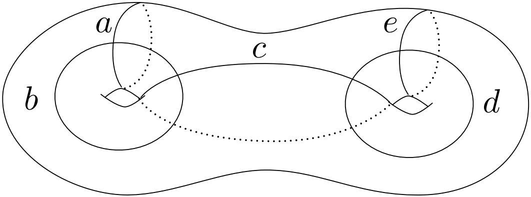
The mapping class group is now generated by 5 twists (A, B, C, D, E) with:

Disjointness: $[A, C] = [A, D] = [A, E] = \cdots = 0$

Braidness: $ABA = BAB\,, BCB = CBC\,, \ldots$

3-chain: $(ABC)^4 = E^2$, Hyperellitic: $[H, A] = 0\,, H^2 = 1\,.$

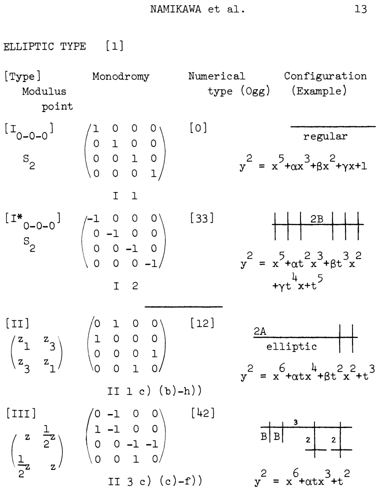
A simple set is given by Dehn twists around the following cycles:



Namikawa-Ueno classification

We can again classify surface diffeomorphisms, and obtain explicit factorisations of the charges.

A subset arises as a monodromy of degenerating family of curves. These can be classified with algebraic geometry tools, generalizing the Kodaira analysis [Namikawa, Ueno].


NAMIKAWA et al. 13

ELLIPTIC TYPE [1]

[Type] Modulus point	Monodromy	Numerical type (Ogg)	Configuration (Example)
$[I_{0-0-0}]$ S_2	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ I 1	[0]	regular; $y^2 = x^5+\alpha x^3+\beta x^2+\gamma x+1$
$[I^*_{0-0-0}]$ S_2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ I 2	[33]	2B; $y^2 = x^5+\alpha t^2x^3+\beta t^3x^2 +\gamma t^4x+t^5$
[II] $\begin{pmatrix} z_1 & z_3 \\ z_3 & z_1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ II 1 c) (b)-h))	[12]	2A elliptic; $y^2 = x^6+\alpha tx^4+\beta t^2x^2+t^3$
[III] $\begin{pmatrix} z & \frac{1}{2}z \\ \frac{1}{2}z & z \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ II 3 c) (c)-f))	[42]	B B 3 2 2; $y^2 = x^6+\alpha tx^3+t^2$

Genus-2 curves

A genus-2 curve is described by a sextic

$$y^2 = \sum_{i=0}^{6} c_i x^i = \prod_{i=1}^{6} (x - \theta_i)\,.$$

From the coefficients, we can obtain the dual K3 surface

$$y^2 = x^3 + a u^4 x + b u^6 + c u^3 x + d u^5 + u^7\,,$$

where (x, y) are coordinate of the fiber and u a coordinate on a $\mathbb{P}^1$ base.

Genus-2 curves

We need to introduce the analogous of f and g for genus one curves. These are the Igusa-Clebsh invariants:

$$I_2 = c_6^2 \sum (12)^2(34)^2(56)^2$$
$$I_4 = c_6^4 \sum (12)^2(23)^2(31)^2(45)^2(56)^2(64)^2$$
$$I_6 = c_6^6 \sum (12)^2(23)^2(31)^2(45)^2(56)^2(64)^2(14)^2(25)^2(36)^2$$
$$I_{10} = c_6^{10} \sum (ij)^2 \,,$$

where $(ij) = (\theta_i - \theta_j)$. The F-theory coefficients then read:

$$a = -3I_4 \,,\ b = 2(I_2 I_4 - 3I_6) \,,\ c = -1944 I_{10} \,,\ d = 486 I_2 I_{10} \,.$$

Example I: $[I_N - I_P - 0]$ singularity

We consider a local model from the Namikawa-Ueno list:

$$y^2 = (x^2 - z^N)((x-\alpha)^2 - z^M)(x-\beta)(x-1)\,,$$

with monodromy

$$\tau \to \tau + N\,, \quad \rho \to \rho + M\,.$$

This describes a stack of M NS5 branes on a $\mathbb{C}^2/\mathbb{Z}_N$ singularity in the $E_8 \times E_8$ theory. By duality, we can read the dynamics from F-theory. We obtain a 6d (1,0) theory that has been much studied recently.
[Aspinwall, Morrison], [Heckman, Morrison, Vafa, ...]

Example II: $[III - III]$ singularity

We consider a local model

$$y^2 = x(x^2 - z)(x - 1)((x - 1)^2 - z)\,,$$

with non-geometric monodromy

$$\tau \to \frac{\rho}{\beta^2 - \rho\tau}\,, \quad \rho \to \frac{\tau}{\beta^2 - \rho\tau}\,, \quad \beta \to -\frac{\beta}{\beta^2 - \rho\tau}\,.$$

If $\beta = 0$ this describes the double elliptic example discussed before. From the duality we map this to a local F-theory model:

$$y^2 = x^3 + au^4x + du^5 + u^7\,, \quad \text{with}$$

$$a = -36(z + 3)z^2(z - 1)^2\,, \quad d = 46656z^6(z - 1)^8\,.$$

From this we can again obtain the corresponding 6d theory.

Conclusions

A large class of Heterotic non-geometric defects can be studied by constructing 6d (1,0) theories from Heterotic/F-theory duality. The backreaction of such defects should be approximated by the class of local semi-flat (non) geometries obtained by solving the corresponding Cauchy-Riemann equations.

Many open questions:

- Geometric framework for such solutions?
- Full Heterotic duality group, and beyond T^2.
- Application to the U-duality group in IIB. [Candelas et al.][Martucci, Morales, Pacifici]

Thank you!